L1 – Exponential Growth
MCR3U
Jensen

DO IT NOW!

A type of bacteria grows so that it triples in number every day. On the day we begin observations, the bacteria has a population of 100.

a) Make a table to show the population over 5 days.

Day	Population
0	100
1	300
2	900
3	2700
4	8100
5	24300

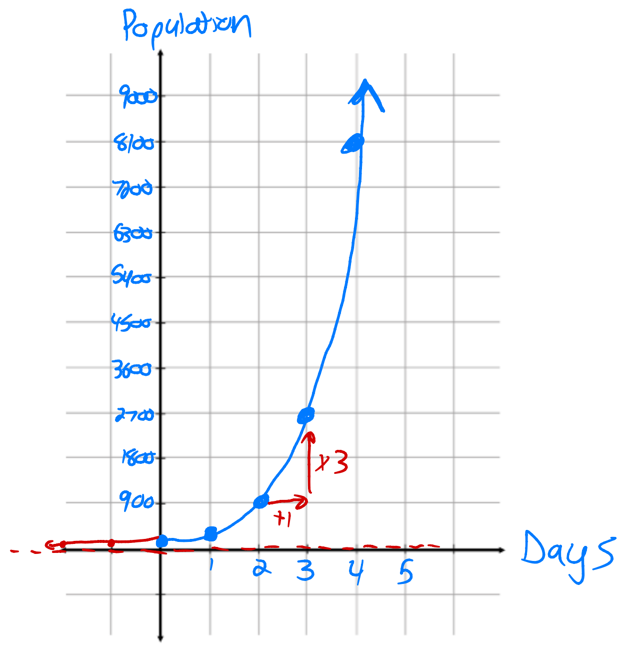
b) Calculate finite differences and indicate any patterns you see

Day	Population	1st Differences	2nd Differences
0	100	[redacted]	[redacted]
1	300	300 - 100 = 200	[redacted]
2	900	900 - 300 = 600	600 - 200 = 400
3	2700	2700 - 900 = 1800	1800 - 600 = 1200
4	8100	8100 - 2700 = 5400	5400 - 1800 = 3600
5	24300	24300 - 8100 = 16200	16200 - 5400 = 10800

×3 ×3 ×3 ×3 ×3 +200 +600 +1800 +5400 +16200 +400 +1200 +3600 +10800 ×3 ×3 ×3 ×3

The finite differences for an exponential relationship have a common RATIO.

c) Graph the relation



d) Write an equation to model this growth

Day (x)	Population (y)
0	$100 \times 3^0 = 100$
1	$100 \times 3^1 = 300$
2	$100 \times 3^2 = 900$
3	$100 \times 3^3 = 2700$
4	$100 \times 3^4 = 8100$
5	$100 \times 3^5 = 24300$

The relationship between days and population is easier to see when we look at the number of times the population has been tripled.

$$\boxed{y = 100(3)^x}$$

General Properties of Exponential Growth

Equation: $y = a(b)^x$

$a =$ initial amount

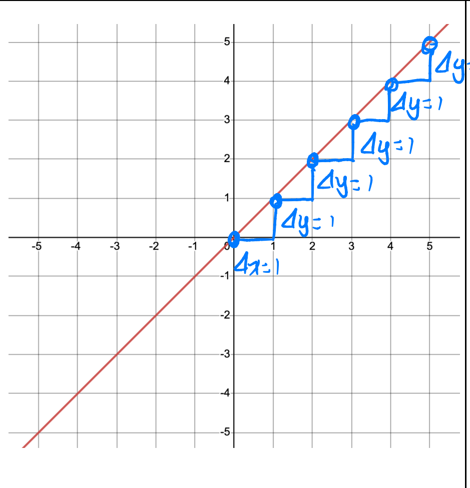
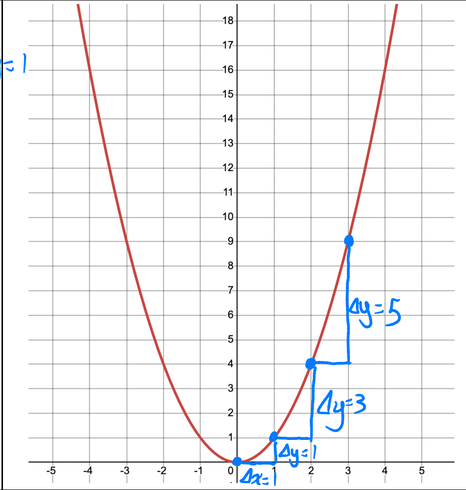
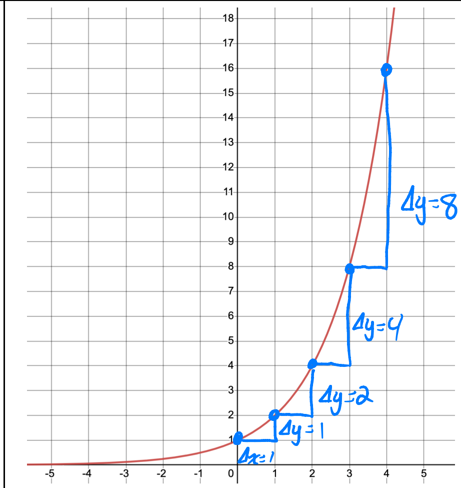
$b =$ growth factor

