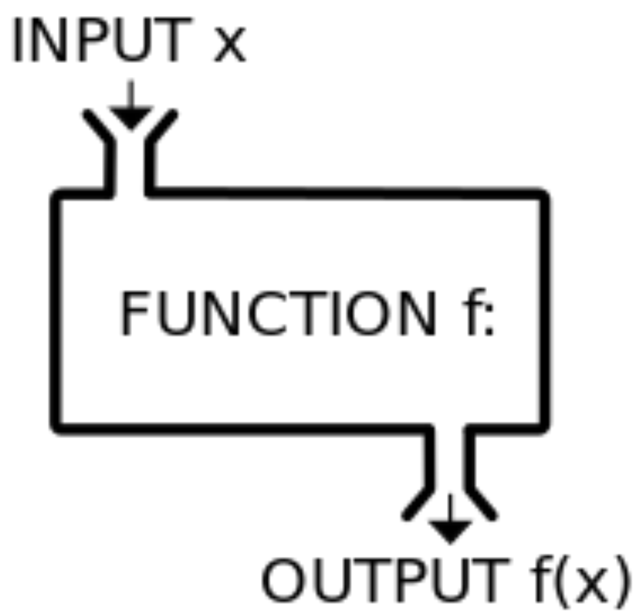
Chapter 1- Functions

Lesson Package

MCR3U



Chapter 1 Outline

Unit Goal: By the end of this unit, you will have an understanding of what a function is and their different representations. You will be able to determine the zeros and the max or min of a quadratic function. You will also be able to simplify expressions involving radicals.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Functions, Domain, and Range	- distinguish a function from a relation - explain meanings of the terms domain and range	A1.1
L2	Function Notation	- explain the meaning of the term function - represent functions using function notation	A1.1, A1.2
L3	Max or Min of a Quadratic	- determine the max or min value of a quadratic function using completing the square and partial factoring	A2.2, A2.3
L4	Radical Expressions	- simplify radical expressions by adding, subtracting, and multiplying	A3.2
L5	Solve Quadratics by Factoring	- Determine the zeros of a quadratic by factoring	A2.1, A2.3
L6	Solve Quadratics using Quaratic Formula	- Determine the zeros of a quadratic using the quadratic formula	A2.1, A2.3
L8	Linear Quadratic Systems	- Solve problems involving the intersection of a linear function and a quadratic function	A2.5

Assessments	**F/A/O**	**Ministry Code**	**P/O/C**	**KTAC**
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Max/Min of Quadratic	F		P	
PreTest Review	F/A		P	
Test - Functions	O	A1.1, A1.2, A2.1, A2.2, A2.3, A2.5, A3.2	P	K(21%), T(34%), A(10%), C(34%)

1.1 Functions, Domain, and Range - Lesson
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Section 1: Relation vs. Function

Definitions

Relation – an identified pattern between two variables that may be represented as a table of values, a graph, or an equation.

Functions – a relation in which each of value of the independent variable (x), corresponds to exactly one value of the dependent variable (y)

Note: *All functions are relations but not all relations are functions. For a relation to be a function, there must be only one 'y' value that corresponds to a given 'x' value.*

Function or Relation Investigation

1) Complete the following tables of values for each relation:

$y = x^2$

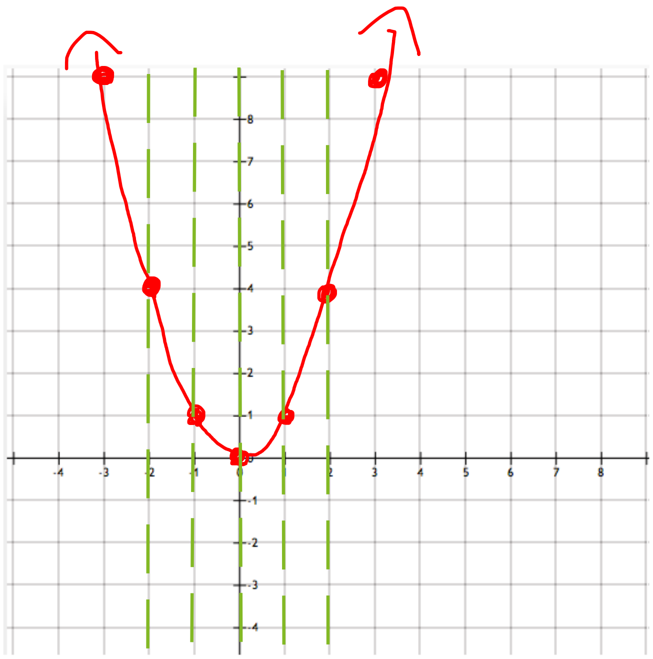
x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$x = y^2$

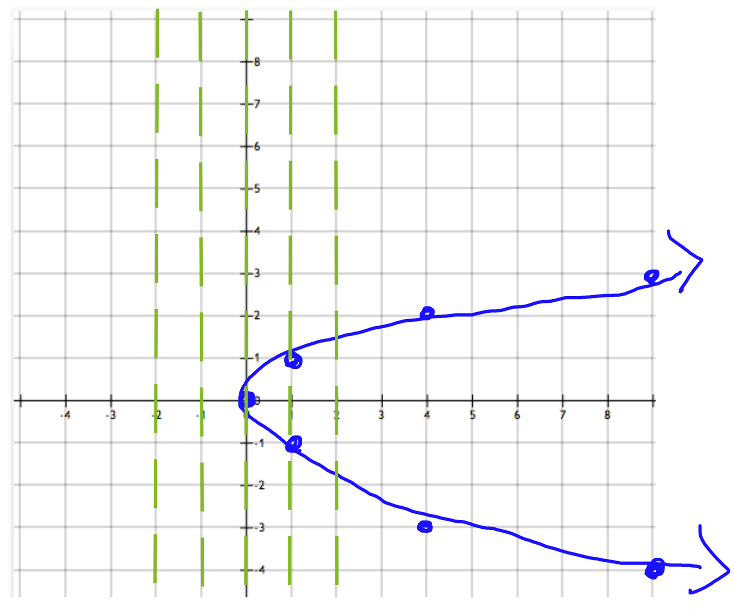
x	y
9	-3
4	-2
1	-1
0	0
1	1
4	2
9	3

2) Graph both relations

$y = x^2$



$x = y^2$



3) Draw the vertical lines $x = -2,\ x = -1,\ x = 0,\ x = 1$, and $x = 2$ on the graphs above.

4) Compare how the lines drawn in step 3 intersect each of the relations. Which relation is a function? Explain why.

For $y = x^2$, none of the vertical lines drawn intersect the graph at more than one point. That means that for each value of x, there is only 1 corresponding value of y. This means it is a function.

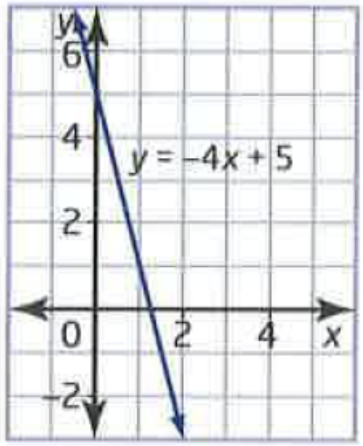
For $x = y^2$, some of the vertical lines drawn intersect the graph at more than one point. That means that some x-values correspond to more than one y-value. This means it is NOT a function.

Section 2: Vertical Line Test

Vertical line test: a method for determining if a relation is a function or not. If every possible vertical line intersects the graph of the relation at only one point, then the relation is a function.

