

## L4 - Properties of Exponential Functions

MCR3U

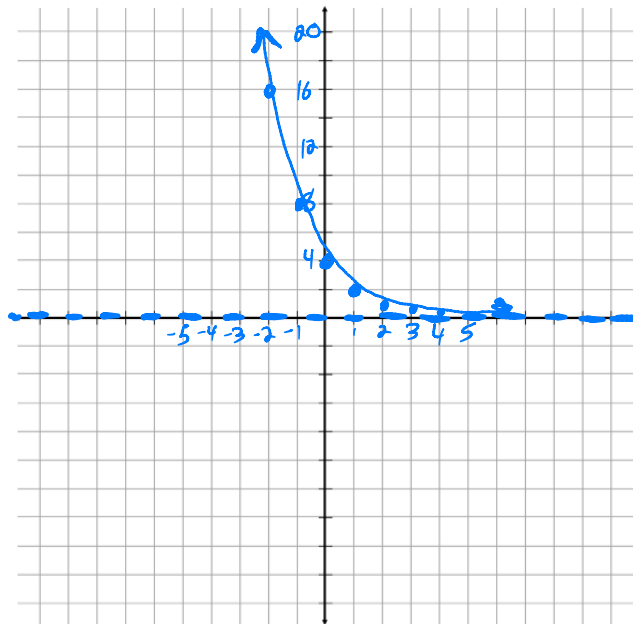
Jensen

### Example 1:

Graph each exponential function. Identify the domain, range, intercepts, intervals of increase/decrease, and the equation of any asymptotes.

a)  $y = 4\left(\frac{1}{2}\right)^x$

| x  | y   |
|----|-----|
| -3 | 32  |
| -2 | 16  |
| -1 | 8   |
| 0  | 4   |
| 1  | 2   |
| 2  | 1   |
| 3  | 0.5 |



Domain:  $\{x \in \mathbb{R}\}$

Range:  $\{y \in \mathbb{R} \mid y > 0\}$

x-int: NONE

y-int:  $(0, 4)$

intervals of increase/decrease:

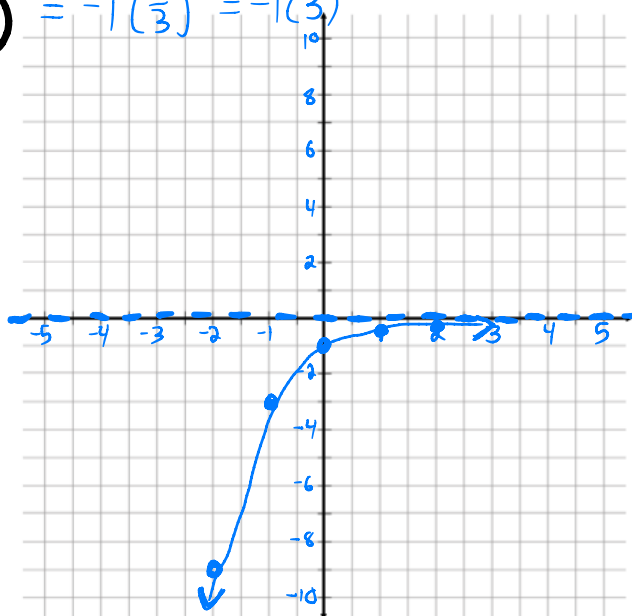
Decreasing

asymptote:

$$y = 0$$

b)  $y = -3^{-x} = -1\left(\frac{1}{3}\right)^x = -1\left(\frac{1}{3}\right)^{-1} = -1(3)^1$

| x  | y     |
|----|-------|
| -2 | -9    |
| -1 | -3    |
| 0  | -1    |
| 1  | -0.33 |
| 2  | -0.11 |



Domain:  $\{x \in \mathbb{R}\}$

Range:  $\{y \in \mathbb{R} \mid y < 0\}$

x-int: NONE

y-int:  $(0, -1)$

intervals of increase/decrease:

Increasing

asymptote:

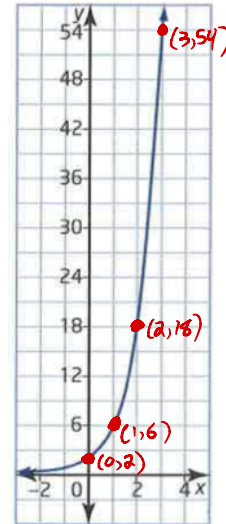
$$y = 0$$

### Example 2:

Write the equation in the form  $y = ab^x$  for the graph shown.

