L2 – Exponential Decay

MCR3U

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General Properties of Exponential Decay

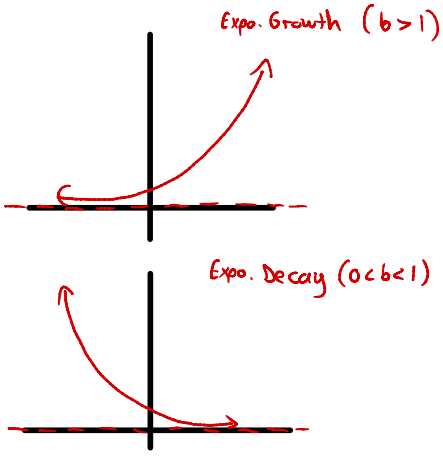
Equation: $y = a(b)^x$

$a =$ initial amount

$b =$ decay factor $(0<b<1)$

$y =$ future amount

$x =$ # of decay periods



To calculate x, use the equation: $x = \frac{\text{total time}}{\text{time of 1 decay period}}$

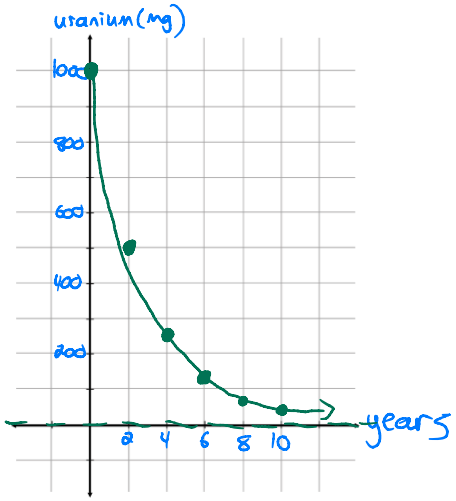
DO IT NOW!

Nuclear power plants use Uranium-239 as a power source. U-239 has a half-life of about 2 years.

a) Complete the chart for the amount of 1000mg sample that will be left after 10 years.

Years (x)	# of half-life periods	Amount of U-239 remaining (y)
0	0	1000
2	1	500
4	2	250
6	3	125
8	4	62.5
10	5	31.25

b) Graph the relation



c) Write an equation to model this growth

$y = a(b)^x \rightarrow x = \frac{t}{2}$

$y = 1000\left(\frac{1}{2}\right)^x$

$y = 1000\left(\frac{1}{2}\right)^{t/2}$

$A(t) = 1000\left(\frac{1}{2}\right)^{t/2}$

d) How much remains after 25 years?

$A(t) = 1000\left(\frac{1}{2}\right)^{t/2}$

$A(25) = 1000\left(\frac{1}{2}\right)^{25/2}$

$A(25) \simeq 0.173 \text{ mg}$

Example 1: Plutonium-239 has a half-life of 24 years. Find the amount of a 50mg sample left after 35 years.

$y = ?$

$a = 50$

$b = \frac{1}{2}$

$x = \frac{35}{24}$

$y = a(b)^x$

$y = 50\left(\frac{1}{2}\right)^{35/24}$

$y \approx 18.2\text{mg}$

If exponential decay is given as a percent use the equation: $y = a(1-r)^x$

$a =$ initial amount

$r =$ rate of decrease (use decimal value)

$x =$ # of decay periods $\left(\frac{\text{total time}}{\text{time of 1 decay period}}\right)$

Example 2:

You buy a new car for $24,000. The value of the car decreases by 16% every year. How much will the car be worth in 8 years?

$y = ?$

$a = 24000$

$r = 0.16$

$x = 8$

$y = a(1-r)^x$

$y = 24000(1-0.16)^8$

$y = 24000(0.84)^8$

$y = \$5949.02$

Example 3: An adult takes 400mg of Advil. Each hour, the amount of Advil in the adult's system decreases by about 29%. How much Advil will be left after 4 hours?

$a = 400$

$r = 0.29$

$x = \frac{4}{1} = 4$

$y = ?$

$$y = a(1-r)^x$$

$$y = 400(1-0.29)^4$$

$$y \approx 101.65 \text{ mg}$$

Example 4: U-239 has a half-life of about 2 years. If you start with a 1000 mg sample, how long will it take to decay to 10 mg?

$a = 1000$

$y = 10$

$b = \frac{1}{2}$

$x = \frac{t}{2}$

$$y = a(b)^x$$

$$10 = 1000\left(\frac{1}{2}\right)^{t/2}$$

$$0.01 = 0.5^{t/2}$$

$$\frac{t}{2} = \log_{0.5}(0.01)$$

$$t = 2\log_{0.5}(0.01)$$

$$t \approx 13.3 \text{ years}$$