L5 – Transformations of Exponential Functions

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Warm-up: Which of the following graphs are the same?

Rule: $(x^a)^b = x^{a \times b}$

$f(x) = 32^x$ $g(x) = 9^x$ $h(x) = 2^{3x} = (2^3)^x = 8^x$ $n(x) = 2^{5x} = (2^5)^x = 32^x$

$p(x) = 3^{3x}$ $q(x) = 3^{2x} = (3^2)^x = 9^x$ $r(x) = 8^x$

Exponential functions can be transformed in the same way as other function. The graph of can be found by performing transformations on the graph of $f(x) = b^x$

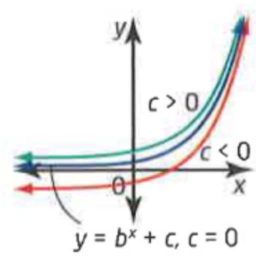
$g(x) = a\, b^{k(x-d)} + c$

Changes to the *y*-coordinates (vertical changes)

***c*: vertical translation** $g(x) = b^x + c$

The graph of $g(x) = b^x + c$ is a vertical translation of the graph of b^x by c units.

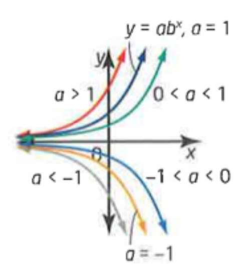
If $c > 0$, the graph shifts **UP**
If $c < 0$, the graph shifts **DOWN**



***a*: vertical stretch/compression** $g(x) = a \cdot b^x$

The graph of $g(x) = a \cdot b^x$ is a vertical stretch or compression of the graph of b^x by a factor of a.

If $a > 1$ OR $a < -1$, **vertical stretch** by a factor of $\lvert a \rvert$
If $-1 < a < 1$, **vertical compression** by a factor of $\lvert a \rvert$
If $a < 0$, **vertical reflection** (reflection over the x-axis)

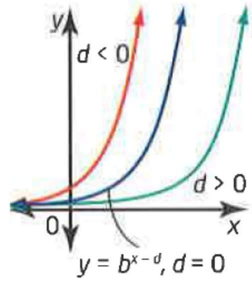


<u>Changes to the x-coordinates (horizontal changes)</u>

***d*: horizontal translation** $\quad g(x) = b^{x-d}$

The graph of $g(x) = b^{x-d}$ is a horizontal translation of the graph of b^x by d units.

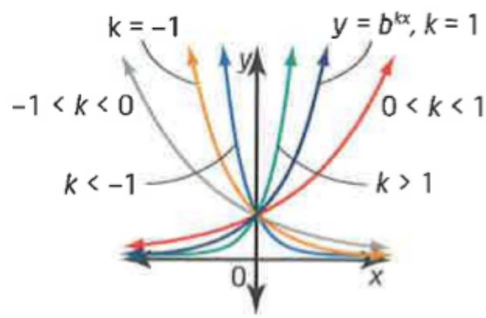
If $d > 0$, the graph shifts **RIGHT**
If $d < 0$, the graph shifts **LEFT**



k: horizontal stretch/compression $\quad g(x) = b^{kx}$

The graph of $g(x) = b^{kx}$ is a horizontal stretch or compression of the graph of b^x by a factor of $\frac{1}{k}$

If $k > 1$ OR $k < -1$, **horizontal compression** by a factor of $\frac{1}{\lvert k \rvert}$
If -1< $k < 1$, **horizontal stretch** by a factor of $\frac{1}{\lvert k \rvert}$
If $k < 0$, **horizontal reflection** (reflection over the y-axis)



Don't forget that the order of the transformations matters!!!

Do the reflections, stretches, and compressions first. Then do the horizontal and vertical shifts.

k d

Example 1: Graph the function $g(x) = 2(2)^{\frac{1}{2}(x-1)}$

a

Step 1: What is the base function?

$y = 2^x$

Step 2: Describe the transformations made to the base function.

