

# L1 - Modeling Periodic Behaviour

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## Section 1: Definitions

**PERIODIC FUNCTION:** a function that has a pattern of  $y$ -values that repeats at regular intervals.

**CYCLE:** one complete repetition of a pattern.

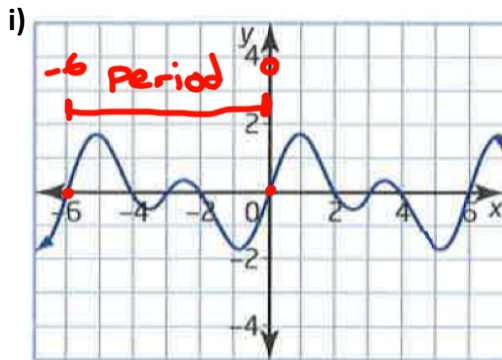
**PERIOD:** the horizontal length of one cycle on a graph.

**AMPLITUDE:** half the distance between the maximum and minimum values of a periodic function.

## Section 2: Recognizing Properties of Periodic Functions

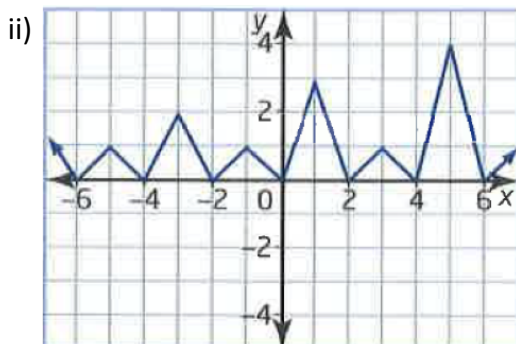
*How to find the PERIOD of a function: choose a convenient  $x$ -coordinate to start at and then move to the right and estimate the  $x$ -coordinate of where the next cycle begins. Find the difference of these  $x$ -coordinates to calculate the period of the function.*

**Example 1:** Determine whether the functions are periodic or not. If it is, state the period of the function.



The pattern of  $y$  – values in one section of the graph repeats in the next section. Therefore, the function IS periodic.

$$\begin{aligned} \text{period} &= 0 - (-6) \\ &= 6 \end{aligned}$$



The pattern of  $y$  – values in one section of the graph does NOT repeat in the next section. Therefore, the function is NOT periodic.

**Example 2:** Is the function periodic? If so, what is the amplitude?

*How to find the AMPLITUDE of a function: the amplitude is half the difference between the max and min values. Use the formula:*

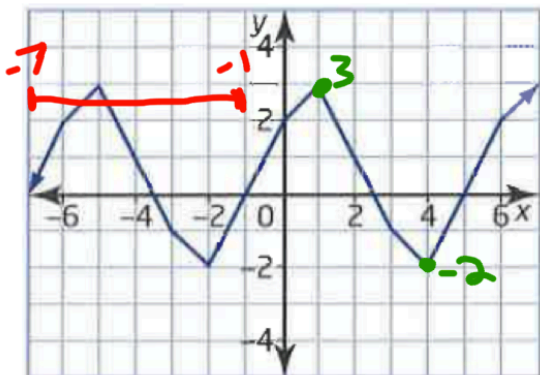
$$\text{amplitude} = \frac{y_{\max} - y_{\min}}{2}$$



Yes, the function is periodic.

$$\text{amplitude} = \frac{3 - (-1)}{2} = \frac{4}{2} = 2 \text{ units}$$

**Example 3:** In the following periodic function, determine the period and amplitude.

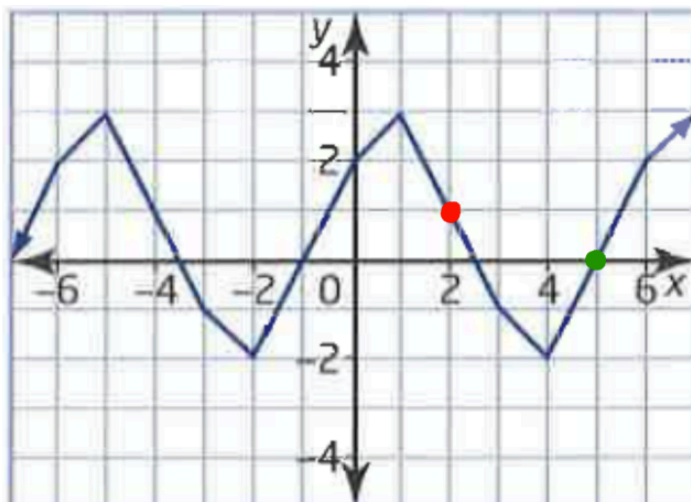


$$\text{period} = -1 - (-7) = 6 \text{ units}$$

$$\text{amplitude} = \frac{3 - (-2)}{2} = \frac{5}{2} \text{ units}$$

### Section 3: Predicting Values of a Periodic Function

**Example 4:** For the following function...



a) determine  $f(2)$  and  $f(5)$

$$f(2) = 1$$

$$f(5) = 0$$

b) determine  $f(8)$ ,  $f(-10)$ , and  $f(14)$

*period = 6 units*

$$\begin{aligned} f(8) &= f(8 - 6) \\ &= f(2) \\ &= 1 \end{aligned}$$

$$\begin{aligned} f(-10) &= f(-10 + 6) \\ &= f(-4) \\ &= 1 \end{aligned}$$

$$\begin{aligned} f(14) &= f(14 - 6) \\ &= f(8) \\ &= 1 \end{aligned}$$

**Hint:**

i) figure out the period of the function

ii) add or subtract the period of the function until you get back to a value on the graph that you know.

c) determine 4 values of  $x$  so that  $f(x) = 2$

From graph:  $f(0) = 2$

$$\begin{aligned} f(0 + 6) &= 2 \\ f(6) &= 2 \end{aligned}$$

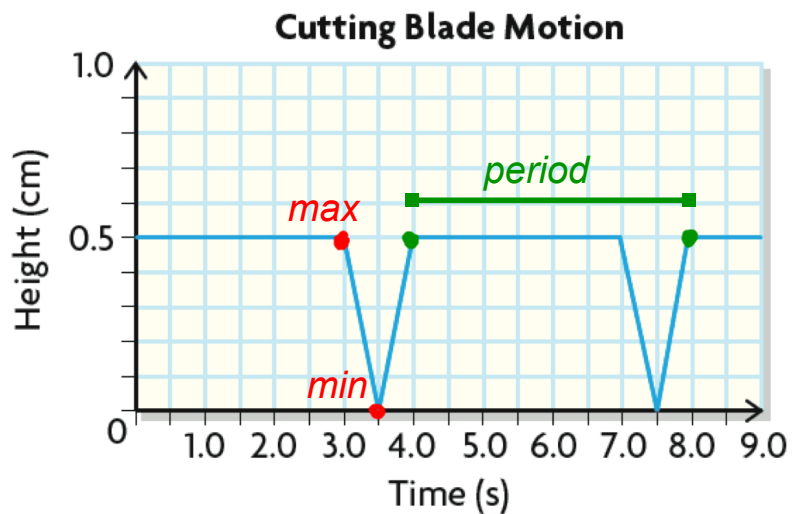
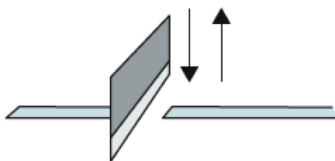
$$\begin{aligned} f(6 + 6) &= 2 \\ f(12) &= 2 \end{aligned}$$

$$\begin{aligned} f(0 - 6) &= 2 \\ f(-6) &= 2 \end{aligned}$$

**Hint:**

Keep adding/subtracting the period value to the  $x$ -value where  $y = 2$ .

**Example 5:** A cutting machine chops strips of plastic into their appropriate lengths. The following graph shows the motion of the cutting blade on the machine in terms of time.



a) State the max height of the blade, the minimum height, and the amplitude of the function.

$$\text{max height} = 0.5 \text{ cm}$$

$$\text{min height} = 0 \text{ cm}$$

$$\text{amplitude} = \frac{y_{\max} - y_{\min}}{2} = \frac{0.5 - 0}{2} = 0.25 \text{ cm}$$

b) What is the period of this function?

$$\text{period} = 8 - 4 = 4 \text{ seconds}$$

c) State the next two times that the blade will strike the cutting surface?

$$\text{Last strike was at 7.5 seconds} \rightarrow f(7.5) = 0$$

$$f(7.5 + 4) = 0$$

$$f(11.5) = 0$$

$$f(11.5 + 4) = 0$$

$$f(15.5) = 0$$

Therefore, the next strikes will be at 11.5 seconds and 15.5 seconds.