5.5/5.6 Application of Sine and Cosine Functions Worksheet #2

MCR3U
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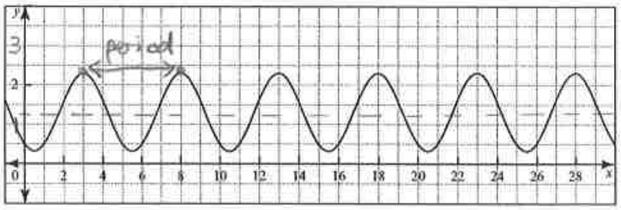
SOLUTIONS

1) A motion sensor recorded the motion of a child on a swing. The data was graphed, as shown.



a) Find the max and min values.

max = 2.25 min = 0.25

b) Find amplitude $a = \frac{2.25 - 0.25}{2} = 1$

c) Determine the vertical shift of the function.

C = max - amp = 2.25 - 1 = 1.25

d) Find the period of the function

period = 8 - 3 = 5

e) Determine the phase shift, if the motion were to be modelled using a sine function.

rising midline is $\frac{90}{k} = \frac{90}{72} = 1.25$ to the left of the max.

$\therefore d = 3 - 1.25 = 1.75$

2) The height of the blade of a wind turbine as it turns through an angle of θ is given by the function $h(\theta) = 8.5 \sin(\theta + 180°) + 40$, with height measured in metres.

a) Find the maximum and minimum positions of the blade.

max = 40 + 8.5 = 48.5 m min = 40 - 8.5 = 31.5 m

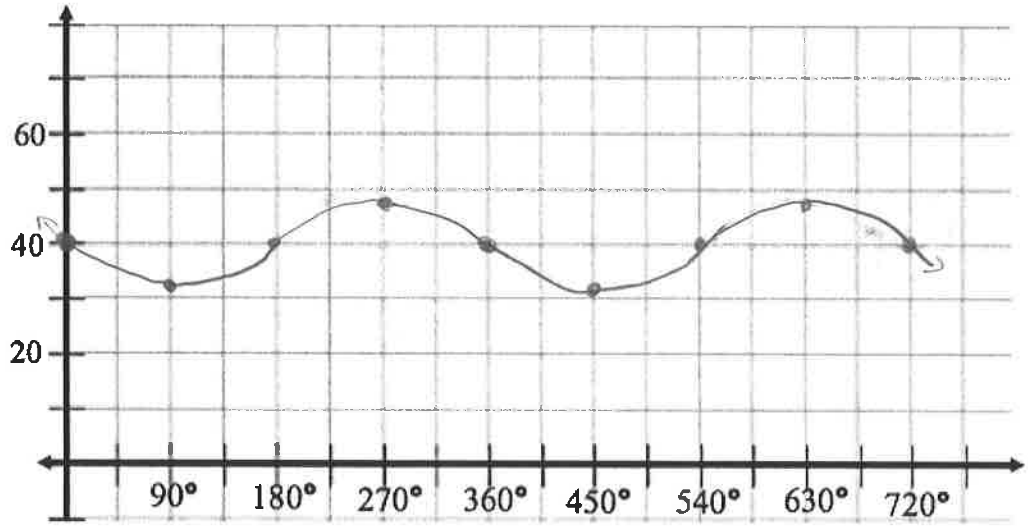
b) Explain what the value of 40 in the equation represents.

Vertical shift; OR height of the center of the turbine.

c) Explain what the value of the amplitude represents.

Length of the blade.

d) Sketch the function over two cycles.



3) The height, h, in meters, of the tide in a given location on a given day at t hours after midnight can be modeled using the sinusoidal function $h(t) = 5\sin[30(t-5)] + 7$.

a) Find the max and min values for the depth of water.

Max = 7+5 = 12 m

Min = 7−5 = 2 m

b) What time is high tide? What time is low tide?

rising midline for sine function is moved 5 units right.

Max = $5 + \frac{90}{k} = 5 + \frac{90}{30} = 8$ (8:00 am)

Min = $5 - \frac{90}{k} = 5 - \frac{90}{30} = 2$ (2:00 am)

Because of the 12 hour period, there is also a max at 8pm and min at 8am.

c) What is the depth of the water at 9:00 am?

$h(9) = 5\sin[30(9-5)] + 7$

$= 5\sin(120) + 7$

$= 11.3$ m

d) Find all the times during a 24-h period when the depth of the water is 3 meters.

