L5 – Transformations of Exponential Functions

MCR3U

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Warm-up: Which of the following graphs are the same?

$f(x) = 32^x$ $\quad g(x) = 9^x$ $\quad h(x) = 2^{3x}$ $\quad n(x) = 2^{5x}$

$p(x) = 3^{3x}$ $\quad q(x) = 3^{2x}$ $\quad r(x) = 8^x$

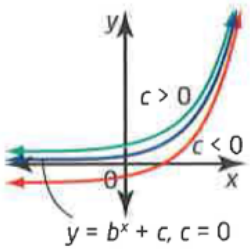
Exponential functions can be transformed in the same way as other function. The graph of can be found by performing transformations on the graph of $f(x) = b^x$

Changes to the *y*-coordinates (vertical changes)

***c*: vertical translation** $\quad g(x) = b^x + c$

The graph of $g(x) = b^x + c$ is a vertical translation of the graph of b^x by c units.

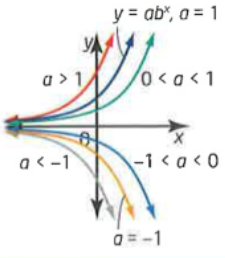
If $c > 0$, the graph shifts **UP**
If $c < 0$, the graph shifts **DOWN**



***a*: vertical stretch/compression** $\quad \boldsymbol{g(x) = a \cdot b^x}$

The graph of $g(x) = \boldsymbol{a} \cdot b^x$ is a vertical stretch or compression of the graph of b^x by a factor of a.

If $a > 1$ OR $a < -1$, **vertical stretch** by a factor of $\lvert a \rvert$
If -1< a < 1, **vertical compression** by a factor of $\lvert a \rvert$
If $a < 0$, **vertical reflection** (reflection over the x-axis)

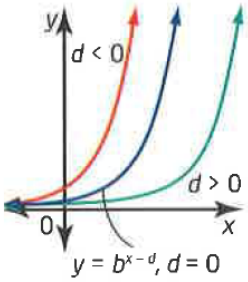


Changes to the *x*-coordinates (horizontal changes)

d: horizontal translation $g(x) = b^{x-d}$

The graph of $g(x) = b^{x-d}$ is a horizontal translation of the graph of b^x by d units.

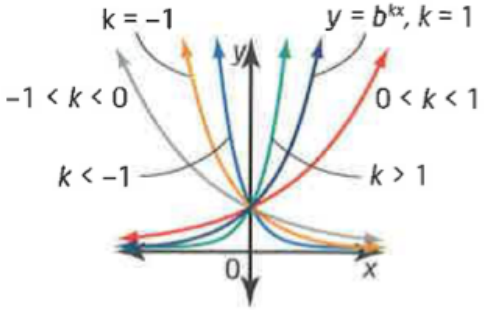
If $d > 0$, the graph shifts **RIGHT**
If $d < 0$, the graph shifts **LEFT**



k: horizontal stretch/compression $g(x) = b^{kx}$

The graph of $g(x) = b^{kx}$ is a horizontal stretch or compression of the graph of b^x by a factor of $\frac{1}{k}$

If $k > 1$ OR $k < -1$, **horizontal compression** by a factor of $\frac{1}{\lvert k \rvert}$
If -1< k < 1, **horizontal stretch** by a factor of $\frac{1}{\lvert k \rvert}$
If $k < 0$, **horizontal reflection** (reflection over the y-axis)



Don't forget that the order of the transformations matters!!!

Do the reflections, stretches, and compressions first. Then do the horizontal and vertical shifts.

Example 1: Graph the function $g(x) = 2(2)^{\frac{1}{2}(x-1)}$

Step 1: What is the base function?

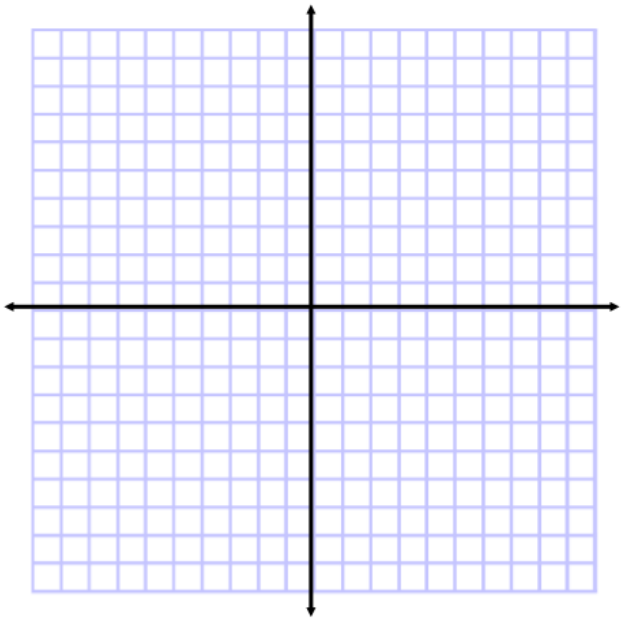
Step 2: Describe the transformations made to the base function.

Step 3: Make a table of values for the base function and the transformed function $g(x)$

x	y

x	y

Step 4: Graph both functions



Example 2: Graph the function $g(x) = 3^{2x-4} + 1$

Hint 1: *The 'k' value must be common factored out.*

Hint 2: *'c' value is the horizontal asymptote.*

Step 1: What is the base function?

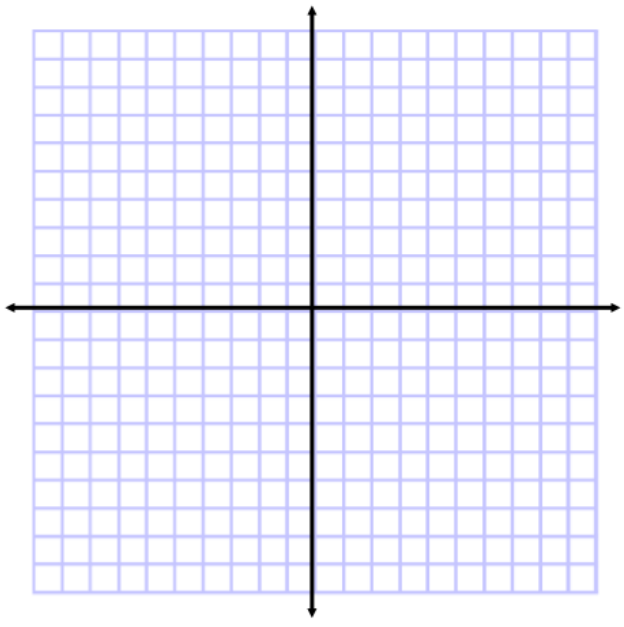
Step 2: Describe the transformations made to the base function.

Step 3: Make a table of values for the base function and the transformed function $g(x)$

x	y

x	y

Step 4: Graph the transformed function



Example 3: Graph the function $g(x) = -2\left(\frac{1}{2}\right)^{x-3} - 2$

Step 1: What is the base function?

Step 2: Describe the transformations made to the base function.

Step 3: Make a table of values for the base function and the transformed function $g(x)$

x	y

x	y

Step 4: Graph the transformed function