

# Arithmetic and Geometric Series - Worksheet #2

MCR3U

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SOLUTIONS

1) The first and last terms in each arithmetic series are given. Determine the sum of the series.

a)  $a = \frac{1}{2}$  and  $t_8 = 4$

$$S_n = \frac{n}{2} (a + t_n)$$

$$S_8 = \frac{8}{2} \left( \frac{1}{2} + 4 \right)$$

$$S_8 = 18$$

b)  $a = 11$  and  $t_{20} = 101$

$$S_{20} = \frac{20}{2} (11 + 101)$$

$$S_{20} = 1120$$

2) Determine the sum of the arithmetic series  $-1 + 2 + 5 + \dots + 164$ .

$$t_n = a + (n-1)d$$

$$164 = -1 + (n-1)(3)$$

$$165 = (n-1)(3)$$

$$55 = n-1$$

$$n = 56$$

$$S_{56} = \frac{56}{2} (-1 + 164)$$

$$= 4564$$

3) The 15<sup>th</sup> term in an arithmetic sequence is 43 and the sum of the first 15 terms of the series is 120. Determine the first three terms of the sequence.

$$t_n = a + (n-1)d$$

$$43 = a + (15-1)d$$

$$\textcircled{1} 43 = a + 14d$$

$$S_{15} = \frac{15}{2} [2a + (15-1)d]$$

$$120 = 7.5(2a + 14d)$$

$$\textcircled{2} 120 = 15a + 105d$$

$$t_n = -27 + (n-1)(5)$$

∴ first three terms

$$\text{are: } -27, -22, -17$$

$$15 \times \textcircled{1} \quad 645 = 15a + 210d$$

$$\textcircled{2} \quad 120 = 15a + 105d$$

$$\underline{525 = 105d}$$

$$d = 5$$

$$43 = a + 14d$$

$$43 = a + 14(5)$$

$$43 - 70 = a$$

$$a = -27$$

4) A toy car is rolling down an inclined track and picking up speed as it goes. The car travels 4 cm in the first second, 8 cm in the second second, 12 cm in the next second, and so on. Determine the total distance travelled by the car in 30 seconds.

$$a = 4$$

$$d = 4$$

$$S_{30} = \frac{30}{2} [2(4) + (30-1)(4)]$$

$$= 15 [8 + 29(4)]$$

$$= 1860$$

∴ The car travels

1860 cm in 30 seconds.

5) For each geometric series, determine the values of  $a$  and  $r$ . Then, determine the indicated sum.

a)  $S_8$  for  $2 + 6 + 18 + \dots$

$$\begin{aligned} a &= 2 \\ r &= 3 \\ S_8 &= \frac{2(3^8 - 1)}{3 - 1} \\ &= \frac{2(6560)}{2} \\ &= 6560 \end{aligned}$$

b)  $S_{10}$  for  $24 - 12 + 6 - \dots$

$$\begin{aligned} a &= 24 \\ r &= -\frac{1}{2} \\ S_{10} &= \frac{24 \left[ \left(-\frac{1}{2}\right)^{10} - 1 \right]}{-\frac{1}{2} - 1} \\ &= \frac{24 \left( \frac{1}{1024} - \frac{1024}{1024} \right)}{-\frac{3}{2}} \\ &= \frac{-24552}{1024} \cdot -\frac{2}{3} \\ &= \frac{49104}{3072} \\ &= \frac{1023}{64} \end{aligned}$$

6) Determine the sum of the geometric series  $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \dots + \frac{128}{6561}$

$$\begin{aligned} a &= \frac{1}{3} \\ r &= \frac{2}{3} \\ t_n &= ar^{n-1} \\ \frac{128}{6561} &= \frac{1}{3} \left(\frac{2}{3}\right)^{n-1} \\ \frac{384}{6561} &= \left(\frac{2}{3}\right)^{n-1} \\ \frac{128}{2187} &= \left(\frac{2}{3}\right)^{n-1} \\ \left(\frac{2}{3}\right)^7 &= \left(\frac{2}{3}\right)^{n-1} \\ 7 &= n-1 \\ n &= 8 \\ S_8 &= \frac{\left(\frac{1}{3}\right) \left[ \left(\frac{2}{3}\right)^8 - 1 \right]}{\frac{2}{3} - 1} \\ &= \frac{\frac{1}{3} \left( \frac{256}{6561} - \frac{6561}{6561} \right)}{-\frac{1}{3}} \\ &= -1 \left( \frac{-6305}{6561} \right) \\ &= \frac{6305}{6561} \end{aligned}$$

7) Determine the sum of the geometric series  $5 - 15 + 45 - \dots + 3645$

$$\begin{aligned} a &= 5 \\ r &= -3 \\ t_n &= ar^{n-1} \\ 3645 &= 5(-3)^{n-1} \\ 729 &= (-3)^{n-1} \\ (-3)^6 &= (-3)^{n-1} \\ 6 &= n-1 \\ n &= 7 \\ S_7 &= \frac{5 \left[ (-3)^7 - 1 \right]}{-3 - 1} \\ &= \frac{5(-2188)}{-4} \\ &= 2735 \end{aligned}$$

8) The sum of  $4 + 12 + 36 + 108 + \dots + t_n$  is 4372. How many terms are in this series?

$$a = 4$$

$$r = 3$$

$$4372 = \frac{4(3^n - 1)}{3 - 1}$$

$$8744 = 4(3^n - 1)$$

$$2186 = 3^n - 1$$

$$2187 = 3^n$$

$$3^7 = 3^n$$

$$n = 7$$

9) The third term of a geometric series is 24 and the fourth term is 36. Determine the sum of the first 10 terms.

$$t_3 = ar^{3-1}$$

$$24 = ar^2$$

$$a = \frac{24}{r^2}$$

$$t_4 = ar^{4-1}$$

$$36 = ar^3$$

$$36 = \left(\frac{24}{r^2}\right)r^3$$

$$36 = 24r$$

$$r = \frac{36}{24}$$

$$r = \frac{3}{2}$$

$$a = \frac{24}{\left(\frac{3}{2}\right)^2}$$

$$= \frac{24}{\left(\frac{9}{4}\right)}$$

$$= 24 \cdot \frac{4}{9}$$

$$= \frac{32}{3}$$

$$S_{10} = \frac{\frac{32}{3} \left[ \left(\frac{3}{2}\right)^{10} - 1 \right]}{\frac{3}{2} - 1}$$

$$= \frac{\frac{32}{3} \left[ \frac{59049}{1024} - \frac{1024}{1024} \right]}{\frac{1}{2}}$$

$$= \frac{164}{3} \left( \frac{58025}{1024} \right)$$

$$= \frac{58025}{48}$$

### Answers

1) a) 18 b) 1120

2) 4564

3) -27, -22, -17

4) 1860 cm

5) a)  $S_8 = 6560$  b)  $S_{10} = \frac{1023}{64}$

6)  $S_8 = \frac{6305}{6561}$

7)  $S_7 = 2735$

) 7 terms

9)  $S_{10} = \frac{58025}{48}$

