

3.5 Transformations of Exponential Functions - Worksheet

SOLUTIONS

MCR3U

Iensen

1) Describe the transformations that map the function $y = 2^x$ onto each of the following functions...

a) $y = 2^x - 2$

- down 2 units ($y-2$)

b) $y = 2^{x+3}$

- left 3 units ($x-3$)

c) $y = 4^x$

$= 2^{2x}$

- horizontal compression

by a factor of $\frac{1}{2}$ ($\frac{x}{2}$)

d) $y = 3(2^{x-1}) + 1$

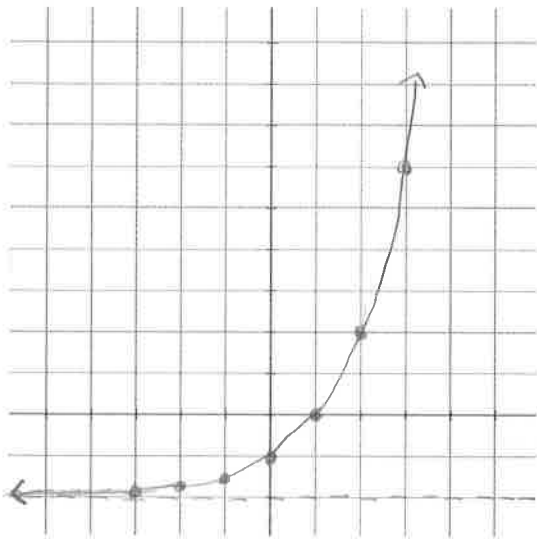
- vertical stretch by a factor of 3 ($3y$)

- right 1 ($x+1$)

- up 1 ($y+1$)

2) Create a sketch of each graph for each equation in question 1. (a table of values may help)

a)



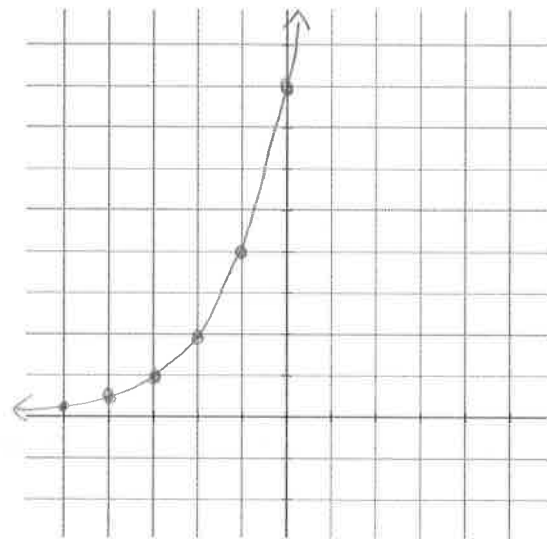
$y = 2^x$

x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

$y = 2^x - 2$

x	y-2
-3	-1.875
-2	-1.75
-1	-1.5
0	-1
1	0
2	2
3	6

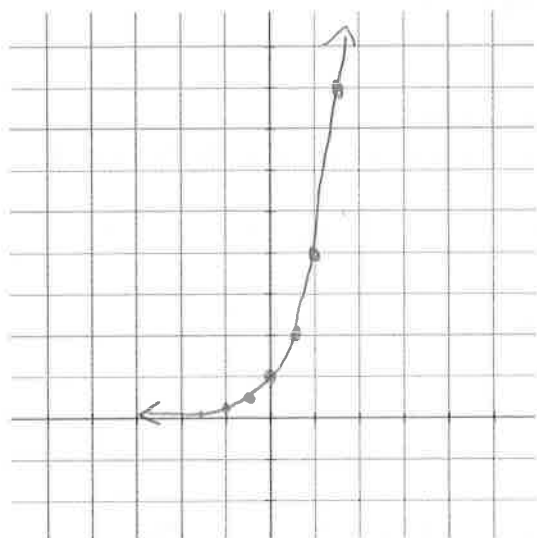
b)



$y = 2^{x+3}$

x-3	y
-6	0.125
-5	0.25
-4	0.5
-3	1
-2	2
-1	4
0	8

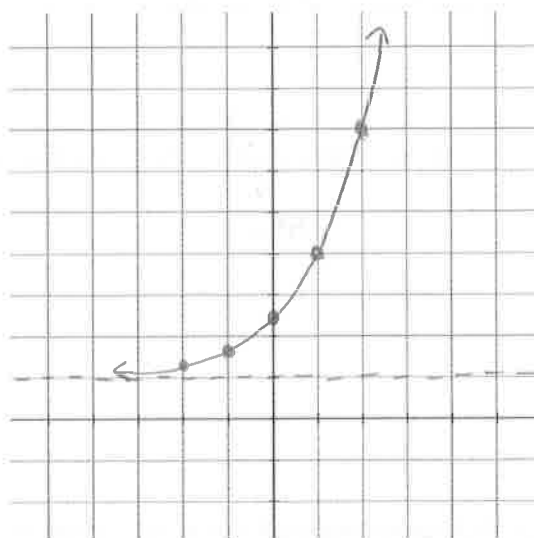
c)



$$y = 2^{2x}$$

$\frac{x}{2}$	y
-1.5	0.125
-1	0.25
-0.5	0.5
0	1
0.5	2
1	4
1.5	8

d)