Start by determining the growth factor ( $b$ ). As  $x$  changes by 1 unit, what factor does  $y$  change by?

| $x$ | $y$ |
|-----|-----|
| 0   | ?   |
| 1   | 6   |
| 2   | 18  |
| 3   | 54  |



Next, determine the initial value ( $a$ ) by plugging in the coordinates of one of the points ( $x, y$ ) on the graph and the growth rate ( $b$ ), then solve for  $a$ .

$$\begin{aligned}y &= a(b)^x \\y &= a(3)^x \\6 &= a(3)^1 \\a &= \frac{6}{3} \\a &= 2\end{aligned}$$

Final Equation:

$$y = 2(3)^x$$

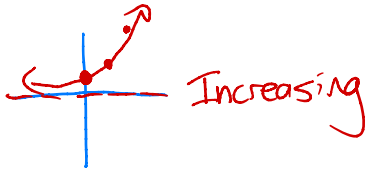
**Example 3:** A radioactive sample has a half-life of 3 days. The initial sample is 200 mg. Write a function to relate the amount remaining, in milligrams, to the time, in days.

$$\begin{aligned}y &= a(b)^x \\y &= 200\left(\frac{1}{2}\right)^{t/3} \\A(t) &= 200\left(\frac{1}{2}\right)^{t/3}\end{aligned}$$

What do you know so far about when a function of the form  $y = a(b)^x$  is increasing and when it is decreasing?

$$a > 0$$

$$b > 1$$



$$a > 0$$

$$0 < b < 1$$



$$a < 0$$

$$b > 1$$



$$a < 0$$

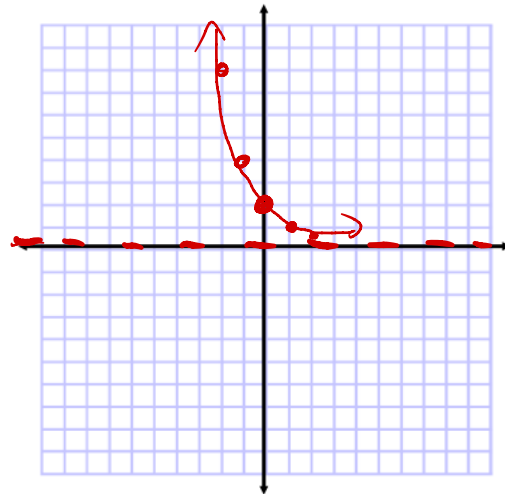
$$0 < b < 1$$



**Example 4:** Make a rough sketch of the graph of the following functions based on your knowledge of whether they are increasing or decreasing.  $y = a(b)^x$

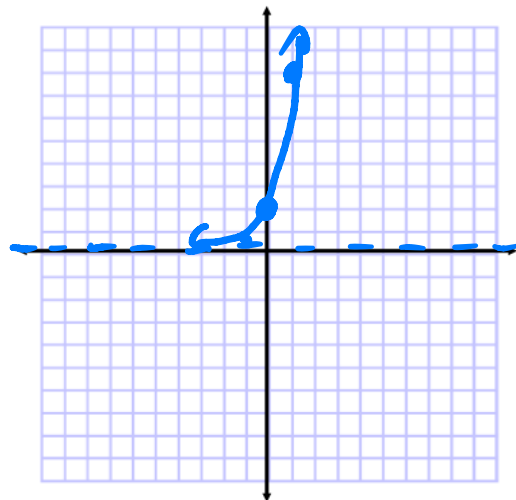
a)  $y = 2\left(\frac{1}{2}\right)^x$

Decreasing



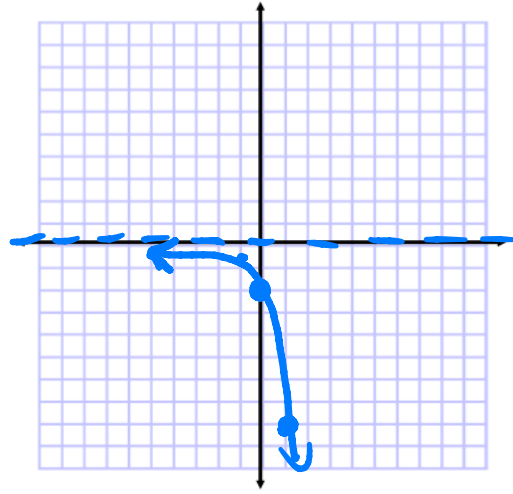
b)  $y = 2(4)^x$

Increasing



c)  $y = -2(4)^x$

Decreasing



d)  $y = -2\left(\frac{1}{2}\right)^x$

Increasing

