L5 – Trig Applications Part 1
MCR3U
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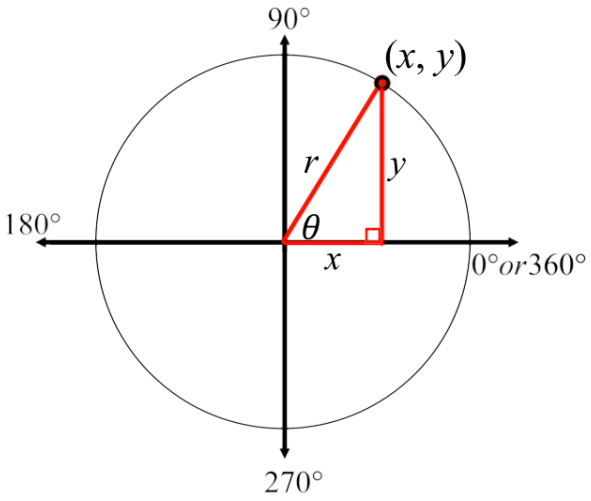
Before we do application questions, it will be good to know the connection between what we learned last chapter and the functions from this chapter:

Desmos - Sine Graph Desmos - Cosine Graph

Section 1: Remembering the Unit Circle

The circle being used has radius r. The radius and the coordinates of a point on the circle (x, y) are related to the primary trig ratios. Study the circle and write expressions for $\sin\theta$, $\cos\theta$, and $\tan\theta$ in terms of x, y, and r.

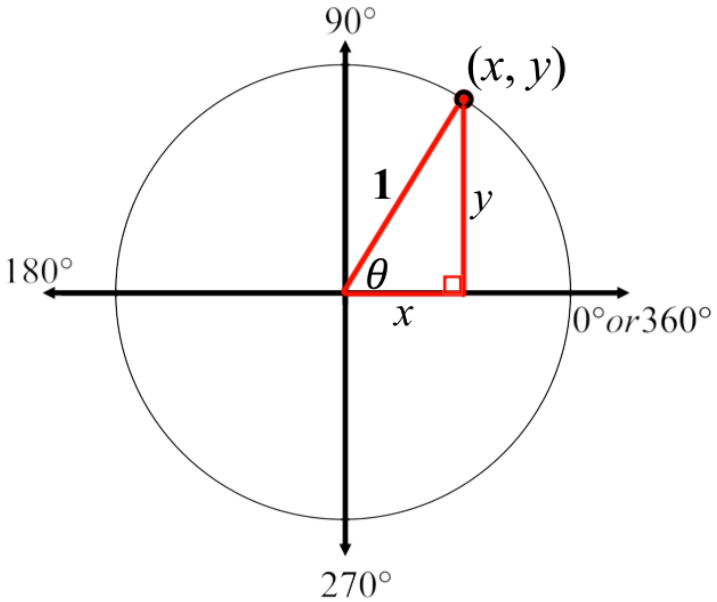


$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

A UNIT CIRLCE has a radius of 1. Use the unit circle to write expressions for $\sin\theta$, $\cos\theta$, and $\tan\theta$ in terms of x, y, and r.



$$\sin\theta = y$$

$$\cos\theta = x$$

$$\tan\theta = \frac{y}{x}$$

Summary of findings for trig ratios using the unit circle:

The sine function:

graphs the relationship between the angle and the **VERTICAL** displacement from the *x*-axis.

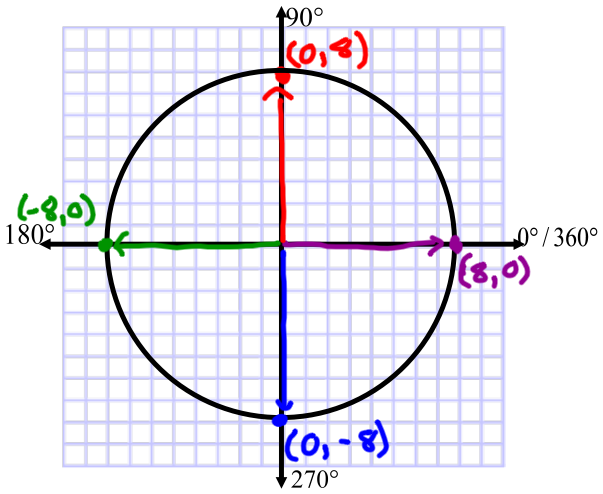
The cosine function:

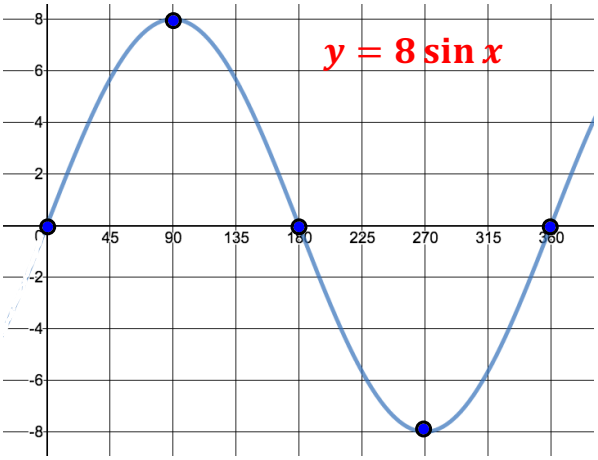
graphs the relationship between the angle and the **HORIZONTAL** displacement from the *y*-axis.