$x+1$	$3y+1$
-2	1.375
-1	1.75
0	2.5
1	4
2	7
3	13
4	25

3) Write the equation for the function that results from each transformation applied to the base function $y = 5^x$.

a) translate down 3 units

$$y = 5^x - 3$$

b) shift right 2 units

$$y = 5^{x-2}$$

c) translate left $\frac{1}{2}$ unit

$$y = 5^{x+0.5}$$

d) shift up 1 unit and left 2.5 units

$$y = 5^{x+2.5} + 1$$

4) Describe the transformations that map the function $y = 8^x$ onto each function.

a) $y = \left(\frac{1}{2}\right) 8^x$

vertical compression
by a factor of $\frac{1}{2}$ $\left(\frac{y}{2}\right)$

b) $y = 8^{4x}$

- horizontal compression by a factor of $\frac{1}{4}$ $\left(\frac{x}{4}\right)$

c) $y = -8^x$

- vertical reflection $(-y)$

d) $y = 8^{-2x}$

- horizontal compression by a factor of $\frac{1}{2}$
and horizontal reflection $\left(\frac{x}{-2}\right)$

5) Write the equation for the function that results from each transformation applied to the base function $y = 7^x$

a) reflect in the x-axis (vertical reflection)

$$y = -7^x$$

b) stretch vertically by a factor of 3

$$y = 3(7)^x$$

c) stretch horizontally by a factor of 2.4

$$y = 7^{\frac{1}{2.4}x}$$

d) reflect in the y-axis and ~~compress~~ ^{stretch} vertically by a factor of 7

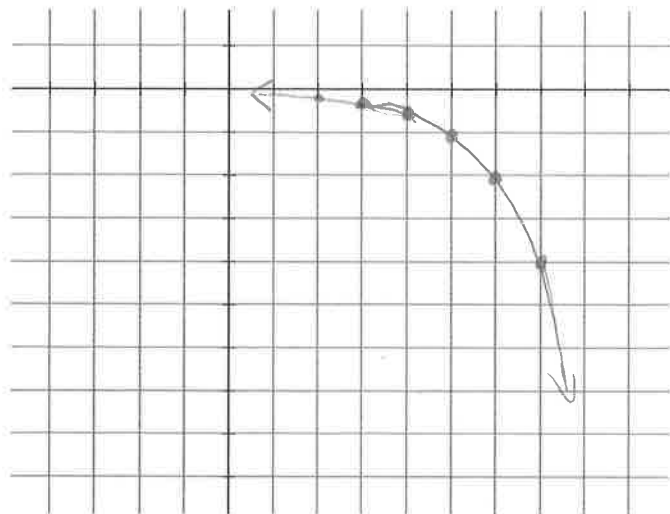
$$y = 7(7)^{-x}$$

6) Sketch the graph of $y = \left(-\frac{1}{2}\right) 2^{x-4}$ by using $y = 2^x$ as the base and applying transformations.

$$y = 2^x$$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$x+4$	$\frac{y}{-2}$
1	-0.0625
2	-0.125
3	-0.25
4	-0.5
5	-1
6	-2
7	-4



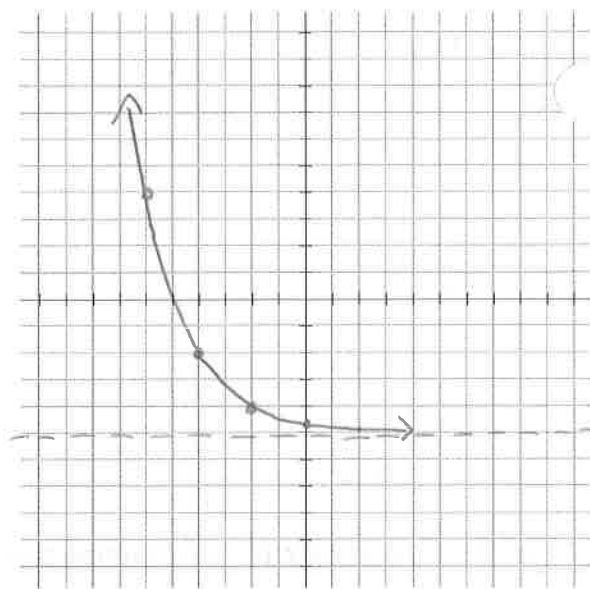
7) Sketch the graph of $y = 3^{-0.5x-1} - 5$ by using $y = 3^x$ as the base and applying transformations.