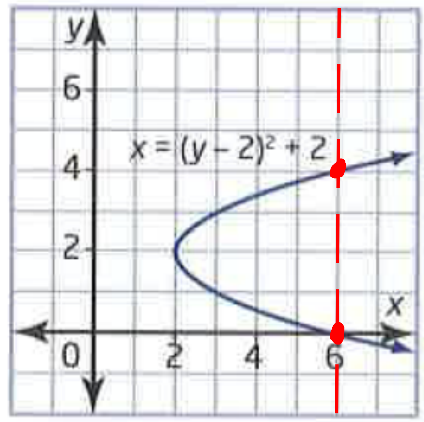
Example 1: Use the vertical line test to determine whether each relation is a function or not.

a)



Function

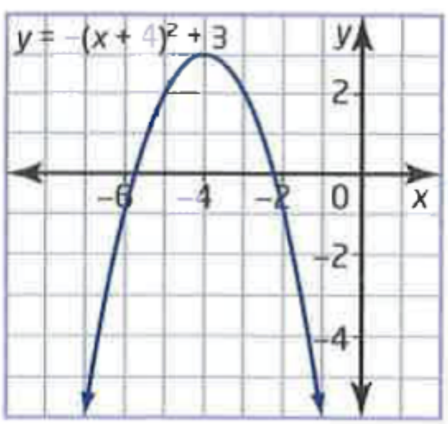
b)



Not a function

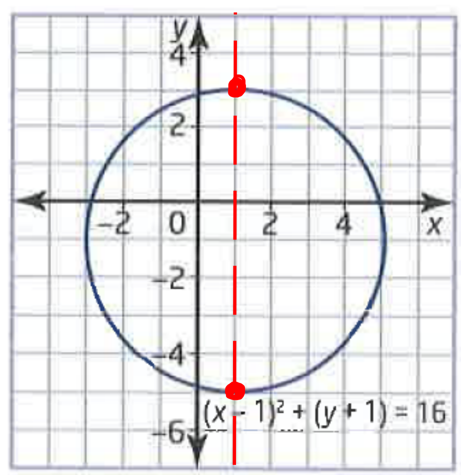
When x = 6, y = 0 and 4

c)



Function

d)



Not a function

When x = 1, y = -5 and 3

Section 3: Domain and Range

For any relation, the set of values of the independent variable (often the *x*-values) is called the domain of the relation. The set of the corresponding values of the dependent variable (often the *y*-values) is called the range of the relation.

Note: *For a function, for each given element of the domain there must be exactly one element in the range.*

Domain: values *x* may take

Range: values *y* may take

General Notation

$D: \{X \in \mathbb{R} \mid \text{restrictions}\}$ or $D: \{x = \#, \#, \ldots\}$

$R: \{Y \in \mathbb{R} \mid \text{restrictions}\}$ or $R: \{y = \#, \#, \ldots\}$

Real number: *a number in the set of all integers, terminating decimals, repeating decimals, non-terminating decimals, and non repeating decimals. Represented by the symbol* $\mathbb{R}$

Example 2: Determine the domain and range of each relation from the data given.

a) { (-3, 4), (5, -6), (-2, 7), (5, 3), (6, -8) }

$D: \{x = -3, -2, 5, 6\}$

$R: \{y = -8, -6, 3, 4, 7\}$

b)

Age	*Number*
4	8
5	12
6	5
7	22
8	14
9	9
10	11

D: $\{x = 4, 5, 6, 7, 8, 9, 10\}$

R: $\{y = 5, 8, 9, 11, 12, 14, 22\}$

Are each of these relations functions?

part a) is NOT a function. There are multiple y-values that correspond to an x-value of 5

part b) is a function. Each value for x has exactly one value for y.

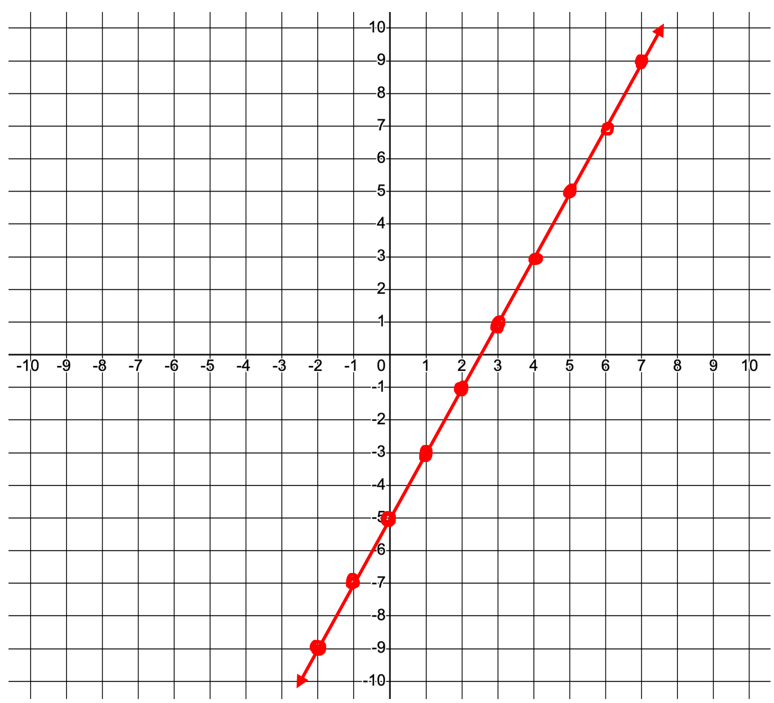
Example 3: Determine the domain and range of each relation. Graph the relation first.

a) $y = 2x - 5$ linear function

slope y-int

D: $\{X \in \mathbb{R}\}$

R: $\{Y \in \mathbb{R}\}$



b) $y = (x-1)^2 + 3$ quadratic function

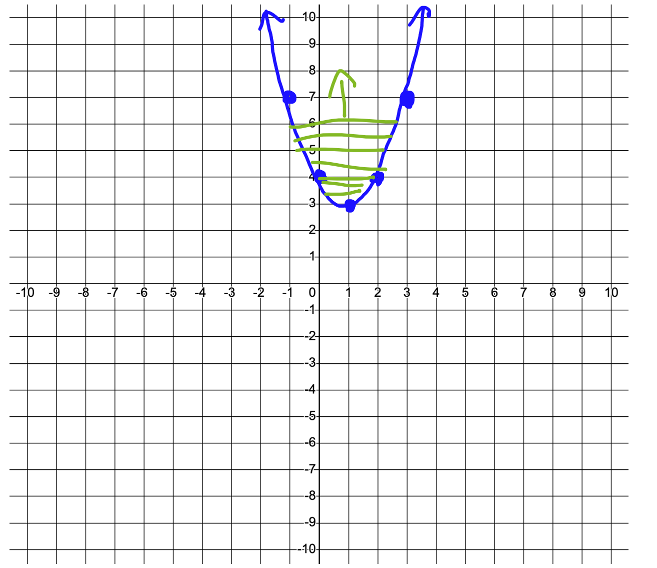
opens up

vertex at (1,3)

x	y
-1	7
0	4
1	3
2	4
3	7

D: $\{X \in \mathbb{R}\}$

R: $\{Y \in \mathbb{R} \mid y \geq 3\}$

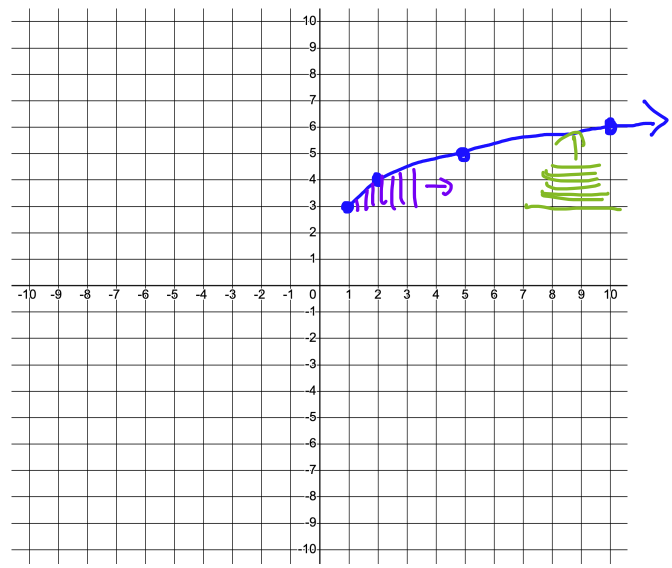


c) $y = \sqrt{x-1} + 3$ radical function

x	y
1	3
2	4
5	5
10	6

D: $\{X \in \mathbb{R} \mid x \geq 1\}$

R: $\{Y \in \mathbb{R} \mid y \geq 3\}$

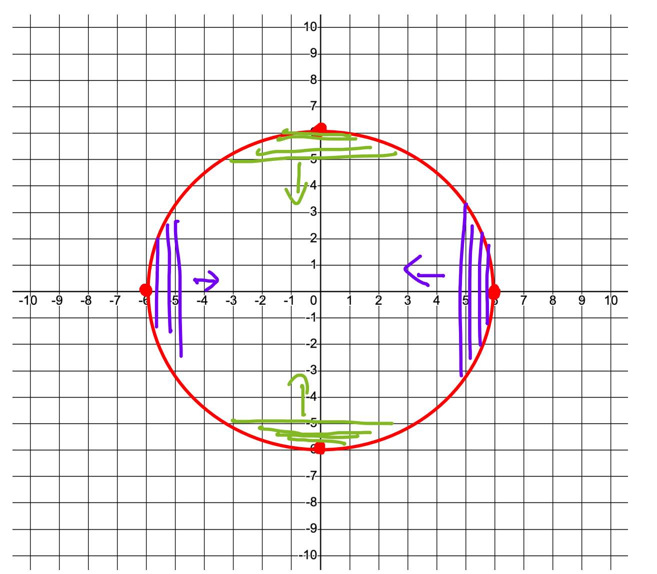


d) $x^2 + y^2 = 36$

circle centered at the origin with a radius of 6.

