5.3 Transformations of Sine and Cosine Worksheet #2
MCR3U
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SOLUTIONS

1) A sinusoidal function has an amplitude of 5 units, a period of 120°, and a maximum at (0, 3).

a) Represent the function with an equation using a sine function

$a = 5$

$k = \frac{360}{period} = \frac{360}{120} = 3$

$c = max - |a| = 3 - 5 = -2$

$d_{sin} = d_{cos} - \frac{90}{k} = 0 - \frac{90}{3} = -30$

$y = 5\sin[3(x+30)] - 2$

b) Represent the function with an equation using a cosine function

$d_{cos} = 0$

$y = 5\cos(3x) - 2$

2) A sinusoidal function has an amplitude of $\frac{1}{2}$ units, a period of 720°, and a maximum at $\left(0, \frac{3}{2}\right)$.

a) Represent the function with an equation using a sine function

$a = 0.5$

$k = \frac{360}{period} = \frac{360}{720} = 0.5$

$c = max - |a| = 1.5 - 0.5 = 1$

$d_{sin} = d_{cos} - \frac{90}{|k|} = 0 - \frac{90}{0.5} = -180$

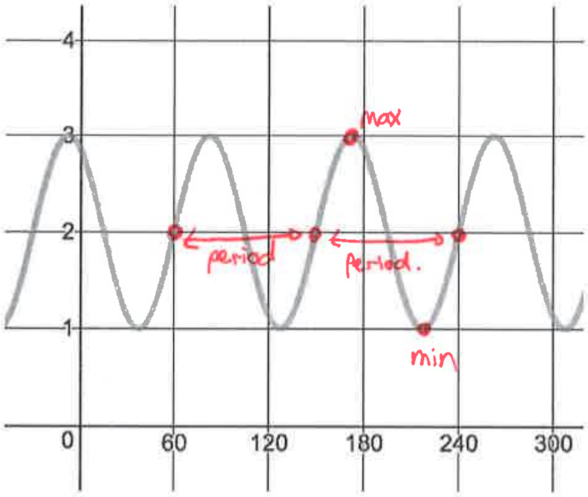
$y = 0.5\sin[0.5(x+180)] + 1$

b) Represent the function with an equation using a cosine function

$d_{cos} = 0$

$y = 0.5\cos[0.5x] + 1$

3) Determine the equation of a cosine function that represents the graph shown.



$$a = \frac{max - min}{2} = \frac{3-1}{2} = 1$$

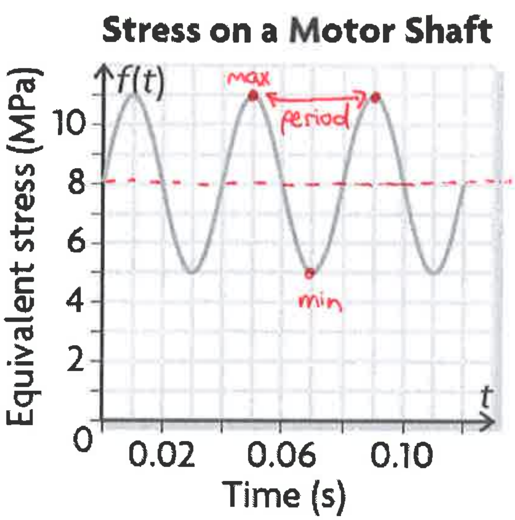
$$k = \frac{360}{period} = \frac{360}{150-60} = \frac{360}{90} = 4$$

$$c = max - |a| = 3 - 1 = 2$$

$$d_{cos} = d_{sin} + \frac{90}{k} = 60 + \frac{90}{4} = 82.5$$

$$y = \cos[4(x - 82.5)] + 2$$

4) The relationship between the stress on the shaft of an electric motor and time can be modelled with a sinusoidal function. Determine an equation of a function that describes stress in terms of time.