$a = 2$; vertical stretch by a factor of 2 $(2y)$

$k = \frac{1}{2}$; horizontal stretch by a factor of 2 $(2x)$

$d = 1$; shift right 1 unit $(x+1)$

Step 3: Make a table of values for the base function and the transformed function $g(x)$

$y = 2^x$	
x	y
-3	0.125
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8

$\times 2$

$\times 2$

$\frac{x}{k} + d$

$g(x) = \overset{a}{2}(2)^{\overset{k}{\frac{1}{2}}(x\overset{d}{-1})}$	
$2x + 1$	$2y$
-5	0.25
-3	0.5
-1	1
1	2
3	4
5	8
7	16

$ay + c$

Step 4: Graph ~~both functions~~

Example 2: Graph the function $g(x) = 3^{2x-4} + 1$ $= \overset{a}{1}\,3^{\overset{k}{2}(x-\overset{d}{2})} + \overset{c}{1}$

Hint 1: *The 'k' value must be common factored out.*

Hint 2: *'c' value is the horizontal asymptote.*

Step 1: What is the base function?

$y = 3^x$

Step 2: Describe the transformations made to the base function.

$k = 2$; horizontal compression by a factor of $\frac{1}{2}$ $\left(\frac{x}{2}\right)$

$d = 2$; shift RIGHT 2 units $(x+2)$

$c = 1$; shift UP 1 unit $(y+1)$

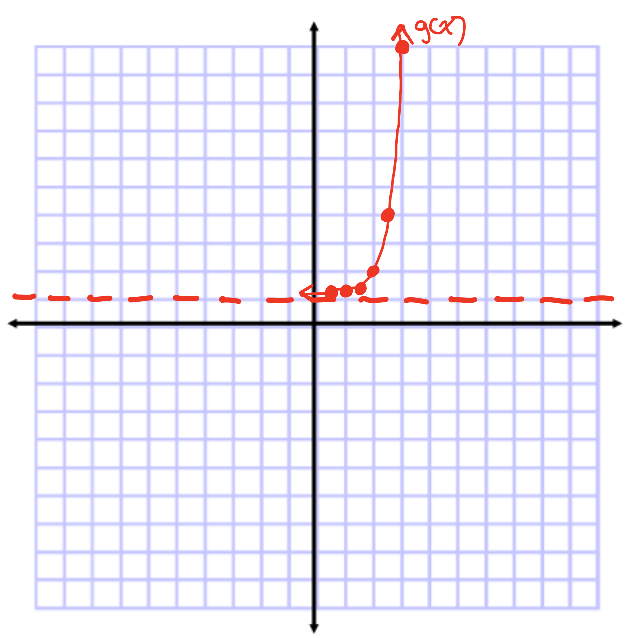
Step 3: Make a table of values for the base function and the transformed function $g(x)$

$y = 3^x$	
x	y
-3	0.04
-2	0.11
-1	0.33
0	1
1	3
2	9
3	27

$\frac{x}{k} + d$; $ay + c$

$g(x) = 3^{2(x-2)} + 1$	
$\frac{x}{2} + 2$	$y + 1$
0.5	1.04
1	1.11
1.5	1.33
2	2
2.5	4
3	10
3.5	28

Step 4: Graph the transformed function



Example 3: Graph the function $g(x) = \underset{a}{-2}\left(\frac{1}{2}\right)^{x\overset{d}{-3}} \underset{c}{-\ 2}$

Step 1: What is the base function?

$y = \left(\frac{1}{2}\right)^x$

Step 2: Describe the transformations made to the base function.

$a = -2$; vertical stretch by a factor of 2 $(2y)$
vertical reflection $(-y)$

$d = 3$; shift right 3 units $(x+3)$

$c = -2$; shift Down 2 units $(y-2)$

Step 3: Make a table of values for the base function and the transformed function $g(x)$

$y = \left(\frac{1}{2}\right)^x$	
x	y
-3	8
-2	4
-1	2
0	1
1	0.5
2	0.25
3	0.125

$\frac{x}{k}+d$ $\qquad$ $ay+c$

$g(x) = -2\left(\frac{1}{2}\right)^{x-3} - 2$	
$x+3$	$-2y-2$
0	-18
1	-10
2	-6
3	-4
4	-3
5	-2.5
6	-2.25

Step 4: Graph the transformed function