L6 – Trig Applications Part 2
MCR3U
Jensen

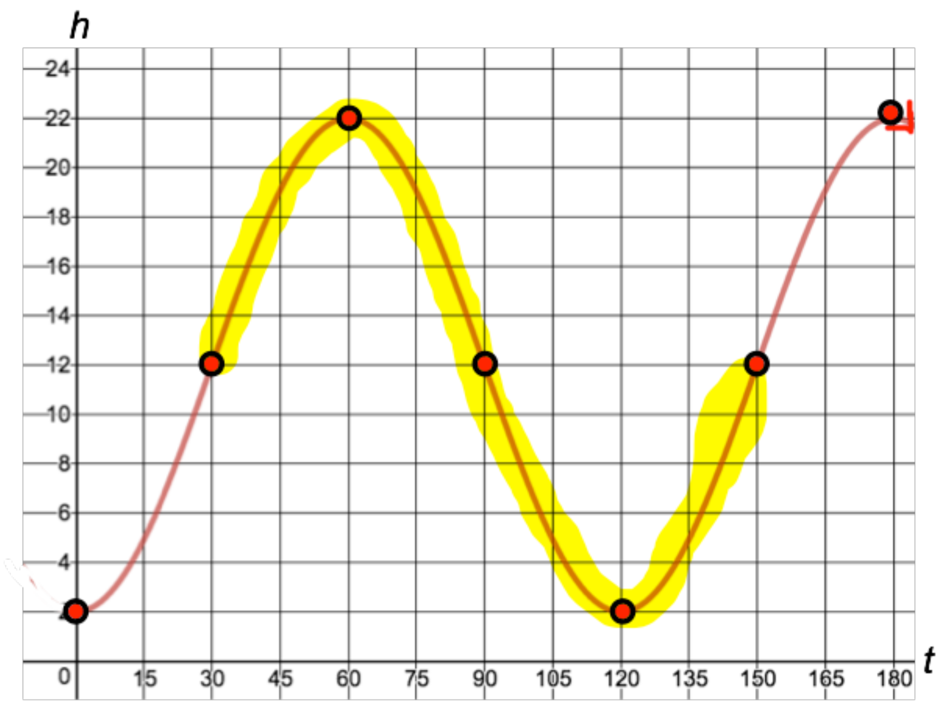
Example 1: The height, h, in meters, above the ground of a rider on a Ferris wheel after t seconds can be modelled by the sine function:

$$h(t) = 10\sin[3(t-30)] + 12$$

a) Graph the function using transformations

$y = \sin x$	
x	y
0	0
90	1
180	0
270	-1
360	0

$h(t) = 10\sin[3(t-30)] + 12$	
$\frac{x}{3} + 30$	$10y + 12$
30	12
60	22
90	12
120	2
150	12



b) Determine the max height, min height, and time for one revolution.

$max = 22$ m
$min = 2$ m
$period = 150 - 30 = 120$ seconds

c) Represent the function using the equation of a cosine function

$a = 10$

$k = 3$

$c = 12$

$$d_{cos} = d_{sin} + \frac{90}{|k|} = 30 + \frac{90}{3} = 60$$

$$\boldsymbol{h(t) = 10\cos[3(t - 60)] + 12}$$

d) What is the height of the rider after 35 seconds? Use both equations to verify your answer.

$$\boldsymbol{h(35) = 10\cos[3(35 - 60)] + 12}$$

$$\boldsymbol{h(35) = 10\cos[-75] + 12}$$

$$\boldsymbol{h(35) = 14.6\ \mathrm{m}}$$

$$\boldsymbol{h(35) = 10\sin[3(35 - 30)] + 12}$$

$$\boldsymbol{h(35) = 10\sin[15] + 12}$$

$$\boldsymbol{h(35) = 14.6\ \mathrm{m}}$$

Example 2: Skyscrapers sway in high-wind conditions. In one case, at $t = 2$ seconds, the top floor of a building swayed 30 cm to the left (-30 cm) and at $t = 12$ seconds, the top floor swayed 30 cm to the right (+30 cm) of its starting position.

a) What is the equation of a cosine function that describes the motion of the building in terms of time?

$$a = \frac{max - min}{2} = \frac{30 - (-30)}{2} = 30$$

$$k = \frac{360}{period} = \frac{360}{20} = 18$$

$c = max - |a| = 30 - 30 = 0$

$d_{cos} = 12$

$$y = 30\cos[18(t - 12)]$$

b) What is the equation of a sine function that describes the motion of the building in terms of time?

$$d_{sin} = d_{cos} - \frac{90}{|k|} = 12 - \frac{90}{18} = 7$$

$$y = 30\cos[18(t - 7)]$$

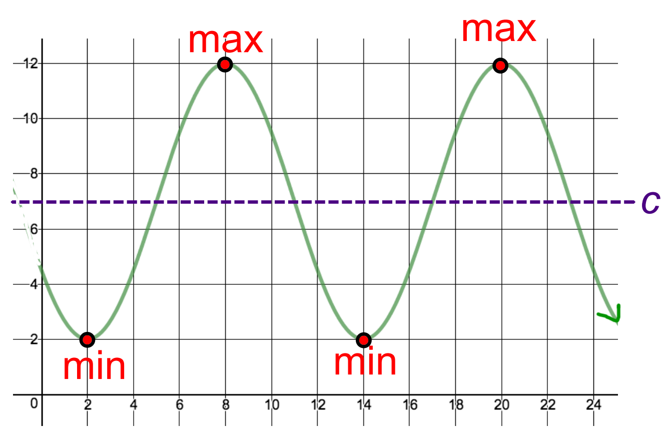
Example 3: The height of the tide on a given day at $'t'$ hours after midnight is modelled by:

$$h(t) = 5\sin[30(t - 5)] + 7$$

a) Find the max and min values for the height of the depth of the water.