D: $\{X \in \mathbb{R} \mid -6 \leq x \leq 6\}$

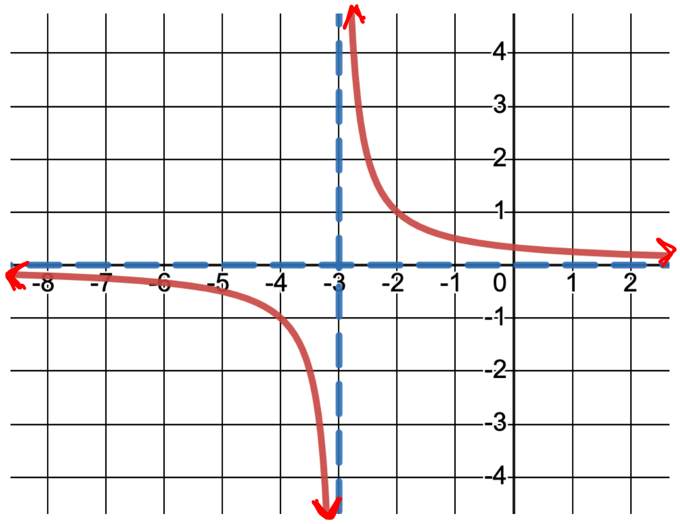
R: $\{Y \in \mathbb{R} \mid -6 \leq y \leq 6\}$



e) $y = \frac{1}{x+3}$ rational function

horizontal asymptote at $y = 0$

vertical asymptote at $x = -3$



D: $\{x \in \mathbb{R} \mid x \neq -3\}$

R: $\{y \in \mathbb{R} \mid y \neq 0\}$

Asymptotes

Asymptote:

The function $\boldsymbol{y = \frac{1}{x+3}}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq -3$. This is why the vertical line $x = -3$ is an asymptote for this function.

Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.

L2 - 1.2 Functions and Function Notation
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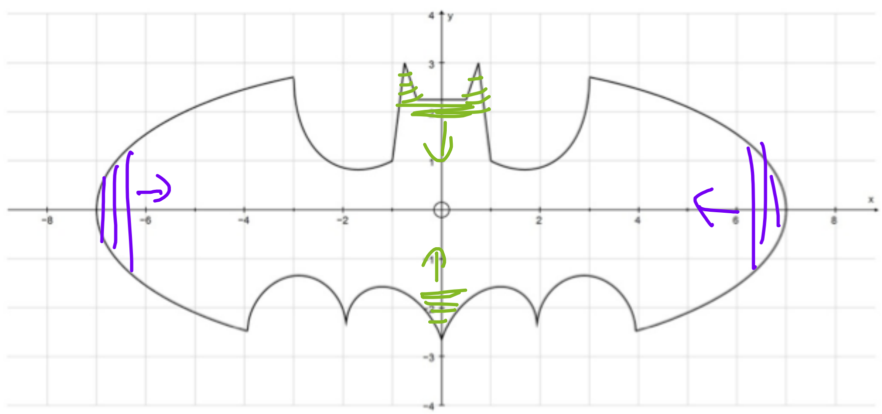
Part 1: Domain & Range Review

a) State the domain and range of the relation shown in the following graph:



D: $\{X \in \mathbb{R} \mid -7 \leq x \leq 7\}$

R: $\{Y \in \mathbb{R} \mid -2.5 \leq y \leq 3\}$

b) Is this a function?

No, it does NOT pass the vertical line test.

c) What determines if a relation is a function or not?

For each value of *x*, there can only be one corresponding value of *y*.

d) How does the vertical line test help us determine if a relation is a function?

If any vertical line touches the graph of the relation in more than one spot, it is NOT a function.

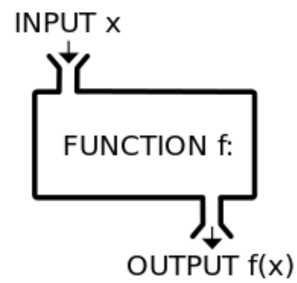
e) What is domain?

The values *x* may take.

f) What is range?

The value *y* may take.

Part 2: Find Values Using Function Notation



What does a function do?

Takes an input (x), performs operations on it and then gives an output (y).

What does function notation look like?

read as 'f of x' or 'f at x'

replaces 'y'

$$f(x) = \text{some operations applied to } x$$

Example 1: For each of the following functions, determine $f(2)$, $f(-5)$, and $f(1/2)$

a) $f(x) = 2x - 4$

$f(2) = 2(2) - 4$
$f(2) = 0$
$(2, 0)$

$f(-5) = 2(-5) - 4$
$f(-5) = -14$
$(-5, -14)$

$f(\frac{1}{2}) = 2(\frac{1}{2}) - 4$
$f(\frac{1}{2}) = -3$
$(\frac{1}{2}, -3)$

b) $f(x) = 3x^2 - x + 7$

$f(2) = 3(2)^2 - 2 + 7$
$f(2) = 17$
$(2, 17)$

$f(-5) = 3(-5)^2 - (-5) + 7$
$f(-5) = 75 + 5 + 7$
$f(-5) = 87$
$(-5, 87)$

$f(\frac{1}{2}) = 3(\frac{1}{2})^2 - \frac{1}{2} + 7$
$f(\frac{1}{2}) = \frac{3}{4} - \frac{2}{4} + \frac{28}{4}$
$f(\frac{1}{2}) = \frac{29}{4}$
$(\frac{1}{2}, \frac{29}{4})$

c) $f(x) = 87$

$f(2) = 87$
$(2, 87)$

$f(-5) = 87$
$(-5, 87)$

$f(\frac{1}{2}) = 87$
$(\frac{1}{2}, 87)$

d) $f(x) = \frac{2x}{x^2-3}$

$f(2) = \frac{2(2)}{(2)^2-3}$

$= 4$

$(2,4)$

$f(-5) = \frac{2(-5)}{(-5)^2-3}$

$= \frac{-10}{22}$

$= -\frac{5}{11}$

$\left(-5, -\frac{5}{11}\right)$

$f\left(\frac{1}{2}\right) = \frac{2\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2-3}$

$= \frac{1}{\frac{1}{4}-\frac{12}{4}}$

$= \frac{1}{\left(-\frac{11}{4}\right)}$

$= -\frac{4}{11}$

$\left(\frac{1}{2}, -\frac{4}{11}\right)$

Part 3: Applications of Function Notation

Example 3: For the function $h(t) = \overset{a}{-3}(t+\overset{h}{1})^2 + \overset{k}{5}$

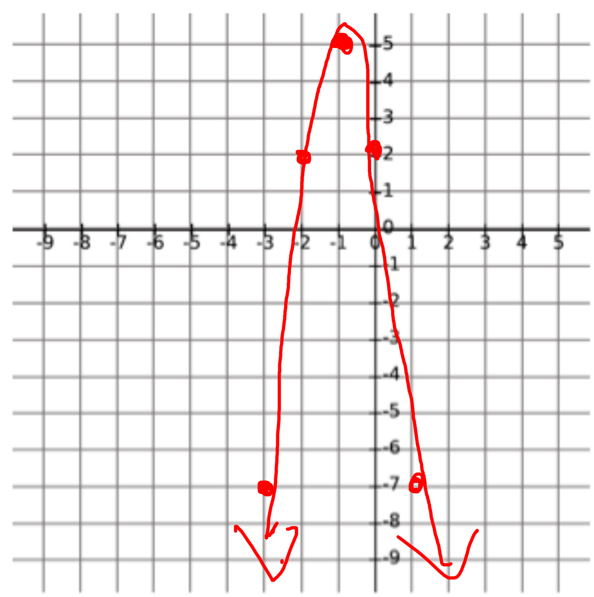
$\boxed{\text{vertex form: } f(x) = a(x-h)^2+k}$

i) Graph it and find the domain and range

opens down $(a<0)$

vertex at $(\overset{h}{-1}, \overset{k}{5})$

t	$h(t)$
-3	-7
-2	2
-1	5
0	2
1	-7



ii) Find $h(-7)$

$h(-7) = -3[(-7)+1]^2+5$

$= -3(-6)^2+5$

$= -3(36)+5$

$= -103$

Example 4: The temperature of the water at the surface of a lake is 22 degrees Celsius. As Geno scuba dives to the depths of the lake, he finds that the temperature decreases by 1.5 degrees for every 8 meters he descends.

a) Model the water temperature at any depth using function notation.

