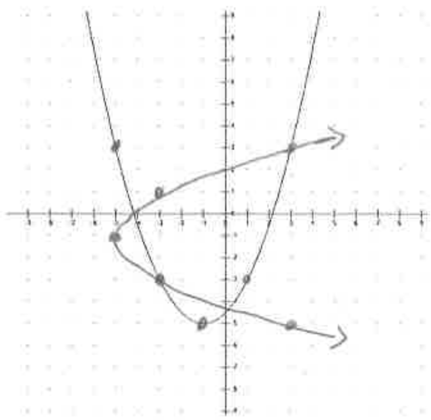
SOLUTIONS

2.7 Inverse of a Function – Worksheet

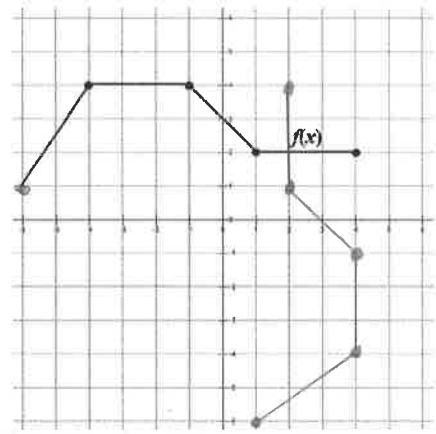
MCR3U
Jensen

1) Sketch the graph of the inverse of each function. Is the inverse of $f(x)$ a function? Explain.

a)



b)



2) Determine the equation of the inverse of each function.

) $f(x) = 2x$

$y = 2x$

$x = 2y$

$\frac{x}{2} = y$

$f^{-1}(x) = \frac{x}{2}$

b) $f(x) = 6x - 5$

$y = 6x - 5$

$x = 6y - 5$

$x + 5 = 6y$

$\frac{x+5}{6} = y$

$f^{-1}(x) = \frac{x+5}{6}$

c) $f(x) = \frac{2x+4}{5}$

$y = \frac{2x+4}{5}$

$x = \frac{2y+4}{5}$

$5x = 2y + 4$

$5x - 4 = 2y$

$\frac{5x-4}{2} = y$

$f^{-1}(x) = \frac{5x-4}{2}$

3) Determine the equation of the inverse of each function

a) $f(x) = x^2 + 6$

$y = x^2 + 6$

$x = y^2 + 6$

$x - 6 = y^2$

$\pm\sqrt{x-6} = y$

$f^{-1}(x) = \pm\sqrt{x-6}$

b) $f(x) = (x+8)^2$

$y = (x+8)^2$

$x = (y+8)^2$

$\pm\sqrt{x} = y + 8$

$\pm\sqrt{x} - 8 = y$

$f^{-1}(x) = \pm\sqrt{x} - 8$

4) For each quadratic function, complete the square and then determine the equation of the inverse.

a) $f(x) = x^2 + 6x + 15$

$f(x) = (x^2+6x+9-9)+15$

$f(x) = (x+3)^2+6$

$x = (y+3)^2+6$

$x-6 = (y+3)^2$

$\pm\sqrt{x-6} - 3 = y$

$f^{-1}(x) = \pm\sqrt{x-6} - 3$

b) $f(x) = 2x^2 + 24x - 3$

$f(x) = 2(x^2+12x+36)-72-3$

$f(x) = 2(x+6)^2-75$

$x = 2(y+6)^2-75$

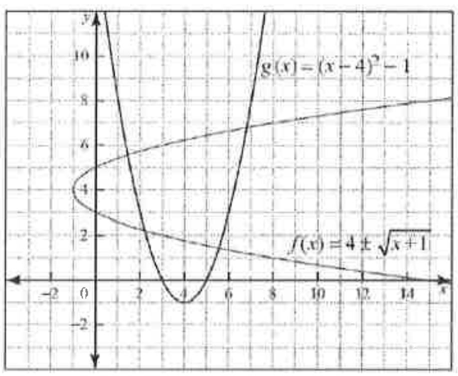
$\frac{x+75}{2} = (y+6)^2$

$\pm\sqrt{\frac{x+75}{2}} - 6 = y$

$f^{-1}(x) = \pm\sqrt{\frac{x+75}{2}} - 6$

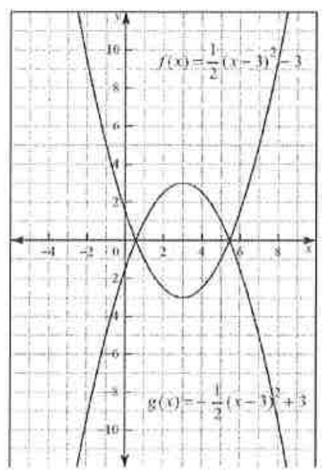
5) Determine if the two relations shown are inverses of each other. Justify your conclusion.

