L4 – Transformations of Sine and Cosine Part 2

Graph → Equation

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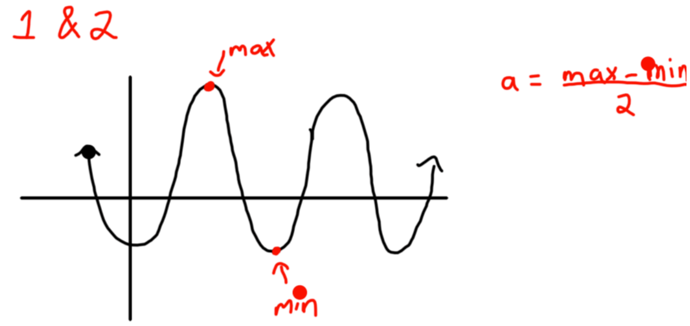
Section 1: How to Determine the Equation of a Sine or Cosine Function Given its Graph

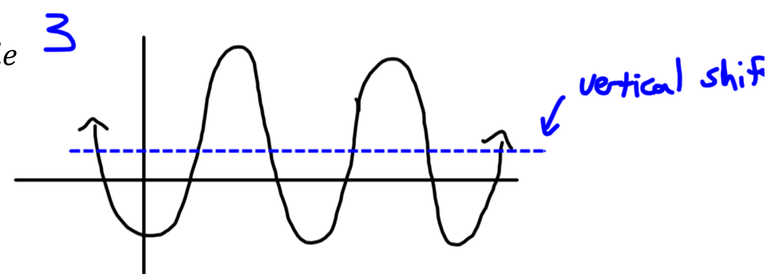
1) Find the max and min of the function

2) Find the amplitude of the function (a-value): $a = \frac{max - min}{2}$



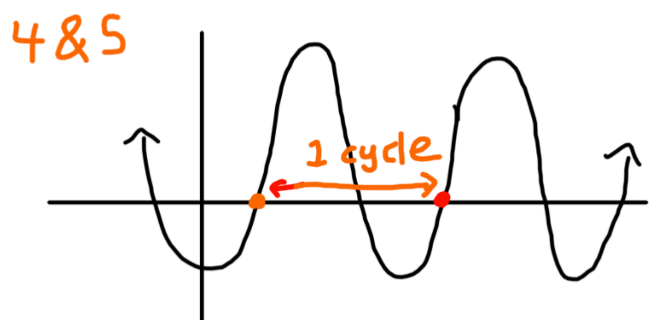
3) Find the vertical shift (c-value): $c = max - amplitude$

(this finds the 'middle' of the function)



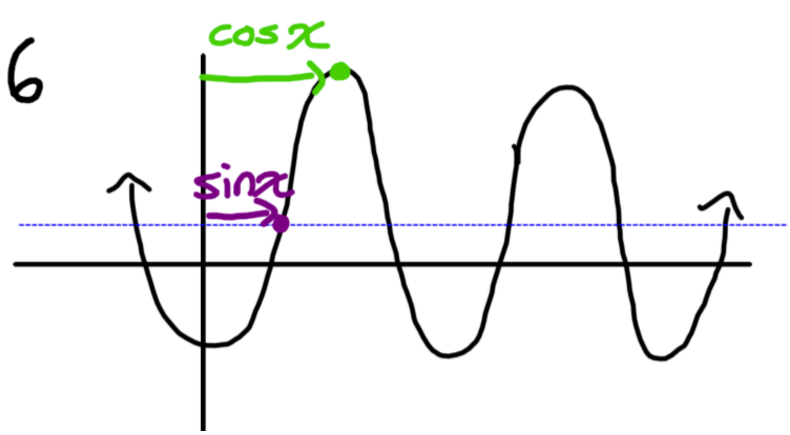
4) Find the period (in degrees) of the function using a starting point and ending point of a full cycle

5) Calculate the k-value. $k = \frac{360}{period}$ → $period = \frac{360}{|k|}$



6) Determine the phase shift (d-value)