Notice it is a constant rate of change making it a linear function of the form y = mx + b

$m = \frac{\Delta T}{\Delta d} = \frac{-1.5}{8} = \frac{-3}{16}$

$T(d) = -\frac{3}{16}d + 22$

$b = 22$

b) What is the water temperature at a depth of 40 meters?

$T(40) = \frac{-3}{16}(40) + 22$

$= 14.5°C$

c) At the bottom of the lake the temperature is 5.5 degrees Celsius. How deep is the lake?

$5.5 = \frac{-3}{16}d + 22$

$(16)\ -16.5 = \frac{-3}{16}d\ (16)$

$-264 = -3d$

$d = 88$ meters deep

L3 - 1.3 Max or Min of a Quadratic Function

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Part 1: Quadratics Review

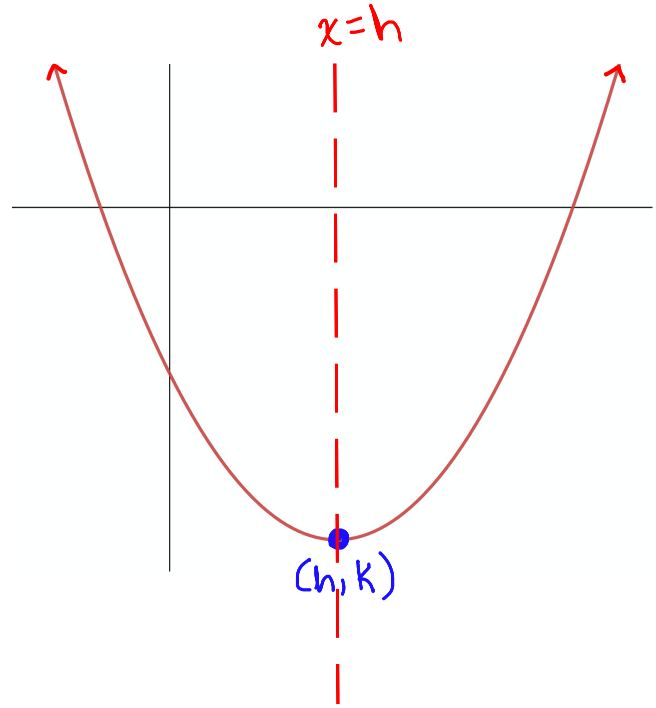
Vertex Form:

$y = a(x - h)^2 + k$

vertex at (h, k)

$a > 0$; opens up
$a < 0$; opens down

axis of symmetry at $x = h$



Factored Form:

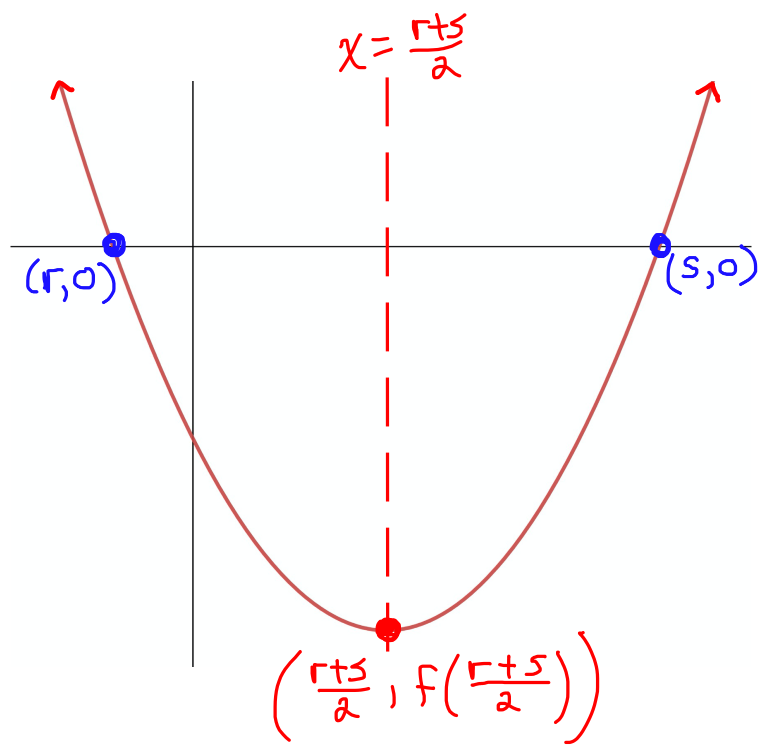
$y = a(x - r)(x - s)$

x-int at $(r, 0)$ and $(s, 0)$

$a > 0$; opens up
$a < 0$; opens down

axis of symmetry at $x = \frac{r+s}{2}$

vertex at $\left(\frac{r+s}{2}, f\left(\frac{r+s}{2}\right)\right)$



Standard Form:

$\boldsymbol{y = ax^2 + bx + c}$

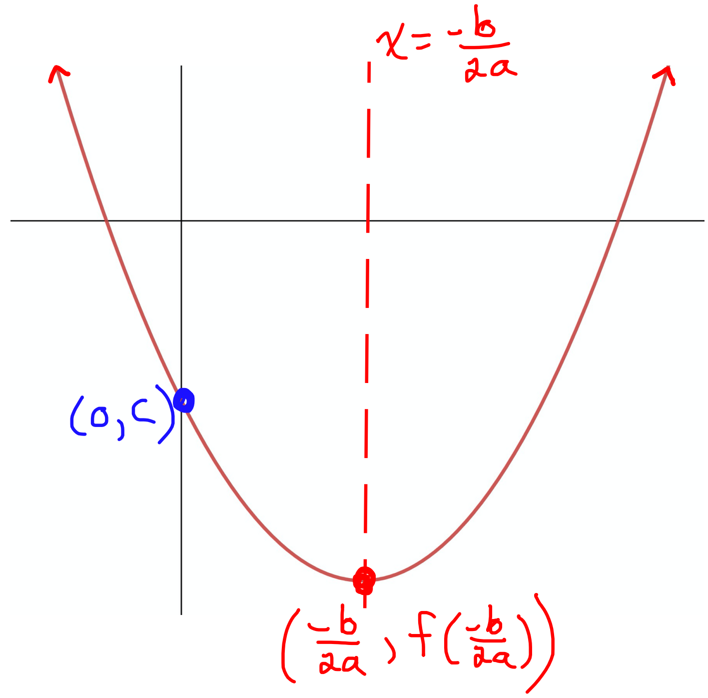
y-int at $(0, c)$

$a > 0$; opens up

$a < 0$; opens down

axis of symmetry at $x = \frac{-b}{2a}$

vertex at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$



Part 2: Perfect Square Trinomials

Completing the square is a process for changing a standard form quadratic equation into vertex form

$$y = ax^2 + bx + c \rightarrow y = a(x - h)^2 + k$$

Notice that vertex form contains a $(x - h)^2$. A binomial squared can be obtained when factoring a perfect square trinomial:

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

The process of completing the square involves creating this perfect square trinomial within the standard form equation so that it can be factored to create the vertex form equation.

Let's start by analyzing the following perfect square trinomials. Specifically notice how the middle term is 2 times the product of the square roots of the first and last terms.

$x^2 + 10x + 25$

$10x = 2(\sqrt{x^2})(\sqrt{25})$

$10x = 2(x)(5)$

$10x = 10x$

$x^2 - 12x + 36$

$12x = 2(\sqrt{x^2})(\sqrt{36})$

$12x = 2(x)(6)$

$12x = 12x$

Example 1: Determine the value of k that would make each quadratic a perfect square trinomial. Then factor the trinomial.

a) $x^2 + 14x + k$

$14x = 2(\sqrt{x^2})(\sqrt{k})$

$14\cancel{x} = 2(\cancel{x})(\sqrt{k})$

$\left(\frac{14}{2}\right)^2 = (\sqrt{k})^2$

$k = 49$

b) $x^2 - 24x + k$

$24x = 2(\sqrt{x^2})(\sqrt{k})$

$24\cancel{x} = 2(\cancel{x})(\sqrt{k})$

$\left(\frac{24}{2}\right)^2 = (\sqrt{k})^2$

$k = 144$

Tip: You can calculate the constant term that makes the quadratic a PST by squaring half of the coefficient of the x term.

