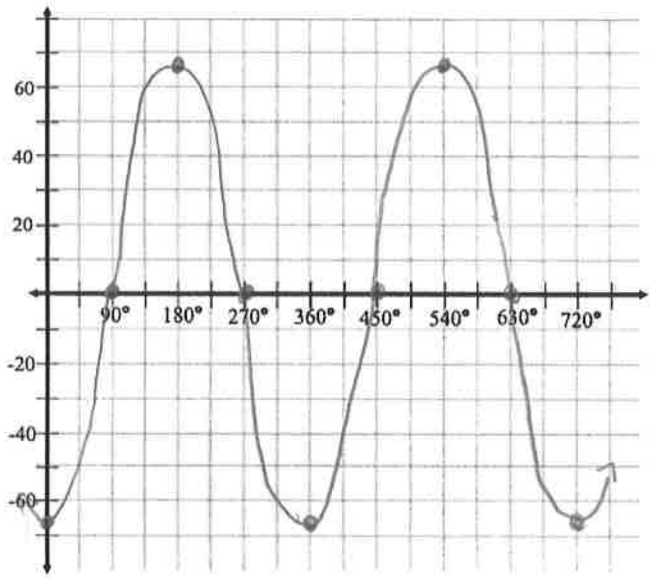
5.5/5.6 Applications of Sine and Cosine Functions Worksheet #1
MCR3U
Jensen

1) At a maximum height of 135 m, the Millennium Wheel, in London, England, is the largest cantilevered structure in the world. It moves so slowly that there is usually no need to stop the wheel to let people on or off. Let the origin be the center of the wheel.

a) Start a sketch of the **vertical displacement** from the center of the wheel of a car on the wheel as a function of the angle through which the wheel rotates, using the bottom of the wheel as the starting point of the trip. Sketch two cycles.

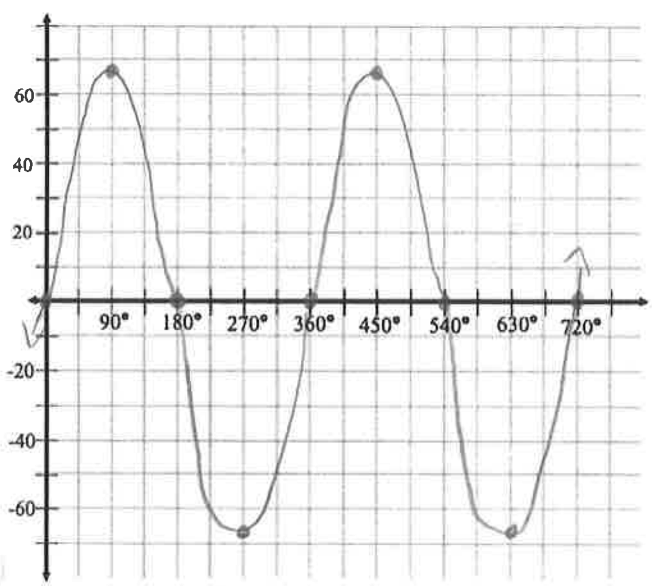


b) Determine the amplitude and period of the function.

$$\text{amplitude} = \frac{67.5-(-67.5)}{2}$$
$$= 67.5$$

period = 360°

2) a) Repeat question 1, except this time graph horizontal displacement instead of vertical displacement.



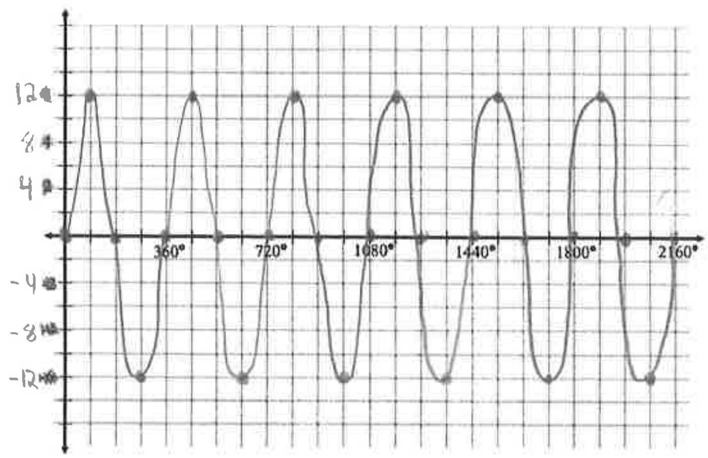
b) Determine the amplitude and period of the function.

a = 67.5

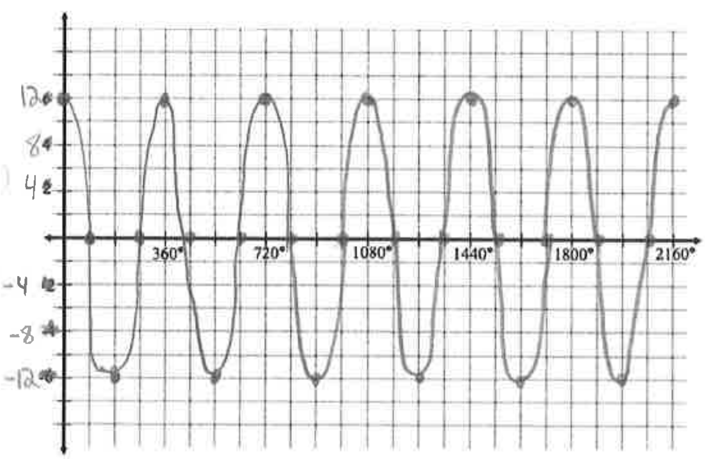
period = 360°

3) The hour hand on a clock has a length of 12cm.

