**2.6 Inverse of a Function – Lesson**
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**Inverse of a function:**

· The inverse of a function *f* is denoted as $f^{-1}$
· The function and its inverse have the property that if f(*a*) = *b*, then $f^{-1}(b) = a$
· So if $f(5) = 13$, then $f^{-1}(13) = 5$
· More simply put: The inverse of a function has all the same points as the original function, except that the *x*'s and *y*'s have been reversed.

It is important to note that $f^{-1}(x)$ is read as "the inverse of *f* at *x*". The -1 does not behave like an exponent.

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

**To draw an inverse, all you need to do is swap the x and y coordinates of each point.**

__Finding Inverses Numerically__

**Example 1:** The table shows ordered pairs belonging to a function $f(x)$. Determine $f^{-1}(x)$, then state the domain and range of $f(x)$ and its inverse.

| $f(x)$ | $f^{-1}(x)$ |
|---|---|
| (-5, 0) | |
| (-4, 2) | |
| (-3, 5) | |
| (-2, 6) | |
| (0, 7) | |

**Example 2:**

**a)** Graph the function $f(x) = x^2$ and its inverse $f^{-1}(x)$.

| $f(x)$ | $f^{-1}(x)$ |
|---|---|
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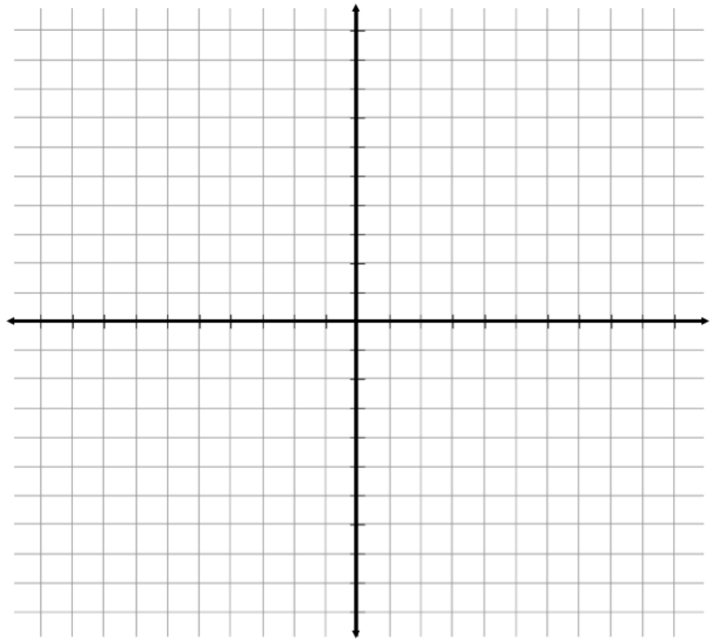

