3.2 Exponential Decay – Worksheet

MCR3U
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1) During medical treatment, the number of bacterial cells in a patient decreases by a factor of $\frac{1}{2}$ every day. A patient has 1 000 000 bacterial cells on Monday, the first day of treatment.

a) How many bacterial cells will remain on Thursday?

$A(t) = 1000000\left(\frac{1}{2}\right)^t$

$A(3) = 1000000\left(\frac{1}{2}\right)^3$

$A(3) = 125000$

b) How many bacterial cells will remain on Sunday?

$A(6) = 1000000\left(\frac{1}{2}\right)^6$

$A(6) = 15625$

c) On what day will the number of remaining bacterial cells be 1950?

$1950 = 1000000\left(\frac{1}{2}\right)^t$

$0.00195 = \left(\frac{1}{2}\right)^t$

$t = \log_{1/2}(0.00195)$

$t \simeq 9$ days (wednesday)

2) Tungsten-187 (W-187) is a radioactive isotope that has a half-life of 1 day. Suppose you start with a 100-mg sample…

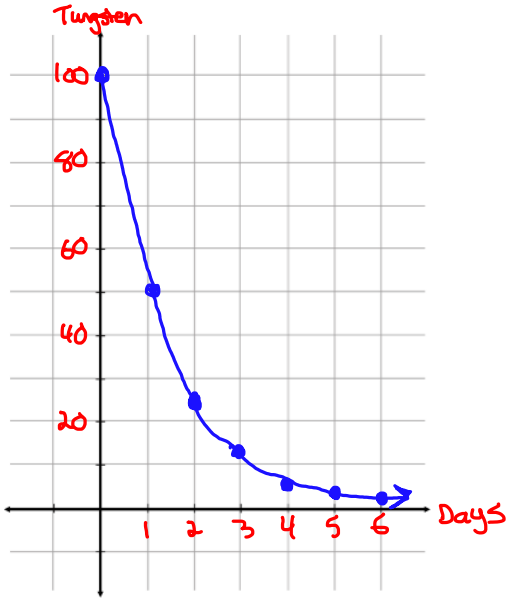
a) make a table of values that gives the amount of tungsten remaining at the end of each day for the next 4 days.

Time (days)	Amount of W-187 remaining
0	100
1	50
2	25
3	12.5
4	6.25

b) Write an equation in the form $f(x) = ab^x$ to relate the amount of W-187 remaining and time.

$$f(x) = 100\left(\frac{1}{2}\right)^x$$

c) Sketch a graph of the relation



d) How much W-187 will remain after 1 week?

$$f(7) = 100\left(\frac{1}{2}\right)^7 = 0.78125 \text{ mg}$$

e) How long will it take for the W-187 to decay to 5% of its initial amount?

$$5 = 100\left(\frac{1}{2}\right)^x$$

$$0.05 = \left(\frac{1}{2}\right)^x$$

$$x = \log_{1/2}(0.05)$$

$$x \simeq 4.32 \text{ days}$$

3) Shylo is very excited about her brand new car! Although she paid $20 000 for the car, its resale value will depreciate (decrease) by 30% of its current value every year.

decay factor of $1 - 0.3 = 0.7$

a) Write an equation relating the car's depreciated value, *v*, in dollars, to the time, *t*, in years since her purchase.

$$V(t) = 20000(0.7)^t$$

b) How much will Shylo's car be worth in…

i) 1 year?

$V(1) = 20000(0.7)^1$

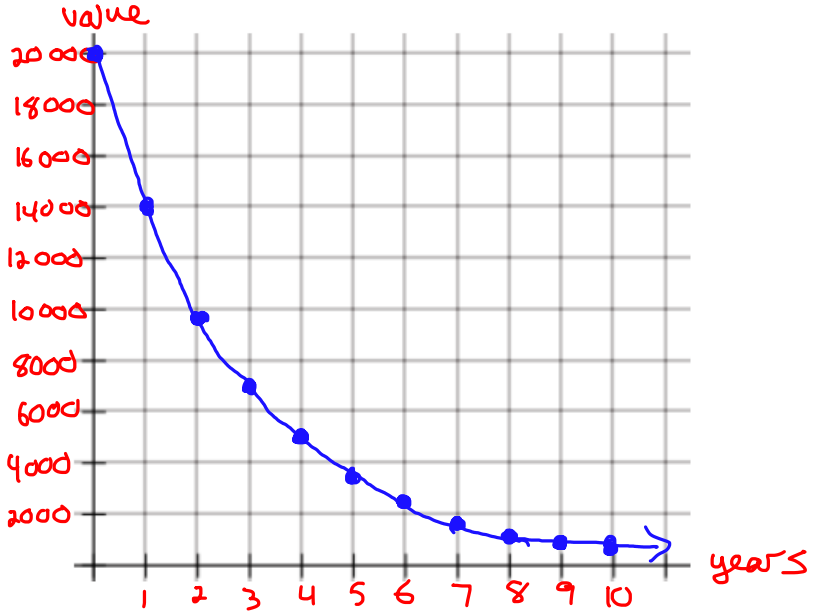
$= \$14000$

ii) 2 years?

$V(2) = 20000(0.7)^2$

$= \$9800$

c) Graph the depreciation function.



d) How long will it take for Shylo's car to depreciate to 10% of its original price?

$2000 = 20000(0.7)^t$

$0.1 = (0.7)^t$

$t = \log_{0.7}(0.1)$

$t \simeq 6.46$ years

4) An isotope of a radioactive substance has a half-life of 23 days. Suppose that you start with an 800-mg sample of the material.

a) Find an equation that models this data

$A(t) = 800\left(\frac{1}{2}\right)^{t/23}$

b) Use the equation to determine the amount of substance left after 100 days

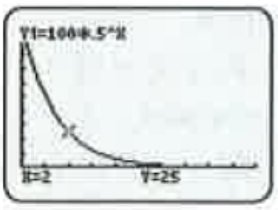
$A(100) = 800\left(\frac{1}{2}\right)^{100/23} \simeq 39.29$ mg

Answers

1) a) 125 000 **b)** 15 625 **c)** next Wednesday

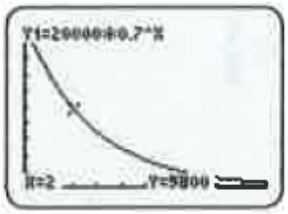
2) a)

Time (days)	Amount of W-187 remaining (mg)
0	100
1	50
2	25
3	12.5
4	6.25

b) $f(x) = 100\left(\frac{1}{2}\right)^x$ c) 

d) 0.781 25 mg

e) 4.3 days

3) a) $v(t) = 20000(0.7)^t$ **b) i)** 14 000 **ii)** 9800 **c)** 

d) about 6.5 years

4) a) $A = 800\left(\frac{1}{2}\right)^{\frac{t}{23}}$ **b)** 39.29 mg