Note: *this only works when the coefficient of x^2 is 1.*

Part 3: Completing the Square

Completing the Square Steps

$$ax^2 + bx + c \rightarrow a(x-h)^2 + k$$

1) Put brackets around the first 2 terms
2) Factor out the constant in front of the x^2 term
3) Look at the last term in the brackets, divide it by 2 and then square it
4) Add AND subtract that term behind the last term in the brackets
5) Move the negative term outside the brackets by multiplying it by the $'a'$ value
6) Simplify the terms outside the brackets
7) Factor the perfect square trinomial

$$a^2 + 2ab + b^2 = (a+b)^2$$

Example 2: Rewrite each quadratic in vertex form by completing the square. Then state the vertex, whether it is a max or min point, and the axis of symmetry.

a) $y = x^2 + 8x + 5$

$y = (x^2+8x)+5$

$y = (x^2+8x+\underline{16} - \underline{16})+5$

$y = (x^2+8x+16)-16+5$ $\quad$ $\underline{4} \times \underline{4} = 16$, $\underline{4} + \underline{4} = 8$

$y = (x+4)(x+4)-11$

$y = (x+4)^2-11$

vertex $(-4,-11)$ is a min point

aos at $x=-4$

b) $y = 2x^2 - 12x + 11$

$y = (2x^2-12x)+11$

$y = 2(x^2-6x)+11$

$y = 2(x^2-6x+\underline{9} - \underline{9})+11$

$y = 2(x^2-6x+9)-18+11$ $\quad$ $\underline{-3} \times \underline{-3} = 9$, $\underline{-3} + \underline{-3} = -6$

$y = 2(x-3)(x-3)-7$

$y = 2(x-3)^2-7$

vertex at $(3,-7)$ is a min point

aos at $x=3$

c) $y = -3x^2 + 9x - 13$

$y = -3(x^2-3x)-13$

$y = -3(x^2-3x+\frac{9}{4} - \frac{9}{4})-13$

$y = -3(x^2-3x+\frac{9}{4})+\frac{27}{4}-\frac{52}{4}$

$y = -3(x-\frac{3}{2})^2-\frac{25}{4}$ $\quad$ $\underline{-3/2} \times \underline{-3/2} = \frac{9}{4}$, $\underline{-3/2} + \underline{-3/2} = -3$

vertex at $(\frac{3}{2}, \frac{-25}{4})$ is a max

d) $y = -\frac{2}{3}x^2 + 8x + 5$

$y = -\frac{2}{3}(x^2-12x)+5$

$y = -\frac{2}{3}(x^2-12x+\underline{36} - \underline{36})+5$

$y = -\frac{2}{3}(x^2-12x+36)+24+5$ $\quad$ $\underline{-6} \times \underline{-6} = 36$, $\underline{-6} + \underline{-6} = -12$

$y = -\frac{2}{3}(x-6)^2+29$

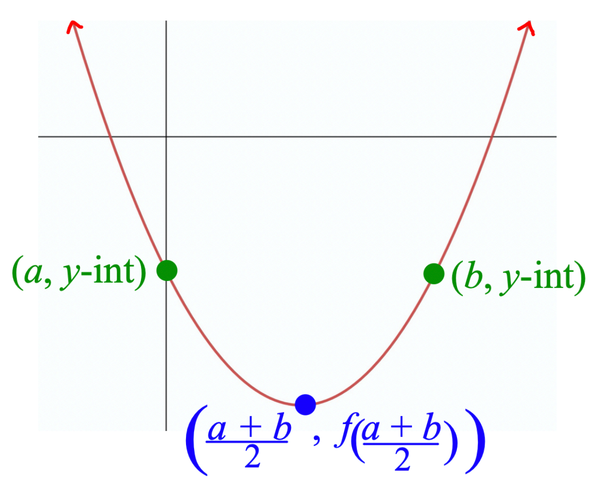
vertex at $(6,29)$ is a max

aos at $x=6$

Part 4: Partial Factoring (another method to find the vertex)

Partial Factoring Steps

1) Set the quadratic equal to the y-intercept
2) Solve the equation for x
3) Find the x-value of the vertex by averaging your answers from the previous step
4) Substitute the x-value of the vertex into the original equation and solve for y-value



Example 3: Use partial factoring to find the vertex. Then state if it is a max or min.

a) $y = x^2 + 2x - 6$

$-6 = x^2 + 2x - 6$

$0 = x^2 + 2x$

$0 = x(x+2)$

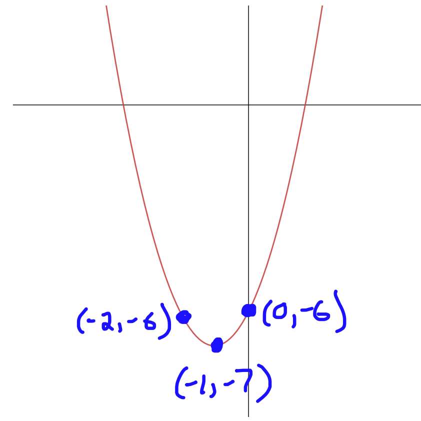
$x_1 = 0 \quad x+2=0$

$x_2 = -2$

$x\text{-vertex} = \frac{0+(-2)}{2} = -1$

$y\text{-vertex} = (-1)^2 + 2(-1) - 6 = -7$

The vertex at $(-1,-7)$ is a min



b) $y = 4x^2 - 12x + 3$

$3 = 4x^2 - 12x + 3$

$0 = 4x^2 - 12x$

$0 = 4x(x-3)$

$4x = 0 \quad x-3=0$

$x_1 = 0 \quad x_2 = 3$

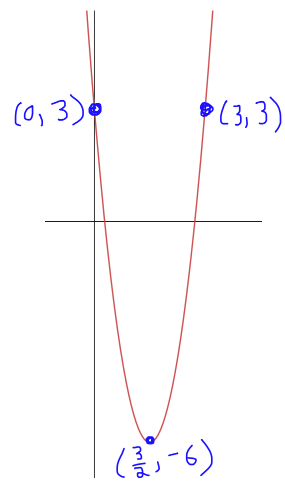
$x\text{-vertex} = \frac{0+3}{2} = \frac{3}{2}$

$y\text{-vertex} = 4\left(\frac{3}{2}\right)^2 - 12\left(\frac{3}{2}\right) + 3$

$= 4\left(\frac{9}{4}\right) - 6(3) + 3$

$= -6$

vertex at $\left(\frac{3}{2}, -6\right)$ is a min.



c) $y = -3x^2 + 9x - 2$

$-2 = -3x^2 + 9x - 2$

$0 = -3x^2 + 9x$

$0 = -3x(x-3)$

$-3x = 0 \quad x-3=0$

$x_1 = 0 \quad x_2 = 3$

$x\text{-vertex} = \frac{0+3}{2} = \frac{3}{2}$

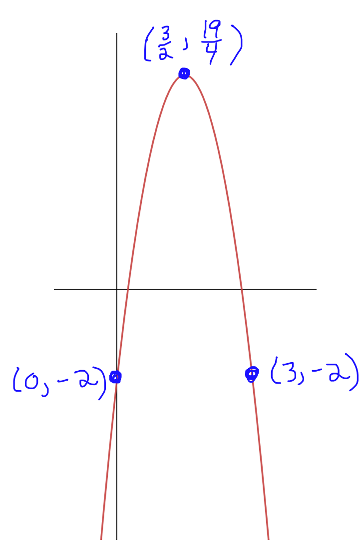
$y\text{-vertex} = -3\left(\frac{3}{2}\right)^2 + 9\left(\frac{3}{2}\right) - 2$

$= -3\left(\frac{9}{4}\right) + \frac{27}{2} - 2$

$= -\frac{27}{4} + \frac{54}{4} - \frac{8}{4}$

$= \frac{19}{4}$

vertex at $\left(\frac{3}{2}, \frac{19}{4}\right)$ is a max



Example 4: Maximizing Revenue

Rachel and Ken are knitting scarves to sell at the craft show. They were planning to sell the scarves for \$10 each, the same as last year when they sold 40 scarves. However, they know that if they adjust the price, they might be able to make mor profit. They have been told that for every 50-cent increase in the price, they can expect to sell four fewer scarves. What selling price will maximize their revenue and what will the revenue be?