a)



yes

b)



NO

6) For the function $f(x) = -5x + 6$

a) determine $f^{-1}(x)$

$x = -5y+6$

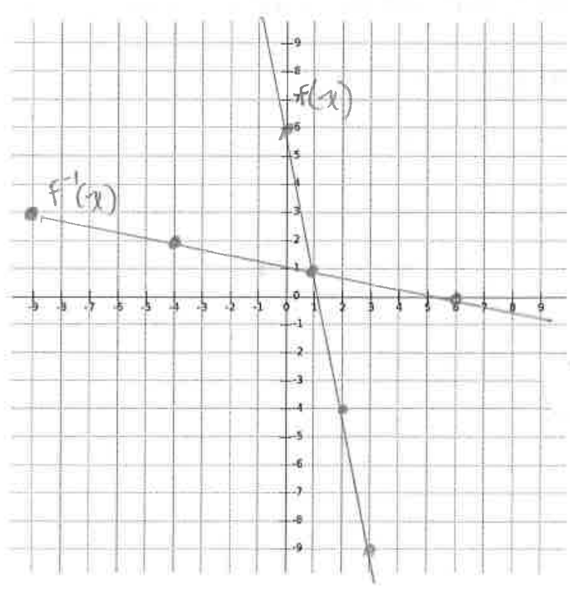
$x-6 = -5y$

$\frac{x-6}{-5} = y$

$\frac{-x+6}{5} = y$

$f^{-1}(x) = \frac{-x+6}{5}$

b) Graph $f(x)$ and its inverse



7) Use transformations to graph the function $f(x) = 2(x-2)^2 + 1$. Find the inverse function $f^{-1}(x)$ and graph it by reflecting $f(x)$ over the line $y = x$ (switch x and y co-ordinates)

1) vertical stretch bafo 2 (2y)
2) shift right 2 units (x+2)
3) shift up 1 unit (y+1)

$y = x^2$

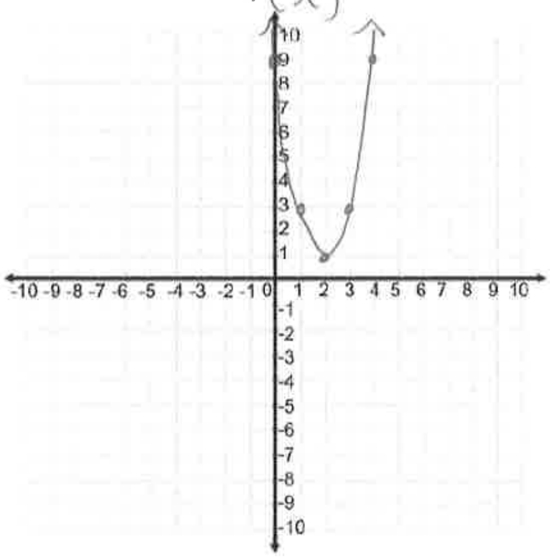
(-3,9)
(-2,4)
(-1,1)
(0,0)
(1,1)
(2,4)
(3,9)

$x+2$	$2y+1$
-1	19
0	9
1	3
2	1
3	3
4	9
5	19

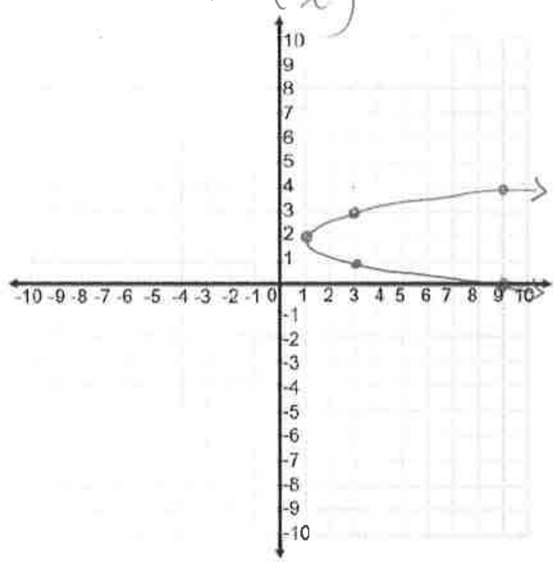
graph these

x	y
19	-1
9	0
3	1
1	2
3	3
9	4
19	5

$f(x)$



$f^{-1}(x)$



8) Determine the equation of the inverse for the given functions and state the domain and range.

a) $f(x) = \sqrt{x+3}$

D: $\{x \in \mathbb{R} \mid x \geq -3\}$
R: $\{y \in \mathbb{R} \mid y \geq 0\}$

$x = \sqrt{y+3}$

$x^2 = y+3$

$x^2 - 3 = y$

$f^{-1}(x) = x^2 - 3$

D: $\{x \in \mathbb{R} \mid x \geq 0\}$
R: $\{y \in \mathbb{R} \mid y \geq -3\}$

b) $f(x) = \frac{3}{x-2} + 2$

D: $\{x \in \mathbb{R} \mid x \neq 2\}$
R: $\{y \in \mathbb{R} \mid y \neq 2\}$

$x = \frac{3}{y-2} + 2$

$x - 2 = \frac{3}{y-2}$

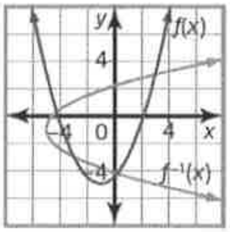
$y - 2 = \frac{3}{x-2}$

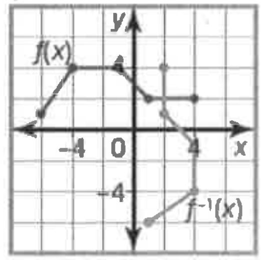
$y = \frac{3}{x-2} + 2$

$f^{-1}(x) = \frac{3}{x-2} + 2$

D: $\{x \in \mathbb{R} \mid x \neq 2\}$
R: $\{y \in \mathbb{R} \mid y \neq 2\}$

Answers

1) a) 

the inverse is NOT a function **b)** 

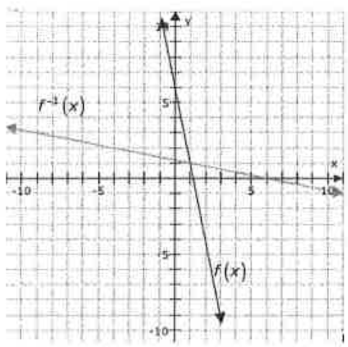
inverse is NOT a function

2) a) $f^{-1}(x) = \frac{x}{2}$ **b)** $f^{-1}(x) = \frac{x+5}{6}$ **c)** $f^{-1}(x) = \frac{5x-4}{2}$

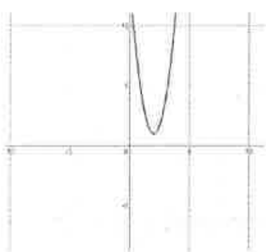
3) a) $f^{-1}(x) = \pm\sqrt{x-6}$ **b)** $f^{-1}(x) = \pm\sqrt{x} - 8$

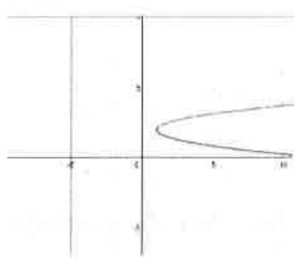
4) a) $f^{-1}(x) = \pm\sqrt{x-6} - 3$ **b)** $f^{-1}(x) = \pm\sqrt{\frac{x+75}{2}} - 6$

5) a) yes **b)** no

6) a) $f^{-1}(x) = \frac{-x+6}{5}$ **b)** 

7) $f^{-1}(x) = 2 \pm \sqrt{\frac{x-1}{3}}$





8) a) $f^{-1}(x) = x^2 - 3$; Domain for $f(x)$: $\{X \in \mathbb{R} | x \geq -3\}$, Range for $f(x)$: $\{Y \in \mathbb{R} | y \geq 0\}$
Domain for $f^{-1}(x)$: $\{X \in \mathbb{R} | x \geq 0\}$, Range for $f(x)$: $\{Y \in \mathbb{R} | y \geq -3\}$

b) $f^{-1}(x) = \frac{3}{x-2} + 2$; Domain for $f(x)$: $\{X \in \mathbb{R} | x \neq 2\}$, Range for $f(x)$: $\{Y \in \mathbb{R} | y \neq 2\}$
Domain for $f^{-1}(x)$: $\{X \in \mathbb{R} | x \neq 2\}$, Range for $f(x)$: $\{Y \in \mathbb{R} | y \neq 2\}$