a) Sketch the graph of the vertical position of the tip of the hour hand from the center of the clock versus the angle through which the hand turns for a time period of 72 h. Assume that the hour hand starts at 9.



b) Sketch the graph of the horizontal position of the tip of the hour hand versus the angle through which the hand turns for a time period of 72 h. Assume that the hour hand starts at 3.



c) How many cycles appear in the graph in part a) and b)?

$$\# \text{ of cycles} = \frac{72 \text{ hours}}{\# \text{of hours per cycle}} = \frac{72}{12} = 6 \text{ cycles.}$$

4) A Ferris wheel has a diameter of 20 m and is 4 m above ground level at its lowest point. Assume that a rider enters a car from a platform that is located 30° around the rim before the car reaches its lowest point.

a) Model the rider's height above the ground versus angle using a transformed sine function

$a = \frac{20}{2} = 10$ $\quad k = \frac{360}{360} = 1$ $\quad$ d-value: must rotate $30+90=120°$ before reaching rising midline; ∴ $d = 120$

$c = \text{max} - \text{amp}$
$= 24 - 10$
$= 14$

$$y = 10\sin(x - 120°) + 14$$

b) Model the rider's height above the ground versus angle using a transformed cosine function.

shift cosine function 90° to right to be equal to sine function.

$y = \sin x = \cos(x - 90)$

∴ $y = 10\sin(x-120°) + 14 = 10\cos(x - 210°) + 14$

OR

$a = 10$
$c = 14$
$k = 1$

$y = 10\cos(x - 210) + 14$

d-value: must rotate $30 + 180 = 210°$ to get to max height. ∴ $d = 210$

c) Suppose that the platform is moved to 60° around the rim from the lowest position of the car. How will the equations in parts a) and b) change? Write the new equations.

Initial position is 30° sooner; ∴ phase shift must increase by 30° for each function

$y = 10\sin(x - 150) + 14$

$y = 10\cos(x - 240) + 14$

5) Suppose that the center of the Ferris wheel in the previous question is moved upward 2 m, but the platform is left in place at a point 30° before the car reaches its lowest point. How do the equations in parts a) and b) change? Write the new equations.

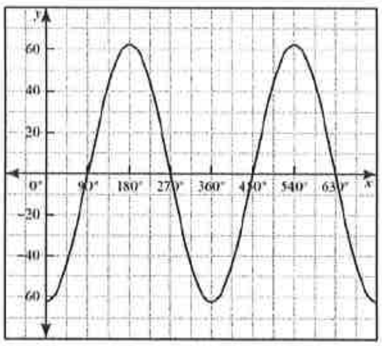
Vertical shift increases by 2; ∴ $c = 16$

$y = 10\sin(x - 120) + 16$

$y = 10\cos(x - 210) + 16$

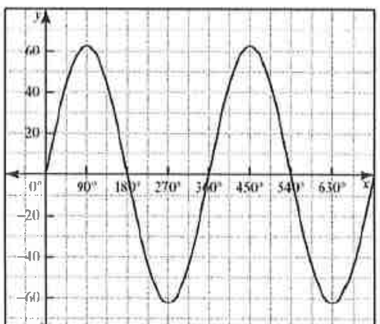
Answers

1) a)



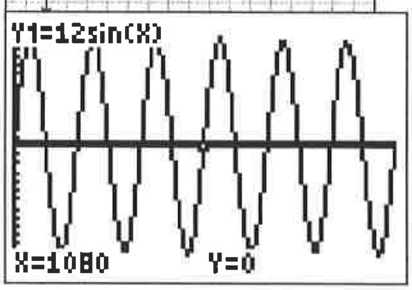
b) amplitude 67.5; period 360°

2) a)

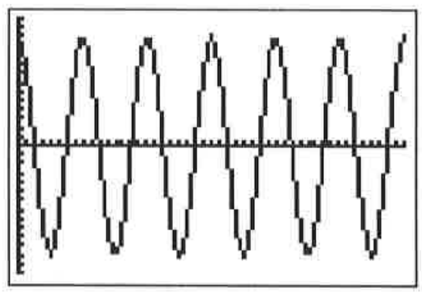


b) amplitude 67.5; period 360°

3) a)



b)



c) The graphs in part a) and b) both show 6 full cycles because there are six 12-h periods in 72 h.

4) a) The amplitude is 10 m and the midline is at 14 m. If the rider begins her ride 30° before the minimum, then she will reach the rising midline point after a rotation of 120° for a phase shift of 120° to the right. The period is 360°. An equation that models the rider's height versus the rotation angle is
$y = 10 \sin (x - 120°) + 14$.

b) For a cosine function, the rider must rotate 210° to reach the first maximum point. This requires a phase shift of 210° to the right. The other parameters remain the same. An equation that models the rider's height versus the rotation angle is $y = 10 \cos (x - 210°) + 14$.

c) If the initial position is placed 30° sooner, then the phase shift of both curves must increase by 30°.
A new sine equation that models the rider's height versus the rotation angle is $y = 10 \sin (x - 150°) + 14$.
A new cosine equation that models the rider's height versus the rotation angle is
$y = 10 \cos (x - 240°) + 14$.

5) If the centre of the Ferris wheel is raised by 2 m, then the vertical shift also increase by 2 from 14 to 16. The relative position of the platform does not change, so the phase shift is not affected.

a) A new sine equation that models the rider's height versus the rotation angle is $y = 10 \sin (x - 120°) + 16$.

b) A new cosine equation that models the rider's height versus the rotation angle is
$y = 10 \cos (x - 210°) + 16$.