Let n = # of \$0.50 increases

cost = $10 + 0.5n$

number sold = $40 - 4n$

Revenue = (cost)(# sold)

$= (10 + 0.5n)(40 - 4n)$

$0 = (10 + 0.5n)(40 - 4n)$

$0 = 10 + 0.5n \qquad 0 = 40 - 4n$

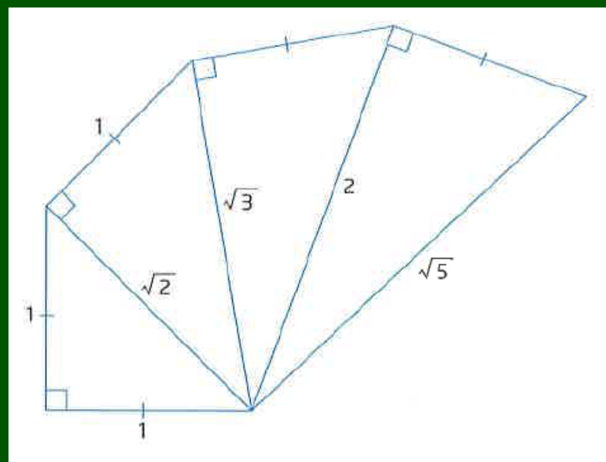
$-10 = 0.5n \qquad 4n = 40$

$n_1 = -20 \qquad n_2 = 10$

n-vertex $= \frac{-20 + 10}{2} = -5$; to get a max revenue they must raise the price by \$0.50 five times.

$R(-5) =$

1.4 Working With Radicals



Lesson Outline

Section 1: Investigation

Section 2: Definitions

Section 3: Entire radicals to mixed radicals

Section 4: Add/Subtract radicals

Section 5: Multiply Radicals

Section 6: Application

Investigation

a) Complete the following table:

A	B
$\sqrt{4}\times\sqrt{4} =$ 4	$\sqrt{4\times4} =$ 4
$\sqrt{81}\times\sqrt{81} =$ 81	$\sqrt{81\times81} =$ 81
$\sqrt{225}\times\sqrt{225} =$ 225	$\sqrt{225\times225} =$ 225
$\sqrt{5}\times\sqrt{5} =$ 5	$\sqrt{5\times5} =$ 5
$\sqrt{31}\times\sqrt{31} =$ 31	$\sqrt{31\times31} =$ 31
$\sqrt{12}\times\sqrt{9} =$ 10.39	$\sqrt{12\times9} =$ 10.39
$\sqrt{23}\times\sqrt{121} =$ 52.75	$\sqrt{23\times121} =$ 52.75

b) What do you notice about the results in each row?

The results are the same in each row.

c) Make a general conclusion about an equivalent expression for $\sqrt{a}\times\sqrt{b}$

$$\sqrt{a}\times\sqrt{b} = \sqrt{a\times b}$$

Definitions

Radicand:

a number or expression under a radical sign

Entire Radical:

a radical in the form $\sqrt{n}$, where $n > 0$, such as $\sqrt{45}$

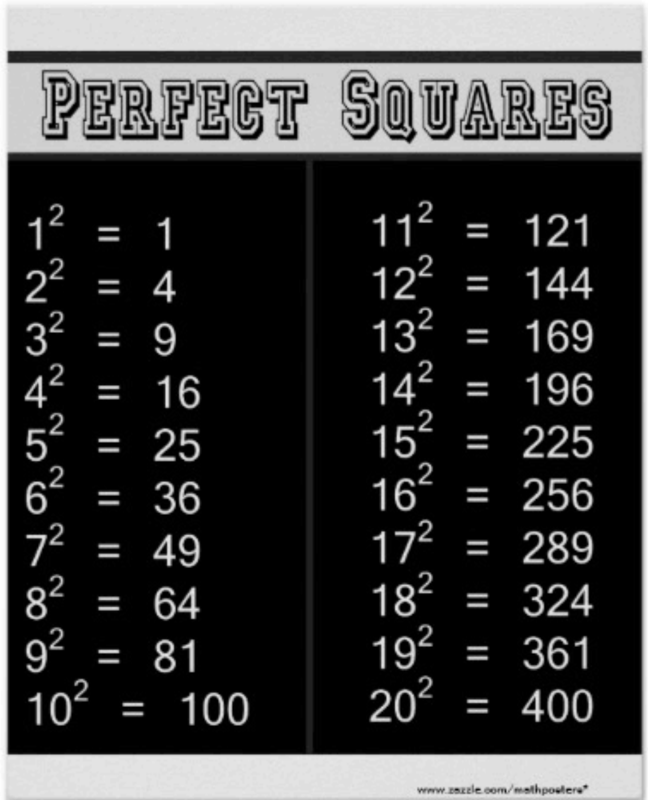
Mixed Radical:

a radical in the form $a\sqrt{b}$, where $a \neq 1$ or -1 and $b > 0$, such as $3\sqrt{5}$

More About Radicals

Some numbers cannot be expressed as fractions. These are called irrational numbers. One type of irrational number is of the form $\sqrt{n}$ where n is not a perfect square. These numbers are sometimes called radicals.

An approximate value can be found for these irrational numbers using a calculator but it is better to work with an exact value. Answers should be left in radical form when an EXACT answer is needed. Sometimes entire radicals can be simplified by removing perfect square factors. The resulting expression is called a mixed radical.



Example 1: Express each radical as a mixed radical in simplest form.

Hint: *remove perfect square factors and then simplify*

a) $\sqrt{50}$

$= \sqrt{25 \times 2}$

$= (\sqrt{25})(\sqrt{2})$

$= 5\sqrt{2}$

b) $\sqrt{27}$

$= \sqrt{9 \times 3}$

$= (\sqrt{9})(\sqrt{3})$

$= 3\sqrt{3}$

c) $\sqrt{180}$

$= \sqrt{36 \times 5}$

$= (\sqrt{36})(\sqrt{5})$

$= 6\sqrt{5}$

Adding and Subtracting Radicals

Adding and subtracting radicals works in the same way as adding and subtracting polynomials. You can only add like terms or, in this case, like radicals.

Example:

$2\sqrt{3}+5\sqrt{7}$ cannot be added because they do not have the same radical.

However, $3\sqrt{5}+6\sqrt{5}$ have a common radical, so they can be added. $3\sqrt{5}+6\sqrt{5}=9\sqrt{5}$

Note, the radical stays the same when adding and subtracting expressions with like radicals.

Example 2: Simplify the following

a) $9\sqrt{7}-3\sqrt{7}$

$=6\sqrt{7}$

b) $4\sqrt{3}-2\sqrt{27}$

$=4\sqrt{3}-2(\sqrt{9})(\sqrt{3})$

$=4\sqrt{3}-2(3)(\sqrt{3})$

$=4\sqrt{3}-6\sqrt{3}$

$=-2\sqrt{3}$

c) $5\sqrt{8} + 3\sqrt{18}$

$= 5(\sqrt{4})(\sqrt{2}) + 3(\sqrt{9})(\sqrt{2})$

$= 5(2)(\sqrt{2}) + 3(3)(\sqrt{2})$

$= 10\sqrt{2} + 9\sqrt{2}$

$= 19\sqrt{2}$

d) $\frac{1}{4}\sqrt{28} - \frac{3}{4}\sqrt{63} + \frac{2}{3}\sqrt{50}$

$= \frac{1}{4}(\sqrt{4})(\sqrt{7}) - \frac{3}{4}(\sqrt{9})(\sqrt{7}) + \frac{2}{3}(\sqrt{25})(\sqrt{2})$

$= \frac{2}{4}\sqrt{7} - \frac{9}{4}\sqrt{7} + \frac{10}{3}\sqrt{2}$

$= \frac{-7}{4}\sqrt{7} + \frac{10}{3}\sqrt{2}$

Multiplying Radicals

Example 3: Simplify fully

Multiply the coefficients together and then multiply the radicands together. Then simplify!

a) $(2\sqrt{3})(3\sqrt{6})$

$= (2)(3)(\sqrt{3})(\sqrt{6})$

$= 6\sqrt{18}$

$= 6(\sqrt{9})(\sqrt{2})$

$= 6(3)(\sqrt{2})$

$= 18\sqrt{2}$

b) $2\sqrt{3}(4 + 5\sqrt{3})$

$= 2\sqrt{3}(4) + 2\sqrt{3}(5\sqrt{3})$

$= 8\sqrt{3} + 10\sqrt{9}$

$= 8\sqrt{3} + 10(3)$

$= 8\sqrt{3} + 30$

Don't forget the distributive property:

$a(x+y) = ax + ay$

c) $-7\sqrt{2}(6\sqrt{8} - 11)$

$= -7\sqrt{2}(6\sqrt{8}) - 7\sqrt{2}(-11)$

$= -42\sqrt{16} + 77\sqrt{2}$

$= -168 + 77\sqrt{2}$

d) $(\sqrt{3}+5)(2-\sqrt{3})$

Don't forget FOIL. Each term in the first binomial must be multiplied by each term in the second binomial.

$= \sqrt{3}(2) + \sqrt{3}(-\sqrt{3}) + 5(2) + 5(-\sqrt{3})$

$= 2\sqrt{3} - \sqrt{9} + 10 - 5\sqrt{3}$

$= 2\sqrt{3} - 5\sqrt{3} - 3 + 10$

$= -3\sqrt{3} + 7$

$(a + b)\ (a - b)$

e) $(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})$

There is a shortcut! This is a difference of squares.