$$y = 3^x$$

x	y
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27

$$y = 3^{-0.5(x+2)} - 5$$

$-2x-2$	$y-5$
4	-4.96
2	-4.88
0	-4.67
-2	-4
-4	-2
-6	4
-8	22



8) a) Rewrite $y = 9^x$ using a base of 3. Describe how you can graph this function by transforming the graph of $y = 3^x$.

$$\begin{aligned} y &= 9^x \\ &= (3^2)^x \\ &= 3^{2x} \end{aligned}$$

horizontal compression base $\frac{1}{2}$

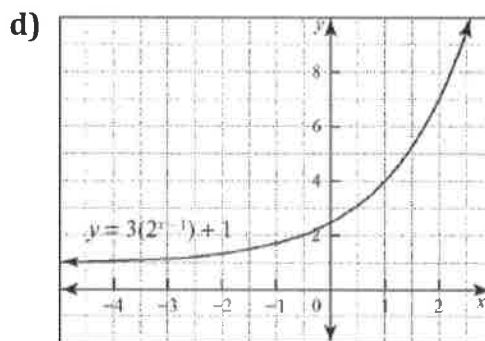
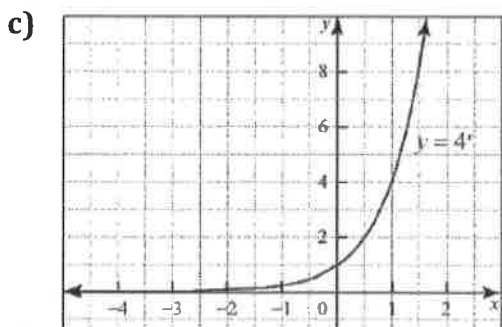
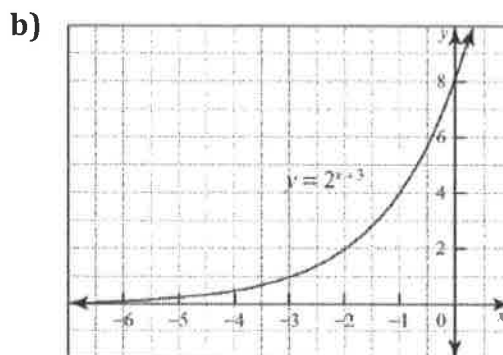
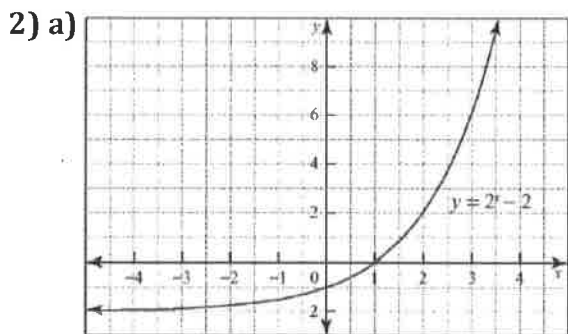
b) Rewrite $y = 9^x$ using a base of 81. Describe how you can graph this function by transforming the graph of $y = 81^x$.

$$\begin{aligned} y &= 9^x \\ &= (81^{1/2})^x \\ &= 81^{1/2 x} \end{aligned}$$

horizontal stretch base 2.

Answers

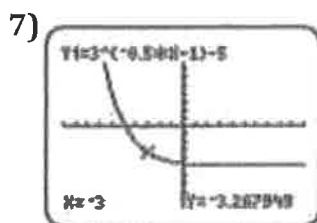
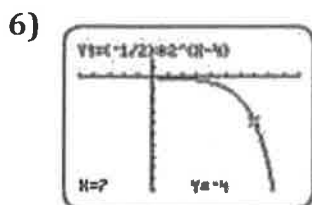
- 1) a) translate 2 units down b) translate 3 units left c) horizontal compression by a factor of $\frac{1}{2}$
 d) vertical stretch by a factor of 3, a translation 1 unit to the right and 1 unit up



3) a) $y = 5^x - 3$ b) $y = 5^{x-2}$ c) $5^{x+\frac{1}{2}}$ d) $y = 5^{x+2.5} + 1$

- 4) a) vertical compression by a factor of $\frac{1}{2}$ b) horizontal compression by a factor of $\frac{1}{4}$
 c) vertical reflection (reflection in the x-axis)
 d) horizontal reflection (reflection in the y-axis) and horizontal compression by a factor of $\frac{1}{2}$

5) a) $y = -7^x$ b) $y = 3(7^x)$ c) $y = 7^{\frac{x}{2.4}}$ d) $y = 7(7^{-x})$



8) a) $y = 3^{2x}$; horizontal compression of the graph of $y = 3^x$ by a factor of $\frac{1}{2}$

b) $y = 81^{\frac{1}{2}x}$; horizontal stretch of the graph of $y = 81^x$ by a factor of 2