Section 2: Modeling with Graphs

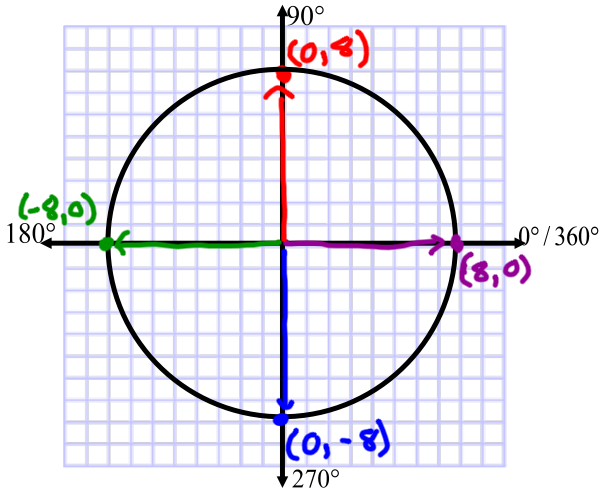
Example 1: You are in a car of a Ferris wheel. The wheel has a radius of 8m and turns counterclockwise. Let the origin be at the center of the wheel. Begin your sketch when the radius from the center of the wheel to your car is along the positive x-axis.

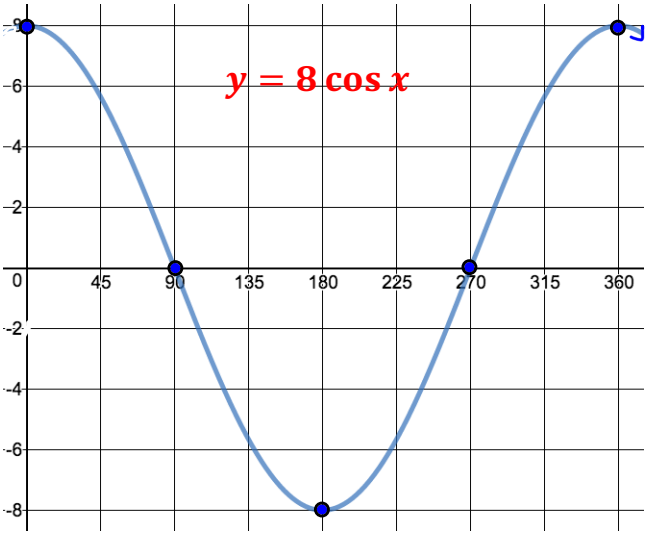
a) Sketch the graph of **vertical displacement** versus the angle of rotation for 1 complete rotation.



