Arithmetic and Geometric Series – Worksheet

MCR3U
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SOLUTIONS

General formula for an arithmetic series:

$$S_n = \frac{n}{2}[2a+(n-1)d] \quad \underline{\text{OR}} \quad S_n = \frac{n}{2}(a+t_n)$$

General formula for a geometric series:

$$S_n = \frac{a(r^n-1)}{r-1}$$

1) Find the designated sum of the arithmetic series

a) S_{14} of $3 \overset{+4}{+} 7 + 11 + 15 + \cdots$

$S_{14} = \frac{14}{2}[2(3)+(14-1)(4)]$

$= 7[6+13(4)]$

$= 406$

b) S_{11} of $-13 \overset{+2}{-} 11 - 9 - 7 - \cdots$

$S_{11} = \frac{11}{2}[2(-13)+(11-1)(2)]$

$= 5.5[-26+10(2)]$

$= -33$

c) S_9 of $22 \overset{-2}{+} 20 + 18 + 16 + \cdots$

$S_9 = \frac{9}{2}[2(22)+(9-1)(-2)]$

$= 4.5[44+8(-2)]$

$= 126$

d) S_{35} of $-2 \overset{-3}{-} 5 - 8 - 11 - \cdots$

$S_{35} = \frac{35}{2}[2(-2)+(35-1)(-3)]$

$= 17.5[-4+34(-3)]$

$= -1855$

2) Determine the sum of each arithmetic series

a) $6 \overset{+7}{+} 13 + 20 + \cdots + 69$

$t_n = a+(n-1)d$

$69 = 6+(n-1)(7)$

$63 = (n-1)(7)$

$9 = n-1$

$n = 10$

$S_{10} = \frac{10}{2}(6+69)$

$= 5(75)$

$= 375$

b) $4 \overset{+11}{+} 15 + 26 + \cdots + 213$

$213 = 4+(n-1)(11)$

$209 = (n-1)(11)$

$19 = n-1$

$n = 20$

$S_{20} = \frac{20}{2}(4+213)$

$= 10(217)$

$= 2170$

c) $5 \overset{-13}{-} 8 - 21 - \cdots - 190$

$-190 = 5+(n-1)(-13)$

$-195 = (n-1)(-13)$

$15 = n-1$

$n = 16$

$S_{16} = \frac{16}{2}(5-190)$

$= 8(-185)$

$= -1480$

d) $100 \overset{-10}{+} 90 + 80 + \cdots - 100$

$-100 = 100+(n-1)(-10)$

$-200 = (n-1)(-10)$

$20 = n-1$

$n = 21$

$S_{21} = \frac{21}{2}(100-100)$

$= 0$

3) Find the designated sum of the geometric series

a) S_7 of $\overset{\times 2}{4 + 8} + 16 + 32 + \cdots$

$S_7 = \dfrac{4(2^7 - 1)}{2-1}$

$= 508$

b) S_{13} of $\overset{\times -6}{1 - 6} + 36 - 216 + \cdots$

$S_{13} = \dfrac{1[(-6)^{13} - 1]}{-6-1} = 1\,865\,813\,431$

c) S_{17} of $\underbrace{486 + 162}_{\times \frac{1}{3}} + 54 + 18 + \cdots$

$S_{17} = \dfrac{486[(\frac{1}{3})^{17} - 1]}{(\frac{1}{3}) - 1} = 729$

d) S_6 of $\overset{\times 5}{3 + 15} + 75 + 375 + \cdots$

$S_6 = \dfrac{3[(5)^6 - 1]}{5-1} = 11718$

4) Determine S_n for each geometric series

a) $a = 6, r = 2, n = 9$

$S_9 = \dfrac{6[(2)^9 - 1]}{2-1} = 3066$

b) $f(1) = 2, r = -2, n = 12$

$S_{12} = \dfrac{2[(-2)^{12} - 1]}{-2-1} = -2730$

c) $f(1) = 729, r = -3, n = 15$

$S_{15} = \dfrac{729[(-3)^{15} - 1]}{-3-1} = 2\,615\,088\,443$

d) $f(1) = 2700, r = 10, n = 8$

$S_8 = \dfrac{2700((10)^8 - 1)}{10-1} \approx 2.999\,999\,97 \times 10^{10}$

5) If the first term of an arithmetic series is 2, the last term is 20, and the increase constant is +2 …

a) Determine the number of terms in the series

$t_n = a + (n-1)d$

$20 = 2 + (n-1)(2)$

$18 = (n-1)(2)$

$9 = n - 1$

$n = 10$

b) Determine the sum of all the terms in the series

$S_{10} = \frac{10}{2}[2 + 20]$

$= 5(22)$

$= 110$

6) A geometric series has a sum of 1365. Each term increases by a factor of 4. If there are 6 terms, find the value of the first term.

S_6 r n

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1365 = \frac{a[(4)^6 - 1]}{4 - 1}$$

$$1365(3) = a(4^6 - 1)$$

$$4095 = a(4095)$$

$$a = 1$$

Answers

1) a) 406 **b)** -33 **c)** 126 **d)** -1855

2) a) 375 **b)** 2170 **c)** -1480 **d)** 0

3) a) 508 **b)** 1 865 813 431 **c)** 729 **d)** 11 718

4) a) 3066 **b)** -2730 **c)** 2 615 088 483 **d)** $2.999\,999\,97 \times 10^{10}$

5) a) $n = 10$ **b)** $S_{10} = 110$

6) $t_1 = 1$