$(a+b)(a-b) = a^2 - b^2$

$a \quad b$

$= (2\sqrt{2})^2 - (3\sqrt{3})^2$

$= 4(2) - 9(3)$

$= 8 - 27$

$= -19$

<u>1.5 Solving Quadratic Equations</u>

Part 1: Solve by Factoring

<u>DO IT NOW!</u>

1. Simplify. $(\sqrt{2}+3\sqrt{3})(5\sqrt{3}-10)$

$= \sqrt{2}(5\sqrt{3}) + \sqrt{2}(-10) + 3\sqrt{3}(5\sqrt{3}) + 3\sqrt{3}(-10)$

$= 5\sqrt{6} - 10\sqrt{2} + 15\sqrt{9} - 30\sqrt{3}$

$= 5\sqrt{6} - 10\sqrt{2} - 30\sqrt{3} + 45$

2. Simplify $\dfrac{2-\sqrt{80}}{4}$

$$= \frac{2-\sqrt{16}(\sqrt{5})}{4}$$

$$= \frac{2-4\sqrt{5}}{4}$$

$$= \frac{2}{4} - \frac{\cancel{4}\sqrt{5}}{\cancel{4}}$$

$$= \frac{1}{2} - \sqrt{5}$$

3. Expand and simplify:

$$4\sqrt{10}\left(3+2\sqrt{2}\right)$$

$$= 4\sqrt{10}(3) + 4\sqrt{10}(2\sqrt{2})$$

$$= 12\sqrt{10} + 8\sqrt{20}$$

$$= 12\sqrt{10} + 8\sqrt{4}(\sqrt{5})$$

$$= 12\sqrt{10} + 16\sqrt{5}$$

Lesson Outline

Section 1: Solve a quadratic with an '*a*' value of 1 or that can be factored out

Section 2: Solve a quadratic with an '*a*' value of not 1 that can't be factored out.

*In all cases we will start with an equation in Standard Form and we will set it equal to 0:

$$ax^2+bx+c = 0$$

NOTE: If it's not in standard form, you must rearrange before factoring.

HOW TO SOLVE QUADRATICS

Solving a quadratic means to find the x-intercepts or roots.

To solve a quadratic equation:

1) It must be set to equal 0. Before factoring, it must be in the form $ax^2+bx+c = 0$
2) Factor the left side of the equation
3) Set each factor to equal zero and solve for 'x'.

zero product rule: if two factors have a product of zero; one or both of the factors must equal zero.

Example 1: Solve the following quadratics by factoring

When factoring $ax^2+bx+c=0$ when 'a' is 1 or can be factored out

Steps to follow:

1) Check if there is a common factor that can be divided out
2) Look at the 'c' value and the 'b' value
3) Determine what factors multiply to give 'c' and add to give 'b'
4) put those factors into (x+r)(x+s) for 'r' and 's'.
5) make sure nothing else can be factored

a) $y = x^2 - 15x + 56$

P: 56
S: -15
-8 and -7

$y = (x-8)(x-7)$

$0 = (x-8)(x-7)$

$x-8=0$ or $x-7=0$

$x=8$ $\quad x=7$

b) $y = x^2 - 5x + 6$

P: 6
S: -5
-2 and -3

$0 = (x-2)(x-3)$

$0 = (x-2)(x-3)$

$x-2=0$ or $x-3=0$

$x=2$ $\quad x=3$

c) $y = 2x^2 - 8x - 42$

P: -21
S: -4
-7 and 3

$0 = 2(x^2-4x-21)$

$0 = 2(x-7)(x+3)$

$0 = (x-7)(x+3)$

$x-7=0$ or $x+3=0$

$x=7$ $\quad x=-3$

Steps to factoring ax^2+bx+c when 'a' cannot be factored out and is not 1.

1) Look to see if there is a common factor that can be divided out
2) Take the 'a' value and multiply it to the 'c' value
3) Determine what factors of THIS number add together to get the 'b' value
4) Break the 'b' value up into THOSE factors!
5) Put parenthesis around the first two variables and the last two
6) Factor by grouping

Example 2: Solve by factoring

a) $8x^2 + 2x - 15 = 0$

P: -120
S: 2

12 and -10

$8x^2 + 12x - 10x - 15 = 0$

(factor by grouping)

$(8x^2 + 12x) + (-10x - 15) = 0$

(common factor each group)

$4x(2x+3) - 5(2x+3) = 0$

(binomial common factor)

$(2x+3)(4x-5) = 0$

(zero product rule)

$2x + 3 = 0$ or $4x - 5 = 0$

$x = -\frac{3}{2}$ $\qquad$ $x = \frac{5}{4}$

b) $2x^2 - 11x = -15$

P: 30 -6 and -5
S: -11

$2x^2 - 11x + 15 = 0$

$2x^2 - 6x - 5x + 15 = 0$

$(2x^2 - 6x) + (-5x + 15) = 0$ (Factor by grouping)

(common Factor each group)

$2x(x-3) - 5(x-3) = 0$ (binomial common Factor)

$(x-3)(2x-5) = 0$

(zero product rule)

$x - 3 = 0$ or $2x - 5 = 0$

$x = 3$ $\quad x = \frac{5}{2}$

Example 3: For the quadratic $y = 2x^2 - 4x - 16$

a) Find the roots of the quadratic by factoring

$0 = 2(x^2 - 2x - 8)$

P: -8 -4 and 2
S: -2

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4^r$ $\quad x = -2^s$

b) Find the axis of symmetry (average of x-intercepts)

a.o.s: $x = \frac{r+s}{2} = \frac{4+(-2)}{2} = \frac{2}{2} = 1$

c) Find the coordinates of the vertex and state if it is a max or min value

$x_{vertex} = 1$

$$y_{vertex} = 2x^2 - 4x - 16$$
$$= 2(1)^2 - 4(1) - 16$$
$$= 2 - 4 - 16$$
$$= -18$$

$\therefore$ vertex is $(1, -18)$

This is a minimum value because the parabola opens up $(a > 0)$.

1.5 Solving Quadratic Equations

Part 2: Solve Using QF

Lesson Outline:

Part 1: Do It Now - QF Refresher

Part 2: Discriminant review

Part 3: Find exact solutions of a quadratic with 2 roots

Part 4: Solve a quadratic with 1 solution

Part 5: Solve a quadratic with 0 solutions

Part 6: Use the discriminant to determine the number of solutions (x-intercepts) a quadratic has

Part 7: Application

DO IT NOW!

a) Do you remember the quadratic formula?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

b) Use the quadratic formula to find the x-intercepts of:

$$0 = \overset{a}{2}x^2 + \overset{b}{7}x - \overset{c}{4}$$

Don't forget that to solve a quadratic, it must be set equal to zero because at an x-intercept, the y-coordinate will be zero.

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{49 + 32}}{4}$$

$$x = \frac{-7 \pm \sqrt{81}}{4}$$

$$x = \frac{-7+9}{4} \quad \text{or} \quad x = \frac{-7-9}{4}$$

$$= \frac{1}{2} \qquad\qquad = -4$$

Part 2: Discriminant Review

Do all parabolas have two x-intercepts?

NO

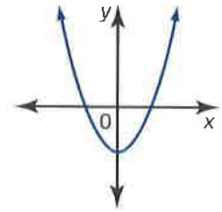
What are the three different scenarios?

0, 1, or 2 solutions

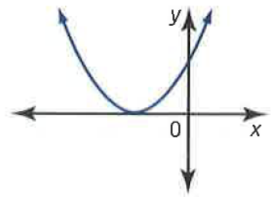
The way to determine how many x-intercepts a parabola might have is by evaluating the $b^2 - 4ac$ part of the quadratic formula (called the "**discriminant**")

Discriminant: the value under the square root

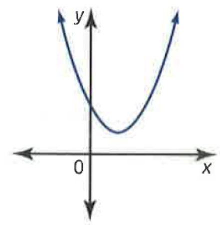
If $b^2 - 4ac > 0$, there are two solutions



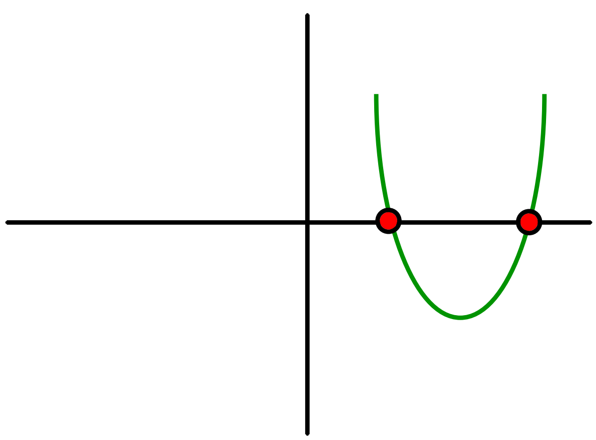
If $b^2 - 4ac = 0$, there is one solution



If $b^2 - 4ac < 0$, there are no solutions



Part 3: Solve a Quadratic With 2 Roots



Objective: Determine the roots of a quadratic using the quadratic formula and leave as EXACT answers

Exact answer: as a radical or fraction. Exact answers do not have decimals.