$max = c + |a| = 7 + 5 = 12$

$min = c - |a| = 7 - 5 = 2$



b) What time is high tide? What time is low tide?

Note: $period = \frac{360}{k} = \frac{360}{30} = 12$; therefore there are 2 cycles in a 24 hour period.

The first rising midline is at $t = 5$. A max will occur $\frac{90}{k}$ to the right of the rising midline.

Therefore, there is a max at $5 + \frac{90}{k} = 5 + \frac{90}{30} = 8$. There will be another high tide in 12 hours (since this is the period of the function).

High tide = 8am AND 8 pm

The first rising midline is at $t = 5$. A min will occur $\frac{90}{k}$ to the left of the rising midline.

Therefore, there is a min at $5 - \frac{90}{k} = 5 - \frac{90}{30} = 2$. There will be another high tide in 12 hours (since this is the period of the function).

Low tide = 2am AND 2 pm

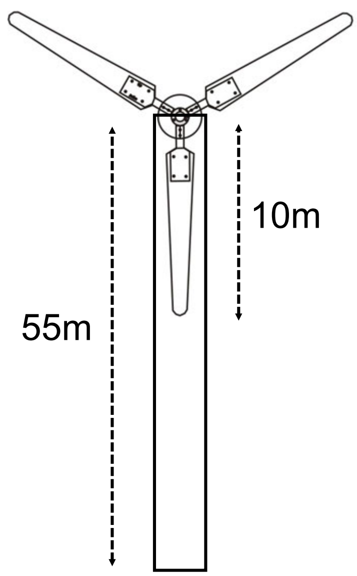
c) What is the depth of the water at 9 am?

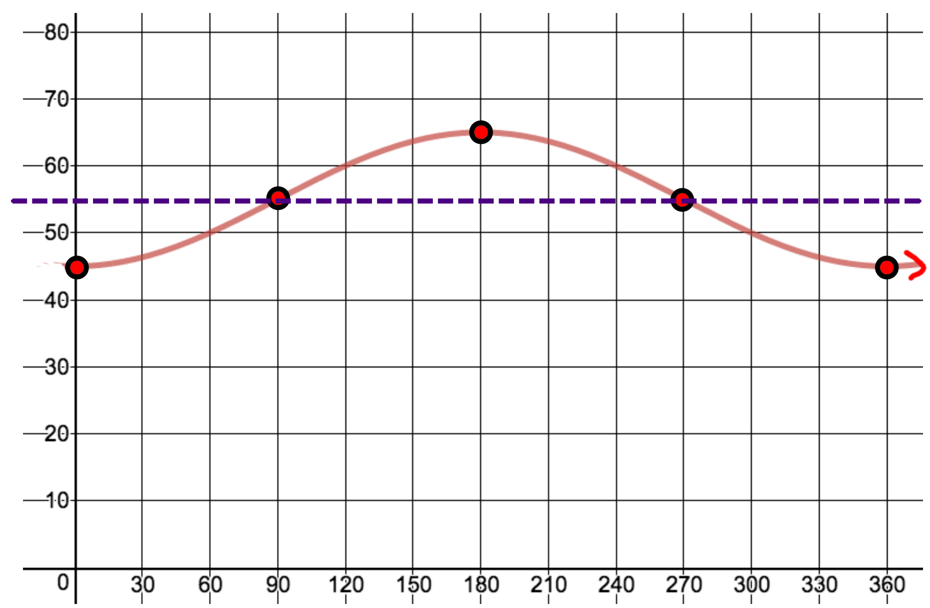
$h(9) = 5\sin[30(9-5)] + 7$

$h(9) = 5\sin[120] + 7$

$h(9) = 11.3 \text{ m}$

Example 4a: A wind turbine has a height of 55m from the ground to the center of the turbine. Graph one cycle of the vertical displacement of a 10m blade turning counterclockwise. Assume the blade starts pointing straight down.





Example 4b: Model the rider's height above the ground versus angle using a transformed sine and cosine function.

$$a = \frac{max - min}{2} = \frac{65-45}{2} = 10$$

$$k = \frac{360}{period} = \frac{360}{360} = 1$$

$$c = max - |a| = 65 - 10 = 55$$

$$d_{cos} = 180$$

$$d_{sin} = 90$$

$h = 10\cos(x-180) + 55$ y= $10\sin(x-90) + 55$