$3 = 5\sin[30(t-5)] + 7$

$\frac{-4}{5} = \sin[30(t-5)]$

$30(t-5) = \sin^{-1}\left(\frac{-4}{5}\right)$

$t-5 = \frac{-53.13}{30}$

→ $t-5 = -1.77$

$t = 3.23$

about 3 hours 14 minutes

This is 1 hour 14 mins after the min; so the height will be the same 1 hour and 14 mins before that; at 12:46 am.

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h(3) at 12:46 am and p.m. (because of 12 hour period)

and

3:14 am and 3:14 p.m.

4) The population, P, of a lakeside town with a large number of seasonal residents can be modeled using the function $P(t) = 5000\sin[30(t-7)] + 8000$, where t is the number of months after New Year's Day.

a) Find the max and min values for the population over a whole year.

max = 8000 + 5000 = 13 000

min = 8000 - 5000 = 3000

b) When is the population a maximum? When is it a minimum?

max of sine at $\frac{90}{k} = \frac{90}{30} = 3$ but you must shift 7 to the right.

min at $-\frac{90}{k} = \frac{-90}{30} = -3$

& max 10 months after New Years and min 4 months after New Years.

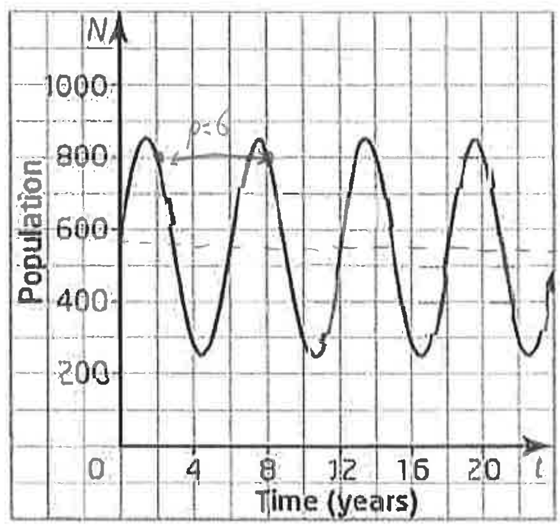
c) What is the population on September 30th?

$P(9) = 5000\sin[30(9-7)] + 8000$

$= 5000\sin(60) + 8000$

$= 12\,330$

5) The population of prey in a predator-prey relation is shown. Time is in years since 1985.



a) Determine the max and min values of the population, to the nearest 50. Use these to find the amplitude.

max = 850

min = 250

$a = \frac{850-250}{2}$

$a = 300$

b) Determine the vertical shift, c.

c = max - amp

c = 850 - 300

c = 550

c) Determine the phase shift, d.

$d = 0$

d) Determine the period. Use the period to determine the value of k.

Period = 8 - 2 = 6

$k = \frac{360}{6} = 60$

e) Model the population versus time with a sinusoidal function.

$P = 300\sin(60t) + 550$

6) The number of millions of visitors that a tourist attraction gets can be modeled using the equation $y = 2.3\sin[30(x+1)] + 4.1$, where $x = 1$ represents January, $x = 2$ represents February, and so on.

a) Determine the period of the function and explain its meaning.

Period $= \frac{360}{30} = 12$ 12 months.

b) Graph the function for 12 months.