Example 1: Find the exact solutions of

$$3x^2 - 10x + 5 = 0$$

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{10 \pm \sqrt{40}}{6}$$ (simplify the radical)

$$= \frac{10 \pm \sqrt{4}(\sqrt{10})}{6}$$

$$= \frac{10 \pm 2\sqrt{10}}{6}$$ (common factor)

$$= \frac{\cancel{2}^{1}(5 \pm \sqrt{10})}{3\cancel{6}}$$ (reduce)

$$= \frac{5 \pm \sqrt{10}}{3}$$

$$x = \frac{5+\sqrt{10}}{3} \quad \text{or} \quad x = \frac{5-\sqrt{10}}{3}$$

Example 2: Find the exact solutions of

$$-2x^2 + 8x - 5 = 0$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(-2)(-5)}}{2(-2)}$$

$$= \frac{-8 \pm \sqrt{24}}{-4}$$ (simplify the radical)

$$= \frac{-8 \pm \sqrt{4}(\sqrt{6})}{-4}$$

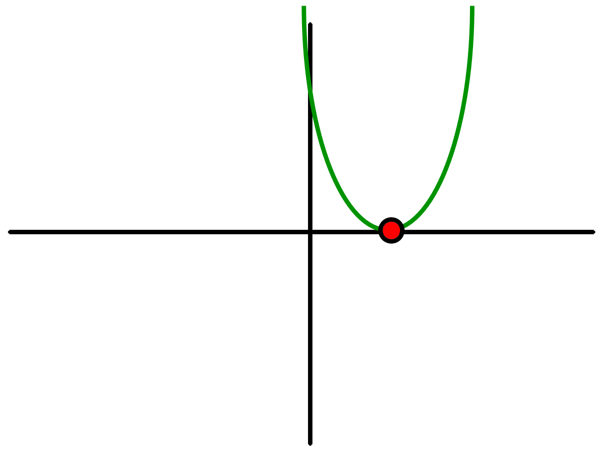
$$= \frac{-8 \pm 2\sqrt{6}}{-4}$$ (common factor)

$$= \frac{\cancel{-2}^{1}(4 \mp \sqrt{6})}{2\,\cancel{-4}}$$ (reduce)

$$= \frac{4 \mp \sqrt{6}}{2}$$

$$x = \frac{4-\sqrt{6}}{2} \quad \text{or} \quad x = \frac{4+\sqrt{6}}{2}$$

Part 4: Solving a Quadratic With 1 Root



Note: *when a quadratic only has 1 solution, the x-intercept is also the vertex*

Example 3: Find the exact roots of

$$4x^2 + 24x + 36 = 0$$

$$4(x^2 + 6x + 9) = 0$$ (common factor)

$$x^2 + 6x + 9 = 0$$

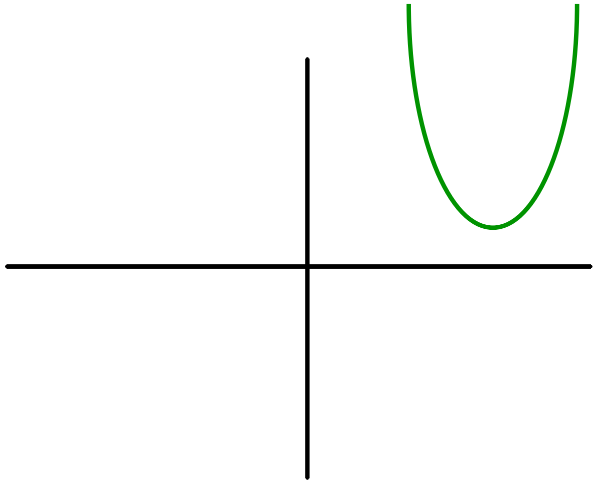
$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{0}}{2}$$

$$x = \frac{-6}{2}$$

$$x = -3$$

Part 5: Solving Quadratics With 0 Roots



2 Scenarios causing 0 roots:
i) vertex is above the x-axis and opens up
ii) vertex is below the x-axis and opens down

Example 4: Find the x-intercepts of

$$\mathbf{8x^2 - 11x + 5 = 0}$$

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(8)(5)}}{2(8)}$$

$$x = \frac{11 \pm \sqrt{-39}}{16}$$

∴ no solutions

Part 6: Use the Discriminant to Determine the Number of Roots

Example 5: For each of the following quadratics, use the discriminant to state the number of roots it will have.

a) $\overset{a}{2}x^2 + \overset{b}{5}x - \overset{c}{5} = 0$

$b^2 - 4ac = 5^2 - 4(2)(-5)$
$= 25 + 40$
$= 60$

$60 > 0$; ∴ 2 solutions

b) $\overset{a}{3}x^2 - \overset{b}{7}x + \overset{c}{5} = 0$

$b^2 - 4ac = (-7)^2 - 4(3)(5)$
$= 49 - 60$
$= -11$

$-11 < 0$; ∴ no solutions

c) $-4x^2 + 12x - 9 = 0$

$b^2 - 4ac = (12)^2 - 4(-4)(-9)$
$= 144 - 144$
$= 0$

$0 = 0$; ∴ 1 solution

Part 7: Application

Example 6: A ball is thrown and the equation below model it's path:

$$h = -0.25d^2 + 2d + 1.5$$

'*h*' is the height in meters above the ground and '*d*' is the horizontal distance in meters from the person who threw the ball.

a) At what height was the ball thrown from? solve for 'h' when d = 0

$h = -0.25(0)^2 + 2(0) + 1.5$
$= 1.5$ meters

b) How far has the ball travelled horizontally when it lands on the ground?

$$0 = -0.25d^2 + 2d + 1.5$$

$$0 = -0.25(d^2 - 8d - 6)$$

$$0 = d^2 - 8d - 6$$

$$d = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{88}}{2}$$

$$d = \frac{8+\sqrt{88}}{2} \quad \text{or} \quad \cancel{d = \frac{8-\sqrt{88}}{2}}$$

$$= 8.7 \text{ meters} \qquad \cancel{= -0.7 \text{ meters}} \quad \text{reject}$$

1.7 Solve Linear-Quadratic Systems

Lesson Outline:

Part 1: Do It Now - review of substitution

Part 2: Possible solutions for a lin-quad system

Part 3: Solve linear-quadratic systems

Part 4: Application

DO IT NOW!

Solve the following linear system using the method of substitution:

① $y = 3x + 7$

② $y = 2x - 5$

$3x+7=2x-5$

$3x-2x=-5-7$

$x=-12$

sub x-value back in to ① or ② and solve for y.

$y=3x+7$

$y=3(-12)+7$

$y=-29$

∴ the POI is $(-12,-29)$

Recall: *solving a linear system means to find the point of intersection (POI)*

Method of Substitution: *solving a linear system by substituting for one variable from one equation into the other equation.*

Steps to Solving A Linear-Quadratic System

1. Set equations equal to each-other

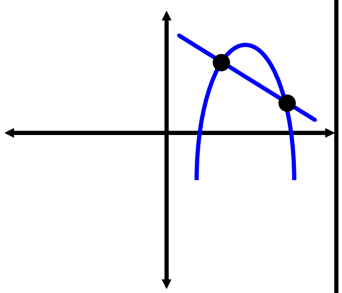
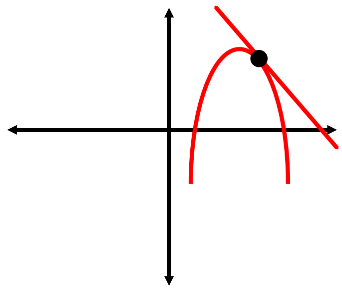
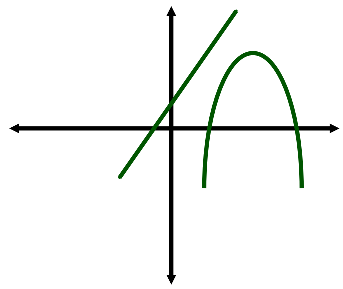
Line = Parabola

2. Rearrange to set the equation equal to zero

3. