## Arithmetic and Geometric Sequences – Worksheet #2

MCR3U

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SOLUTIONS

**1)** For each arithmetic sequence, determine the values of $a$ and $d$. Then, write the next four terms.

**a)** $12, 15, 18, \ldots$ (+3)

$a = 12$

$d = 3$

12, 15, 18, 21, 24, 27, 30

**b)** $\frac{1}{2}, 1, \frac{3}{2}, \ldots$ (+$\frac{1}{2}$)

$a = \frac{1}{2}$

$d = \frac{1}{2}$

$\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$

**2)** Given the values of $a$ and $d$, write the first three terms of the arithmetic sequence. Then, write the formula for the general term.

**a)** $a = 5, d = 2$

5, 7, 9

$t_n = 5 + (n-1)(2)$

**b)** $a = \frac{3}{4}, d = \frac{1}{2}$

$\frac{3}{4}, \frac{5}{4}, \frac{7}{4}$

$t_n = \frac{3}{4} + (n-1)\left(\frac{1}{2}\right)$

**3)** Given the formula for the general term of an arithmetic sequence, determine $t_{12}$.

**a)** $t_n = 1 - 4n$

$t_{12} = 1 - 4(12)$

$= 1 - 48$

$= -47$

**b)** $t_n = \frac{1}{2}n + \frac{3}{2}$

$t_{12} = \frac{1}{2}(12) + \frac{3}{2}$

$= \frac{12}{2} + \frac{3}{2}$

$= \frac{15}{2}$

**4)** Which term in the arithmetic sequence 9, 4, -1, ... has the value -146? (-5)

$t_n = a + (n-1)d$

$-146 = 9 + (n-1)(-5)$

$-155 = (n-1)(-5)$

$31 = n - 1$

$n = 32$

**5)** Determine the number of terms in each arithmetic sequence

**a)** $38, 36, 34, \ldots, -20$ (−2)

$t_n = a + (n-1)d$

$-20 = 38 + (n-1)(-2)$

$-58 = (n-1)(-2)$

$29 = n - 1$

$n = 30$

**b)** $-5, -8, -11, \ldots, -269$ (−3)

$-269 = -5 + (n-1)(-3)$

$-264 = (n-1)(-3)$

$88 = n - 1$

$n = 89$

**6)** Determine $a$ and $d$ and then write the formula for the $n^{th}$ term of each arithmetic sequence with the given terms.

**a)** $t_{10} = 50$ and $t_{27} = 152$

$t_{10} = a + (10-1)d$ $\quad t_{27} = a + (27-1)d$

① $50 = a + 9d$ $\quad$ ② $152 = a + 26d$

② $152 = a + 26d$

① $50 = a + 9d$ −

$102 = 17d$

$d = 6$

$50 = a + 9(6)$

$50 = a + 54$

$a = -4$

$t_n = -4 + (n-1)(6)$

**b)** $t_5 = -20$ and $t_{18} = -59$

$t_5 = a + (5-1)d$ $\quad t_{18} = a + (18-1)d$

① $-20 = a + 4d$ $\quad$ ② $-59 = a + 17d$

① $-20 = a + 4d$

② $-59 = a + 17d$ −

$39 = -13d$

$d = -3$

$-20 = a + 4(-3)$

$-20 = a - 12$

$-8 = a$

$t_n = -8 + (n-1)(-3)$

**7)** In a lottery, the owner of the first ticket drawn receives \$10 000. Each successive winner receives \$500 less than the previous winner.

**a)** How much does the 10th winner receive?

$t_n = 10000 + (n-1)(-500)$

$t_{10} = 10000 + (10-1)(-500)$

$= \$5500$

**b)** How many winners are there in total?

$500 = 10000 + (n-1)(-500)$

$-9500 = (n-1)(-500)$

$19 = n - 1$

$n = 20$

∴ 20 winners.

**8)** At the end of the second week after opening, a new fitness club as 870 members. At the end of the seventh week, there are 1110 members. If the increase is arithmetic, how many members were there in the first week?

$t_2 = a+(2-1)d$ ① $870 = a+d$

$t_7 = a+(7-1)d$ ② $1110 = a+6d$

② $1110 = a+6d$
① $870 = a+d$ −
$240 = 5d$
$d = 48$

$870 = a+48$
$a = 822$

∴ There were 822 members in the first week.

**9)** State the common ratio for each geometric sequence and write the next three terms.

**a)** 1, 2, 4, 8, ... (×2)

$r = 2$

1, 2, 4, 8, 16, 32, 64

**b)** -3, 9, -27, 81, ...

$r = -3$

-3, 9, -27, 81, -243, 729, -2187

**c)** $\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, \ldots$

$r = -1$

$\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, \frac{2}{3}$

**d)** 600, -300, 150, -75, ... (×$-\frac{1}{2}$)

$r = -\frac{1}{2}$

600, -300, 150, -75, 37.5, -18.75, 9.375

**10)** For the geometric sequence 54, 18, 6, ... determine the formula for the general term and then find $t_9$.

$a = 54$

$r = \frac{1}{3}$

$t_n = 54\left(\frac{1}{3}\right)^{n-1}$

$t_9 = 54\left(\frac{1}{3}\right)^{9-1}$

$t_9 = \frac{54}{3^8}$

$t_9 = \frac{54}{6561}$

$t_9 = \frac{2}{243}$

**11)** Write the first four terms of each geometric sequence.

**a)** $t_n = 5(2)^{n-1}$

5, 10, 20, 40

**b)** $a = -1, r = \frac{1}{5}$

$-1, -\frac{1}{5}, -\frac{1}{25}, -\frac{1}{125}$

**12)** Determine the number of terms in the geometric sequence 6, 18, 54, ... , 4374. (×3)

$a = 6$

$r = 3$

$t_n = 6(3)^{n-1}$

$4374 = 6(3)^{n-1}$

$729 = 3^{n-1}$

$3^6 = 3^{n-1}$

$6 = n-1$

$n = 7$

**13)** Which term of the geometric sequence 1, 3, 9, ... has a value of 19 683?

×3

$19\,683 = 1(3)^{n-1}$

$19\,683 = 3^{n-1}$

$\log 19683 = \log 3^{n-1}$

$\log 19683 = (n-1)\log 3$

$\frac{\log 19683}{\log 3} = n-1$

$9 = n-1$

$n = 10$

**Answers**

**1) a)** $a = 12, d = 3$; $12, 15, 18, 21, 24, 27, 30$ **b)** $a = \frac{1}{2}, d = \frac{1}{2}$; $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$

**2) a)** $5, 7, 9; t_n = 5 + (n-1)(2)$ **b)** $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}$; $t_n = \frac{3}{4} + (n-1)\left(\frac{1}{2}\right)$

**3) a)** -47 **b)** $\frac{15}{2}$

**4)** $n = 32$

**5) a)** 30 **b)** 89

**6) a)** $t_n = -4 + (n-1)(6)$ **b)** $t_n = -8 + (n-1)(-3)$

**7) a)** \$5500 **b)** 20 winners

**8)** 822 members

**9) a)** $r = 2$ **b)** $r = -3$ **c)** $r = -1$ **d)** $-\frac{1}{2}$

**10)** $\frac{2}{243}$

**11) a)** 5, 10, 20, 40 **b)** -1, $-\frac{1}{5}, -\frac{1}{25}, -\frac{1}{125}$

**12)** 7 terms

**13)** 10th term