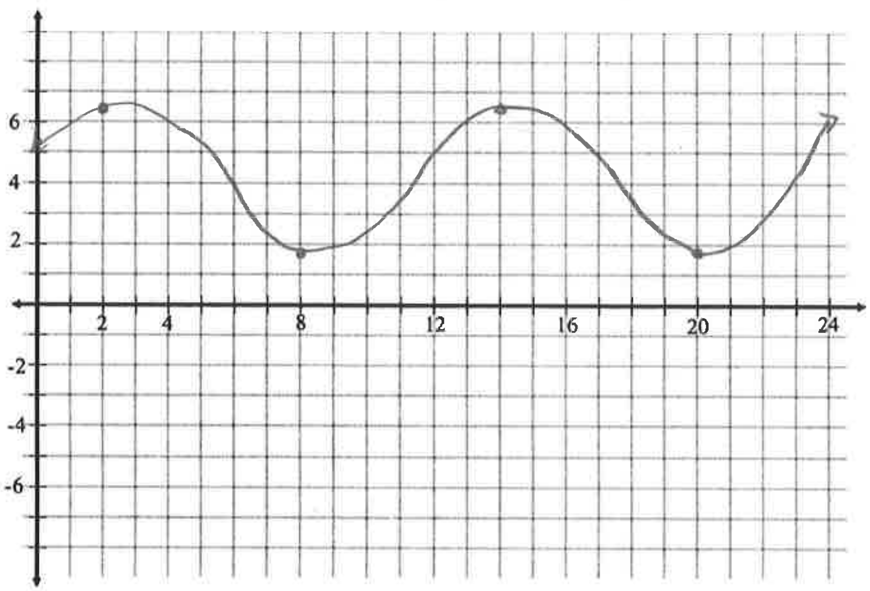
max = 4.1 + 2.3 = 6.4

min = 4.1 − 2.3 = 1.8

Rising midline at −1

First max at $-1 + \frac{90}{30} = -1+3 = 2$

first min is $\frac{180}{30} = 6$ after max.



c) Which month has the most visitors?

month 2: February

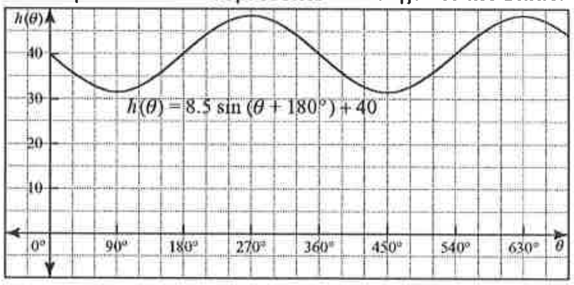
d) Which month has the least visitors?

Month 8: August

Answers

1) a) maximum 2.25, minimum 0.25
b) amplitude 1
c) vertical shift up 1.25
d) period 5
e) horizontal shift 1.75 to the right

2) a) maximum 48.5, minimum 31.5
b) The height of the center of the turbine
c) The amplitude of 8.5 represents the length of the blade.
d)



3) a) From the equation, $c = 7$ and $a = 5$, so the function has a midline value of 7 and an amplitude of 5. The maximum height is 12 m and the minimum height is 2 m.

b) From the equation, $k = 30$ and $d = 5$, so the period is 12 h and the phase shift is 5 h right. The first midline value occurs at 5:00 a.m. The first maximum occurs one-quarter period, or 3 h after this, at 8:00 a.m. The previous minimum is 3 h prior to 5:00 a.m., at 2:00 a.m. Because of the 12-h period, there will also be a maximum at 8:00 p.m. and a minimum at 2:00 p.m.

c) 11.3 m

d) The solution gives a time of approximately 3:14 a.m. This time is 1 h 14 min after the first minimum so the depth should also occur 1 h 14 min before 2:00 a.m, at 12:46 a.m. Because of the 12-h period, the depth will also occur at 12:46 p.m. and 3:14 p.m.

4) a) From the equation, $c = 8000$ and $a = 5000$, so the function has a midline value of 8000 and an amplitude of 5000. The maximum population is 13 000 and the minimum population is 3000.

b) From the equation, $k = 30$ and $d = 7$, so the period is 12 months and the phase shift is 7 months right. The initial midline value occurs at $t = 7$. The maximum occurs 3 months later at $t = 10$ (October) and the minimum 3 months earlier at $t = 4$ (April).

c) 12 330

5) a) From the graph the maximum population is approximately 850 animals and the minimum population is approximately 250 animals.The amplitude is approximately 300 animals, so $a = 300$.

b) The vertical shift is the maximum value minus the amplitude, so $c = 550$.

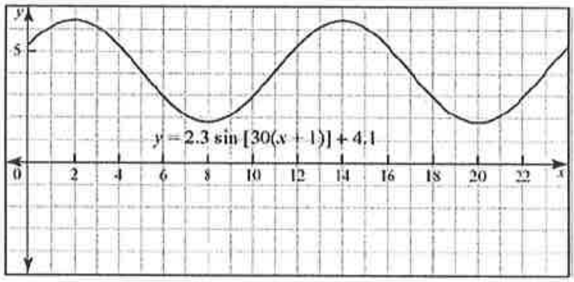
c) The midline intersects the graph at $t = 0$ so no horizontal shift is necessary, so $d = 0$.

d) The pattern repeats every 6 years, so the period is 6 years. $k=60$.

e) A sine function that models the population of prey, N, with respect to time, t, is $N = 300 \sin 60t + 550$.

6) a) 12 months

b)



c) February

d) August