$y =$ future amount

$x =$ # of growth periods

To calculate x, use the equation: $x = \dfrac{\text{time}}{\text{time of 1 growth period}}$

Summary of Patterns in First Differences for Various Functions

<table>
<tr><th>Linear ($y = x$)</th><th>Quadratic ($y = x^2$)</th><th>Exponential ($y = 2^x$)</th></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Constant 1st differences.</td><td>The 1st differences are related by addition.

2nd differences would be constant</td><td>The finite differences are related by MULTIPLICATION.</td></tr>
</table>

Example 1: Your brother tells you a secret. You see no harm in telling two friends. After this second "passing" of the secret, 4 people now know the secret (your brother, you and two friends). If each of these friends now tells two new people, after the third "passing" of the secret, eight people will know. If this pattern of spreading the secret continues, how many people will know the secret after 10 such "passings"?

# of passings (x)	# of ppl that know the secret (y)
0	1
1	2
2	4
3	8

×2 ×2 ×2

$$y = a(b)^x$$

$$y = 1(2)^x$$

$$y = 1(2)^{10}$$

$$\boxed{y = 1024 \text{ people}}$$

Example 2:

a) An insect colony has a current population of 50 insects. Its population doubles every 3 days. What is the population after 12 days?

$y = ?$

$a = 50$

$b = 2$

$x = \frac{12}{3} = 4$

$y = a(b)^x$

$y = 50(2)^4$

$= 50(16)$

$= 800$

$\boxed{y = 800}$

b) The insect colony is actually full of giant, intelligent, mutant insects. They plot that they can overtake the Earth when their population has reached 1 billion. When will we meet our doom? (When does the population reach 1 billion?)

$y = 1\,000\,000\,000$

$a = 50$

$b = 2$

$x = \frac{t}{3}$

$y = a(b)^x$

$\frac{1\,000\,000\,000}{50} = \frac{\cancel{50}(2)^{t/3}}{\cancel{50}}$

$20\,000\,000 = 2^{t/3}$

$\cancel{3} \times \frac{t}{\cancel{3}} = \log_2(20\,000\,000) \times 3$

$\frac{\log(20\,000\,000)}{\log(2)}$

$\boxed{t \simeq 72.76 \text{ days}}$

Note: a logarithm is a function that solves for an unknown exponent.

Ex:

argument

$\log_3 9 = 2$ the exponent that goes on the base to get the argument.

base

because 2 is the exponent that goes on 3 to get 9.

If exponential growth is given as a percent you can use the equation: $y = a(1+r)^x$

$a =$ initial amount

$r =$ rate of increase (as a decimal)

$x =$ # of growth periods

Example 3: In 2005, there were only 285 Pittsburgh Penguins fans in Oakville. The number of Penguins fans increased by 5% per year after 2005 (this is when Crosby was drafted). How many Penguins fans are now in Oakville this year?

$y = ?$

$a = 285$

$r = 0.05$

$x = 19$

$y = a(1+r)^x$

$y = 285(1+0.05)^{19}$

$\boxed{y \simeq 720 \text{ fans}}$