L3 – Transformations of Sine and Cosine Part 1 Equation → Graph

MCR3U

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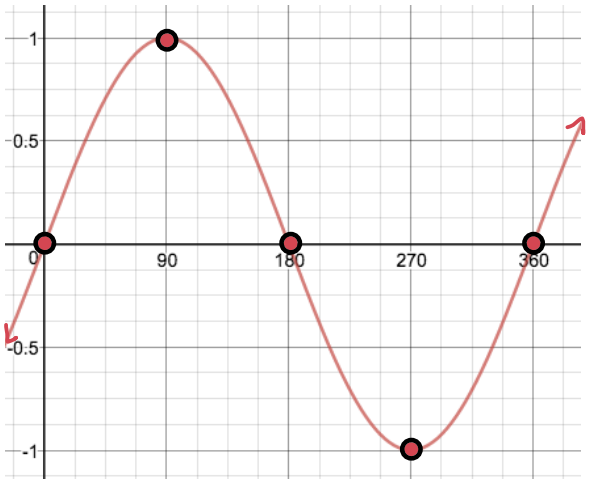
Section 1: Review of Sine and Cosine Functions

$$y = a\sin[k(x-d)] + c \quad \text{OR} \quad y = a\cos[k(x-d)] + c$$

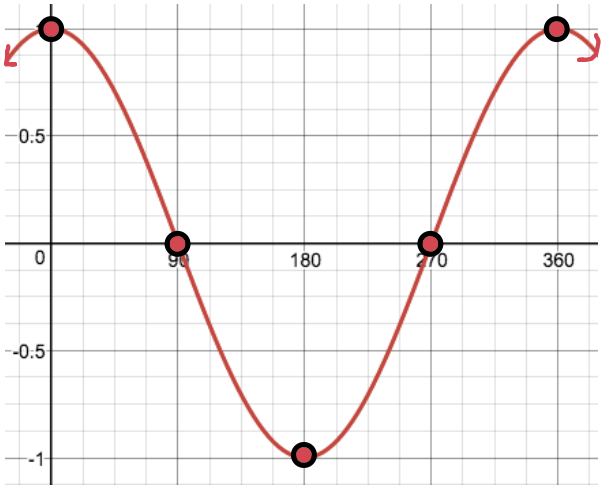
a	k	d	c
Vertical stretch or compression by a factor of a. Vertical reflection if $a < 0$ $\|a\| = amplitude$	Horizontal stretch or compression by a factor of $\frac{1}{k}$. Horizontal reflection if $k < 0$. $\frac{360}{\|k\|} = period$	Phase shift $d > 0; shift\ right$ $d < 0; shift\ left$	Vertical shift $c > 0; shift\ up$ $c < 0; shift\ down$

Graphs of parent functions $y = \sin x$ and $y = \cos x$ using key points:

x	y
0	0
90	1
180	0
270	−1
360	0



x	y
0	1
90	0
180	−1
270	0
360	1



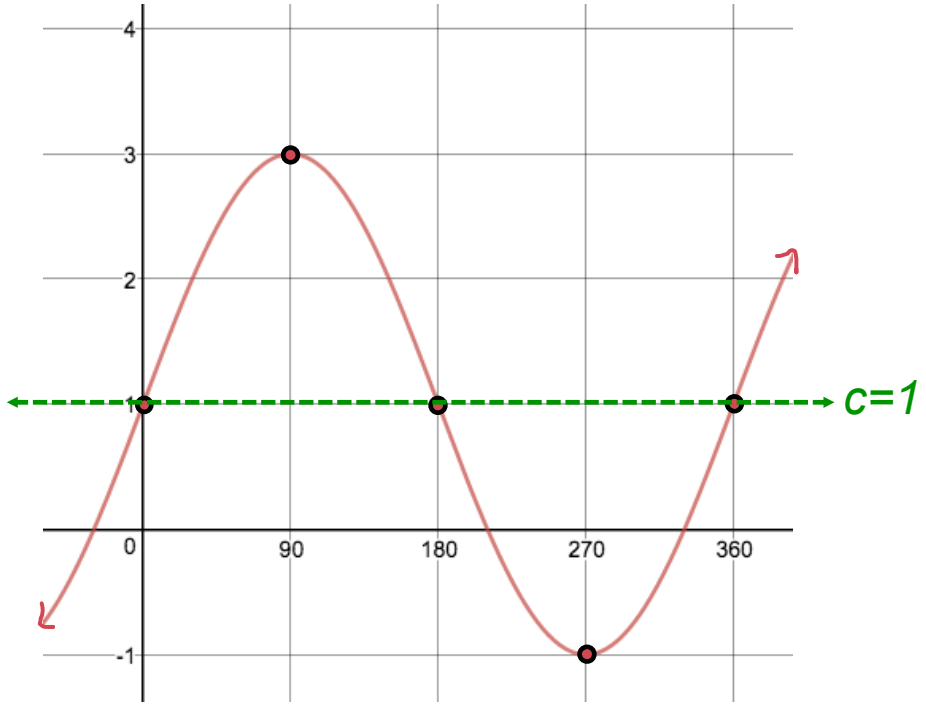
Section 2: Graphing Transformed Sinusoidal Functions

Example 1: Graph $y = 2\sin x + 1$ using transformations. Then state the amplitude, period, and number of cycles between $0°$ and $360°$.

$a = 2$; vertical stretch by a factor of 2 $(2y)$
$c = 1$; vertical shift up 1 unit $(y + 1)$

$y = \sin x$	
x	y
0	0
90	1
180	0
270	−1
360	0

$y = 2\sin x + 1$	
x	$2y + 1$
0	1
90	3
180	1
270	−1
360	1



Amplitude: $amplitude = |a| = 2$

Period: $period = \frac{360}{|k|} = \frac{360}{1} = 360°$

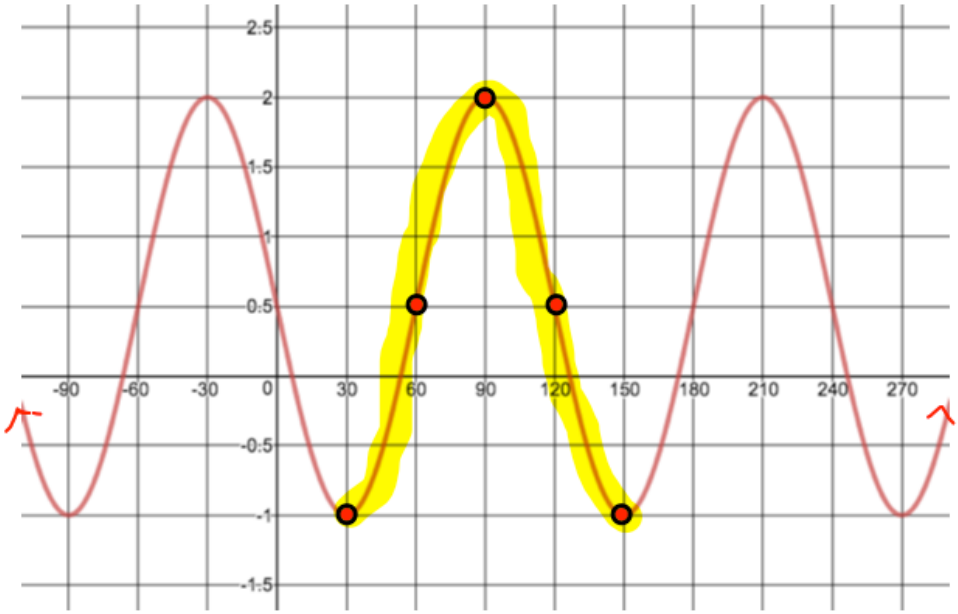
Number of cycles between $0°$ and $360°$: $\#of\ cycles = |k| = 1$

Example 2: Graph $y = -1.5\cos[3(x - 30°)] + 0.5$ using transformations. Then state the amplitude, period, and number of cycles between $0°$ and $360°$.