**b)** State the domain and range of both functions

**Example 3:** Sketch the graph of $g(x) = -2\sqrt{\left(-\frac{1}{2}x\right)} + 3$ , then graph $g^{-1}(x)$.

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

| | |
|---|---|
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| $x$ | $y$ |
|---|---|
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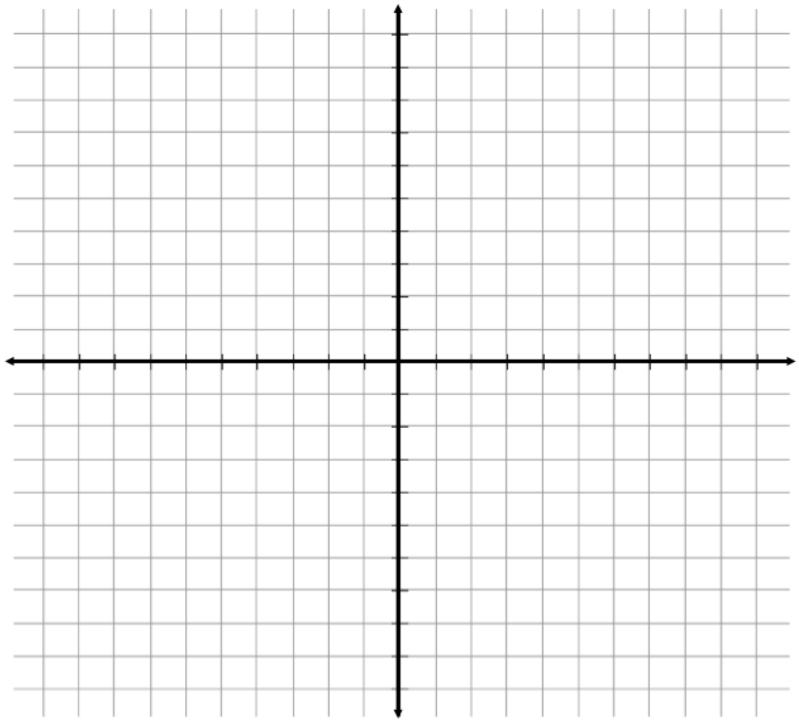
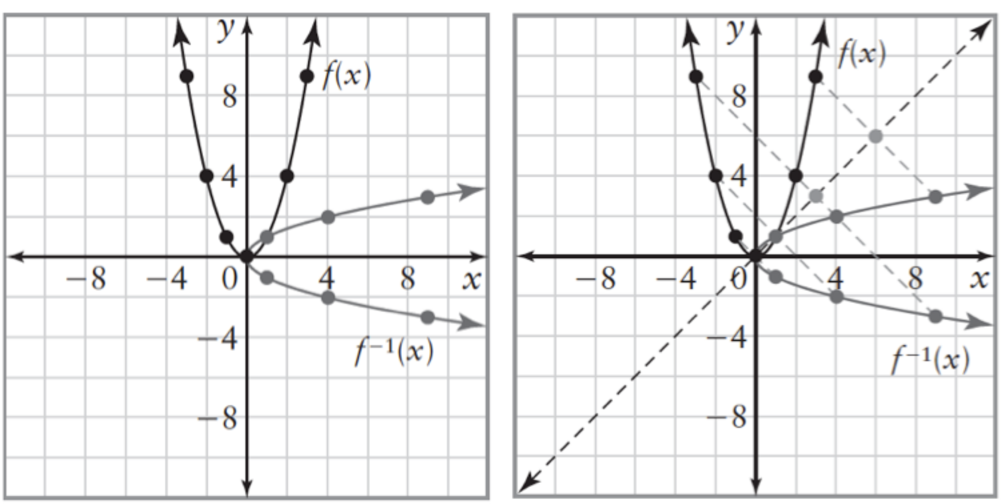
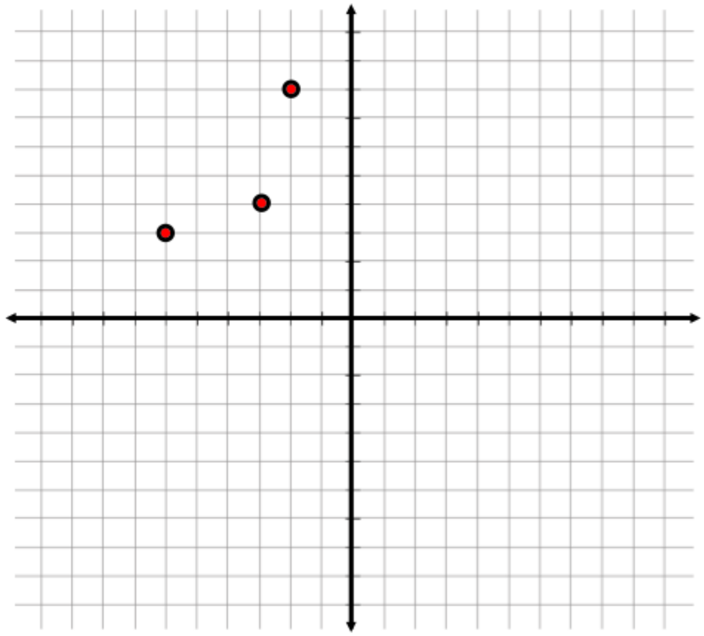


## Finding Inverses by Graphing

The graph of $f^{-1}(x)$ is the graph of $f(x)$ reflected in the line $y = x$. This is true for all functions and their inverses. If you find the midpoint of each pair of points from example 2 and connect them you can prove this theorem.



**Example 4:** Sketch the inverse of $f(x)$



**Finding Inverses Algebraically**

**Algebraic Method for finding the inverse:**

1. Replace $f(x)$ with "$y$"
2. Switch the $x$ and $y$ variables
3. Isolate for $y$
4. replace $y$ with $f^{-1}(x)$

**a)** $g(x) = \frac{3x}{4}$

**b)** $h(x) = 4x + 3$

**c)** $f(x) = x^2 - 1$

**d)** $h(x) = \frac{4x+3}{5}$

**e)** $f(x) = 2x^2 + 16x + 29$

**Note:** for algebraic inverses of quadratic functions, before interchanging *x* and *y*'s you must re-write in vertex form.

**f)** $r(x) = \sqrt{x} + 2$