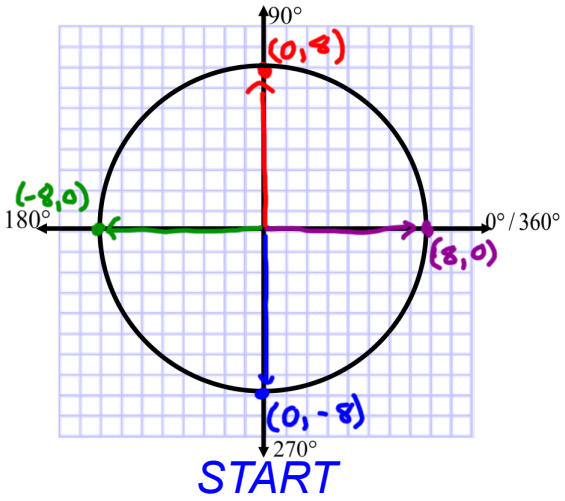


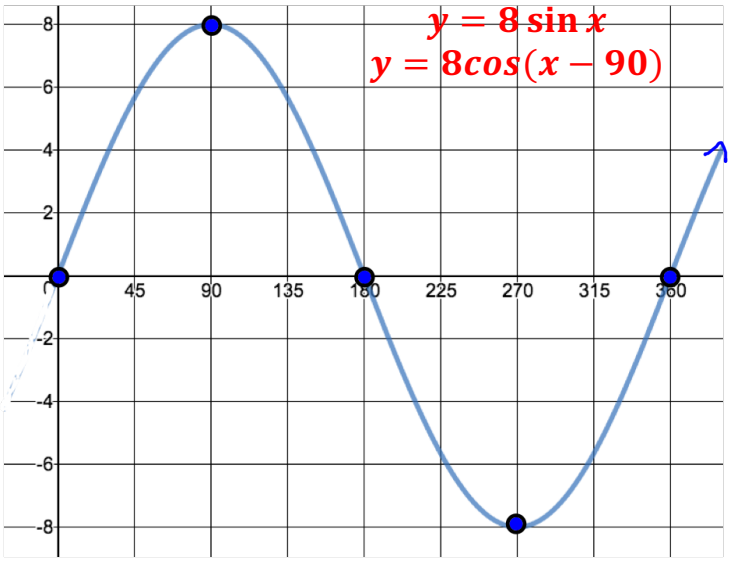
b) Sketch the graph of **horizontal displacement** versus the angle of rotation for 1 complete rotation starting along the positive x-axis.





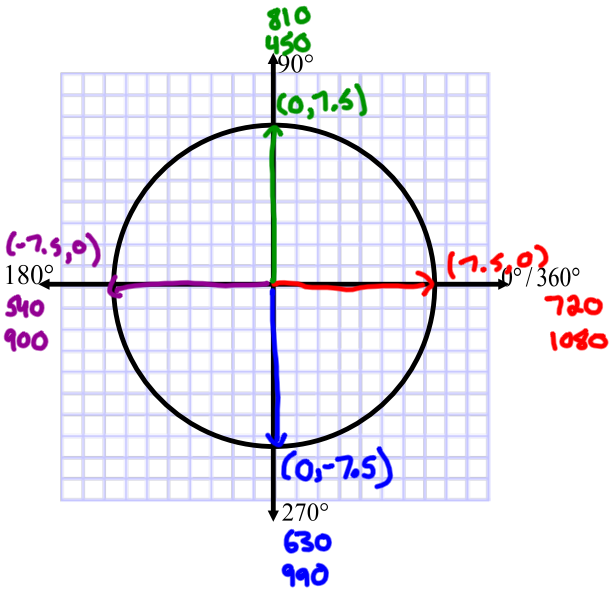
c) Sketch the graph of **<u>horizontal displacement</u>** versus the angle of rotation for 1 complete rotation if your car starts at the **bottom** of the Ferris Wheel.

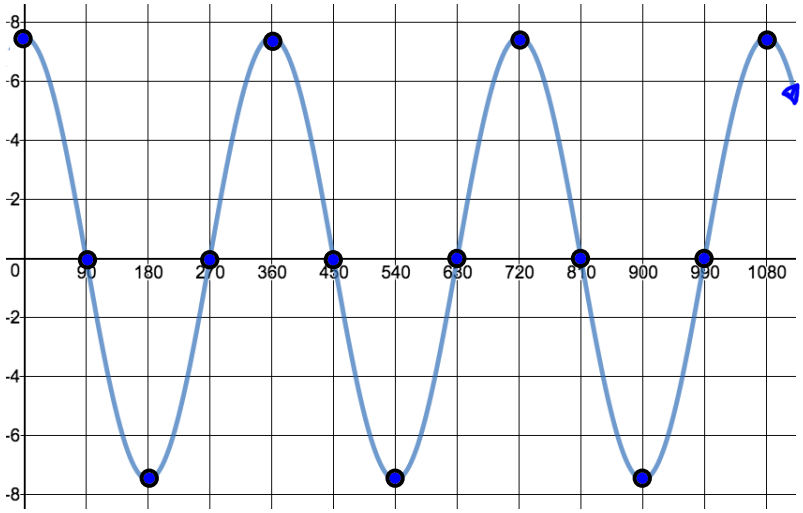




Example 2: A carousel rotates at a constant speed. It has a diameter of 15m. A horse that is directly in line with the center, horizontally, rotates around 3 full times. Create a graph that models the **<u>horizontal distance</u>** from the center as the horse rotates around.