$$a = \frac{max - min}{2} = \frac{11-5}{2} = 3$$

$$k = \frac{360}{period} = \frac{360}{0.04} = 9000$$

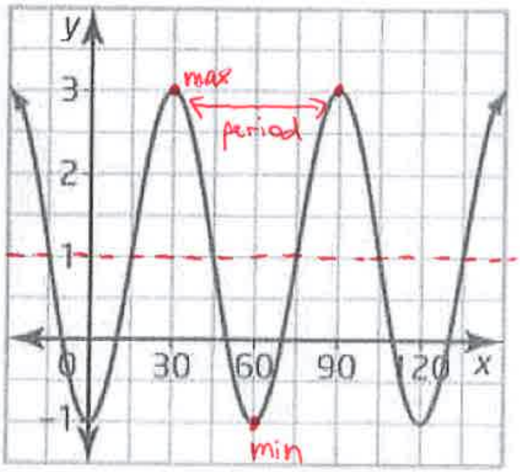
$$c = max - |a| = 11 - 3 = 8$$

$$d_{sin} = 0$$

$$d_{cos} = 0.01$$

$$y = 3\sin[9000x] + 8 \quad \text{OR} \quad y = 3\cos[9000(x - 0.01)] + 8$$

5) Determine the equation of the sine function shown.



$a = \frac{max - min}{2} = \frac{3-(-1)}{2} = 2$

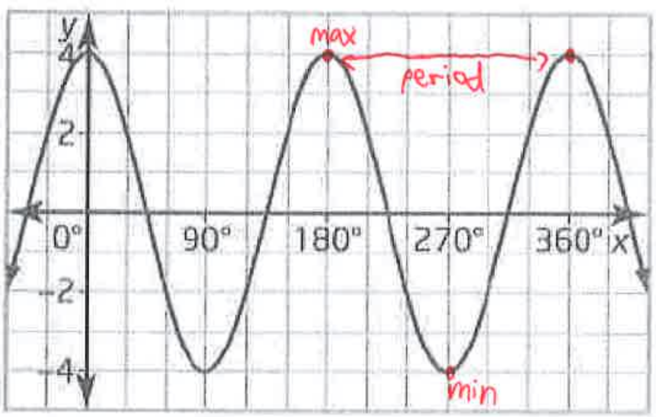
$k = \frac{360}{period} = \frac{360}{60} = 6$

$c = max - |a| = 3 - 2 = 1$

$d_{sin} = 15$

$y = 2\sin[6(x-15)] + 1$

6) Represent the graph of the following functions using a sine and cosine function.



$a = \frac{max - min}{2} = \frac{4-(-4)}{2} = 4$

$k = \frac{360}{period} = \frac{360}{180} = 2$

$c = max - |a| = 4 - 4 = 0$

$d_{cos} = 0$

$d_{sin} = d_{cos} - \frac{90}{k} = 0 - \frac{90}{2} = -45$

$\boxed{\begin{array}{l} y = 4\cos(2x) \\ y = 4\sin[2(x+45)] \end{array}}$

Answers

1) a) $y = 5 \sin [3(x + 30°)] - 2$ **b)** $y = 5 \cos 3x - 2$

2) a) $y = \frac{1}{2}\sin\left[\frac{1}{2}(x+180°)\right]+1$ **b)** $y = \frac{1}{2}\cos\frac{1}{2}x+1$

3) $y = \cos [4(x - 82.5°)] + 2$

4) $y = 3 \sin (9000x) + 8$ OR $y = 3\cos[9000(x - 0.01)] + 8$

5) a) $y = 2 \sin [6(x - 15°)] + 1$
b) If the maximum values are half as far apart, the period of the function is reduced by one-half to 30°. The value of k doubles from 6 to 12. The equation for the new function is $y = 2 \sin [12(x - 15°)] + 1$.

6) $y = 4 \cos 2x$ and $y = 4 \sin [2(x + 45°)]$.