$a = -1.5$; vertical stretch by a factor of 1.5 and a vertical reflection $(-1.5y)$
$k = 3$; horizontal compression by a factor of $\frac{1}{3}$ $\left(\frac{x}{3}\right)$
$d = 30$; phase shift $30°$ to the right $(x + 30)$
$c = 0.5$; vertical shift 0.5 units up $(y + 0.5)$

$y = \cos x$	
x	y
0	1
90	0
180	-1
270	0
360	1

$y = -1.5\cos[3(x - 30°)] + 0.5$	
$\frac{x}{3} + 30$	$-1.5y + 0.5$
30	-1
60	0.5
90	2
120	0.5
150	-1



Amplitude: $amplitude = |a| = 1.5$

Period: $period = \frac{360}{|k|} = \frac{360}{3} = 120°$

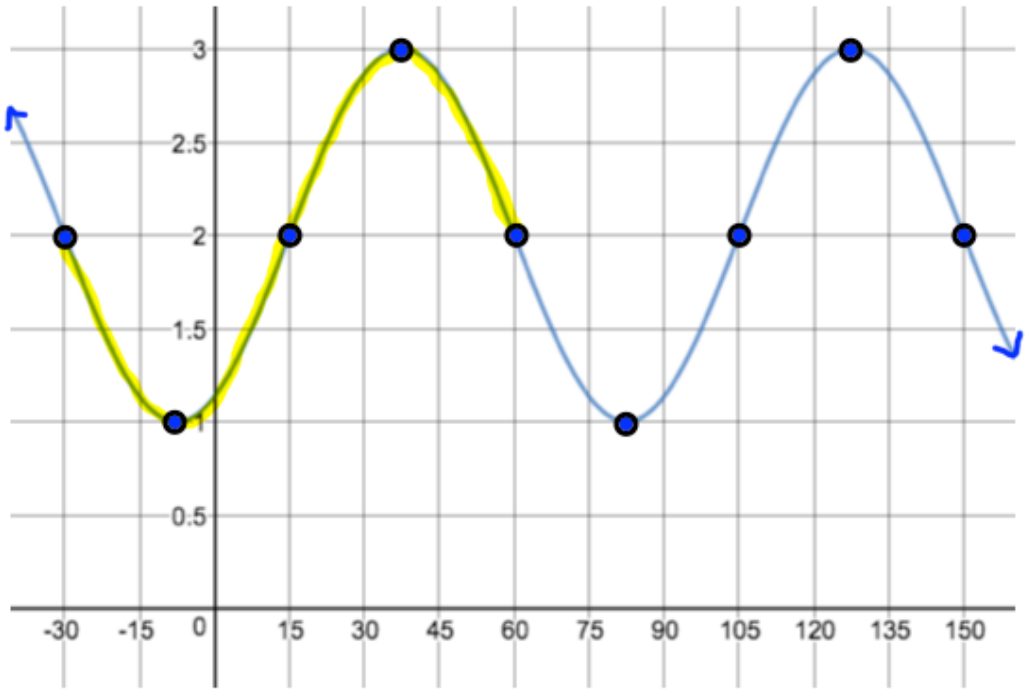
Number of cycles between $0°$ and $360°$: $\#of\ cycles = |k| = 3$

Example 3: Graph $y = \sin[-4(x - 60°)] + 2$ using transformations. Then state the amplitude, period, and number of cycles between $0°$ and $360°$.

$k = -4$; horizontal compression by a factor of $\frac{1}{4}$, and horizontal reflection $\left(\frac{-x}{4}\right)$
$d = 60$; phase shift $60°$ to the right $(x + 60)$
$c = 2$; vertical shift 2 units up $(y + 2)$

$y = \sin x$	
x	y
0	0
90	1
180	0
270	-1
360	0

$y = \sin[-4(x - 60°)] + 2$	
$\frac{-x}{4} + 60$	$y + 2$
60	2
37.5	3
15	2
-7.5	1
-30	2



Amplitude: $amplitude = |a| = 1$

Period: $period = \frac{360}{|k|} = \frac{360}{4} = 90°$

Number of cycles between $0°$ and $360°$: $\#of\ cycles = |k| = 4$