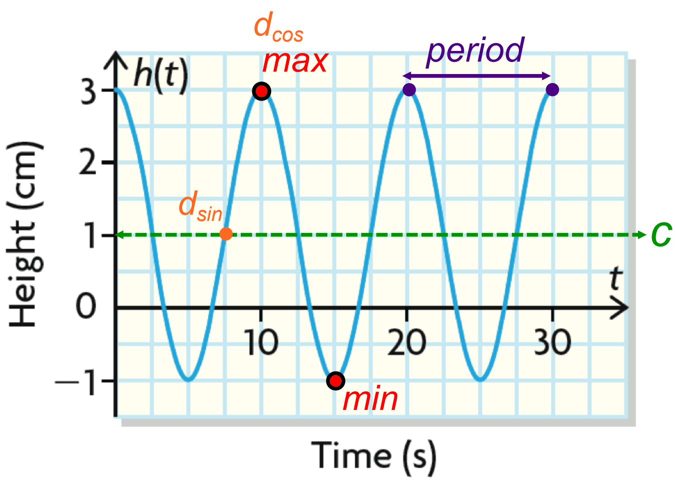
- for $\sin x$: trace along the center line and find the distance between the y-axis and the bottom left of the closest rising midline.
- for $\cos x$: the distance between the y-axis and the closest maximum point



Section 2: Determining the Equation of a Sinusoidal Function Given its Graph

Example 1: For each of the following graphs, determine the equation of a sine and cosine function that represents each graph:

a)



$$a = \frac{max - min}{2} = \frac{3-(-1)}{2} = 2$$

$$k = \frac{360}{period} = \frac{360}{30-20} = \frac{360}{10} = 36$$

$$c = max - |a| = 3 - 2 = 1$$

$$d_{sin} = 7.5$$

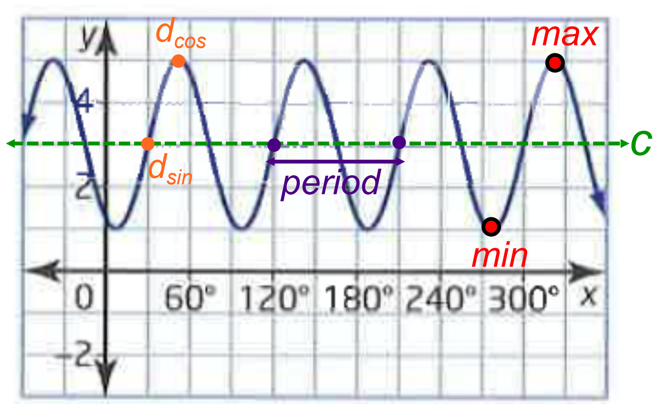
$$d_{cos} = 10$$

d_{sin} → look for x-value of closest rising midline

d_{cos} → look for x-value of closest maximum

$$y = 2\cos[36(x-10)] + 1 \qquad y = 2\sin[36(x-7.5)] + 1$$

b)



$$a = \frac{max - min}{2} = \frac{5-1}{2} = 2$$

$$k = \frac{360}{period} = \frac{360}{210-120} = \frac{360}{90} = 4$$

$$c = max - |a| = 5 - 2 = 3$$

$$d_{sin} = 30$$

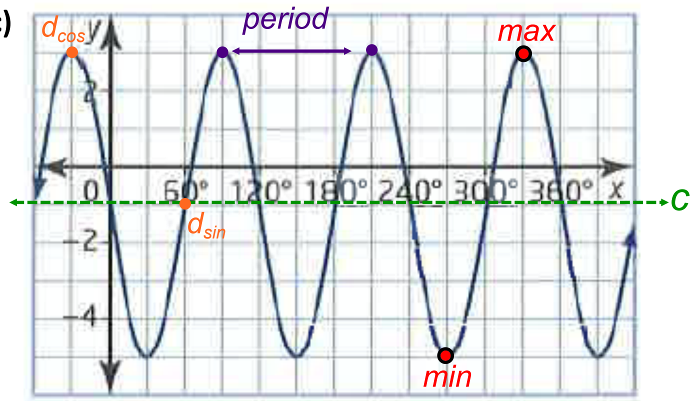
$$d_{cos} = d_{sin} + \frac{90}{|k|} = 30 + \frac{90}{4} = 52.5$$

$$y = 2\cos[4(x-52.5)] + 3 \qquad y = 2\sin[4(x-30)] + 3$$

Note: The $x - value$ of the maximum point was not obvious from the graph. You need to know that maximum points are always $\frac{90}{|k|}$ to the right of the rising midline point. Also, if you knew where the maximum point was, the rising midline point would be $\frac{90}{|k|}$ to the left of the max.

$$d_{cos} = d_{sin} + \frac{90}{|k|} \quad \textbf{OR} \quad d_{sin} = d_{cos} - \frac{90}{|k|}$$

c)



$$a = \frac{max - min}{2} = \frac{3 - (-5)}{2} = 4$$

$$k = \frac{360}{period} = \frac{360}{210 - 90} = \frac{360}{120} = 3$$

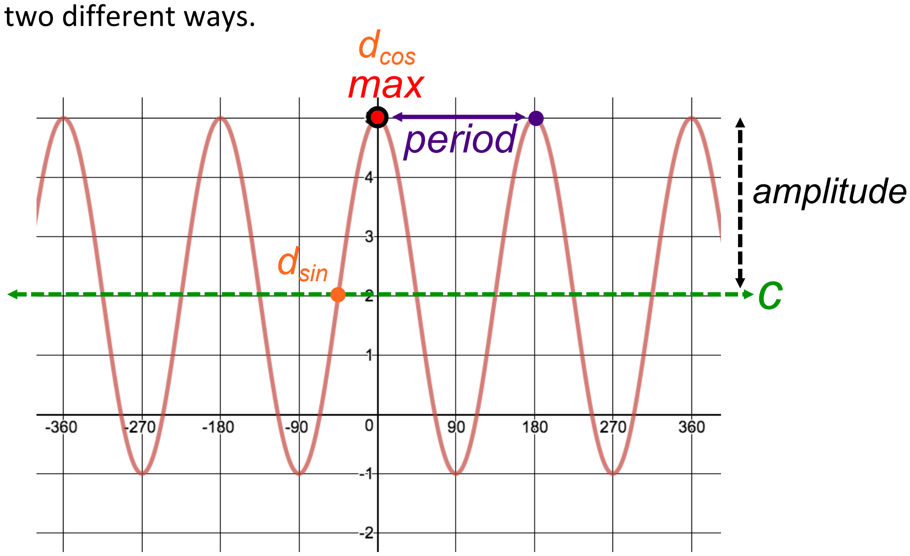
$$c = max - |a| = 3 - 4 = -1$$

$$d_{sin} = 60$$

$$d_{cos} = -30$$

$$y = 4\cos[3(x + 30)] - 1 \qquad y = 4\sin[3(x - 60)] - 1$$

Example 2: A sinusoidal function has an amplitude of 3 units, a period of 180 degrees and a max point at (0, 5). Represent the function with an equation in two different ways.



$$a = 3$$

$$k = \frac{360}{period} = \frac{360}{180} = 2$$

$$c = max - |a| = 5 - 3 = 2$$

$$d_{cos} = 0$$

$$d_{sin} = d_{cos} - \frac{90}{|k|} = 0 - \frac{90}{2} = -45$$

$$y = 3\cos(2x) + 2 \qquad y = 3\sin[2(x + 45)] + 2$$

Example 3: A sinusoidal function has an amplitude of 5 units, a period of 120 degrees and a maximum at (0, 3). Represent the function with an equation in two different ways.

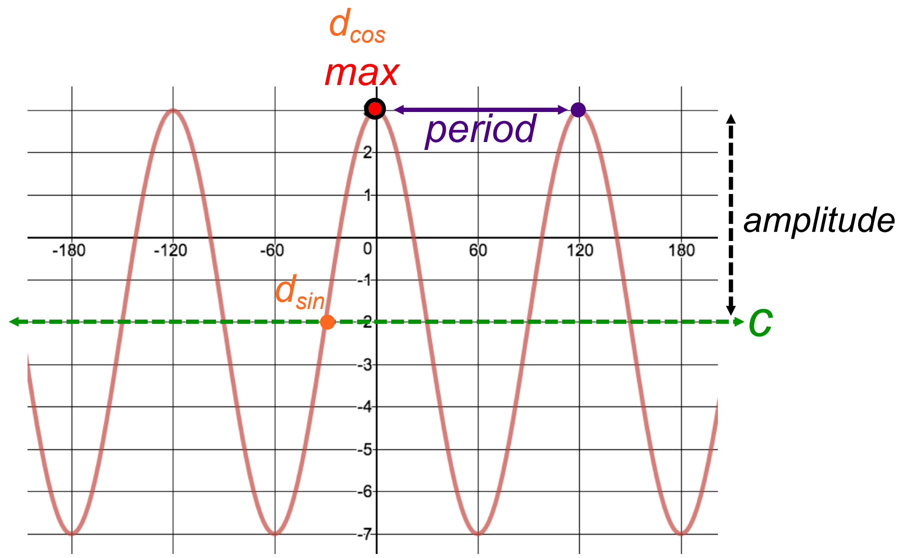
$a = 5$

$k = \frac{360}{period} = \frac{360}{120} = 3$

$c = max - |a| = 3 - 5 = -2$

$d_{cos} = 0$

$d_{sin} = d_{cos} - \frac{90}{|k|} = 0 - \frac{90}{3} = -30$



$y = 5\cos(3x) - 2$

$y = 5\sin[3(x + 30)] - 2$