Note: $radius = \frac{15}{2} = 7.5$

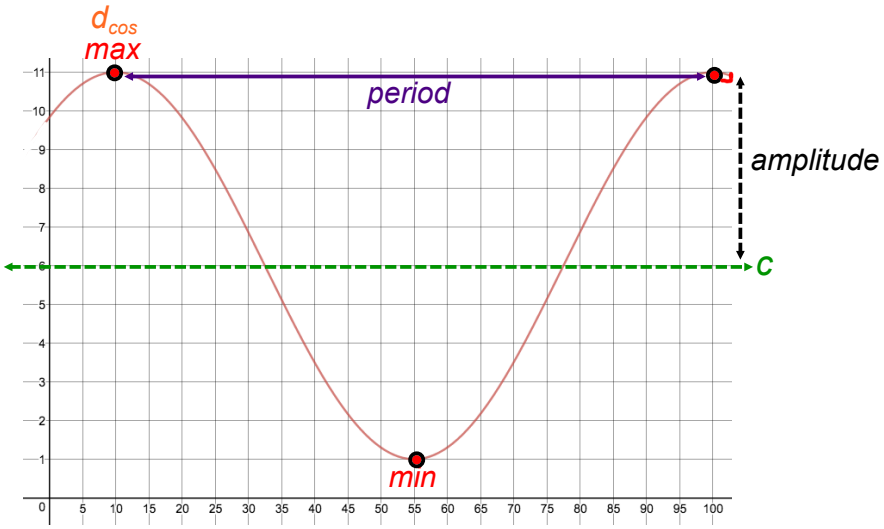




Section 3: Modeling with Equations

Example 3: A group of students is tracking a friend, John, who is riding a Ferris wheel. They know that John reaches the maximum height of 11m at 10 seconds, and then reaches the minimum height of 1m at 55 seconds.

a) Develop an equation of a sine and cosine function that models John's height above the ground.



$$a = \frac{max - min}{2} = \frac{11 - 1}{2} = 5$$

$$k = \frac{360}{period} = \frac{360}{100 - 10} = \frac{360}{90} = 4$$

$$c = max - |a| = 11 - 5 = 6$$

$$d_{cos} = 10$$

$$d_{sin} = d_{cos} - \frac{90}{|k|} = 10 - \frac{90}{4} = -12.5$$

$$y = 5\cos[4(x - 10)] + 6 \qquad y = 5\sin[4(x + 12.5)] + 6$$

b) What is John's height above the ground after 78 seconds?

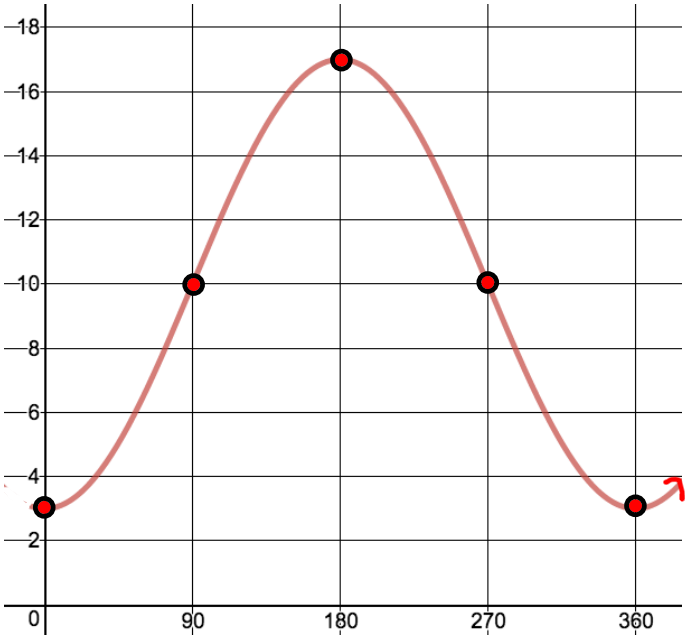
$$y = 5\cos[4(78 - 10)] + 6$$

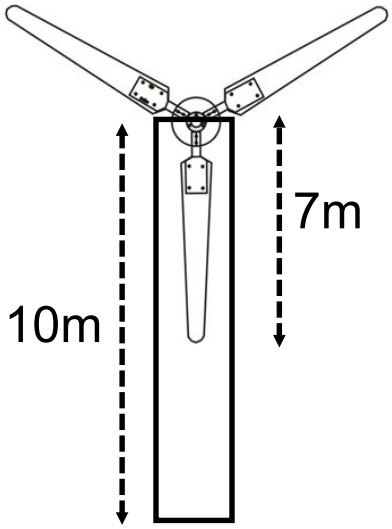
$$y = 5\cos[272] + 6$$

$$y = 6.2\text{m}$$

Example 4: Don Quixote, a fictional character in a Spanish novel, attacked windmills because he thought they were giants. At one point, he got snagged by one of the blades and was hoisted into the air. The center of the windmill is 10 meters off the ground and each blade is 7 meters long. The blade picked him up when it was at its lowest point.

a) Graph Don's height above the ground during one full rotation around the windmill





b) Determine an equation for a sine and cosine function that represents his height above the ground in relation to the angle of rotation.

$$a = \frac{max - min}{2} = \frac{17 - 3}{2} = 7$$

$$k = \frac{360}{period} = \frac{360}{360} = 1$$

$$c = max - |a| = 17 - 7 = 10$$

$$d_{cos} = 180$$

$$d_{sin} = 90$$

$$y = 7\cos(x - 180) + 10 \qquad y = 7\sin(x - 90) + 10$$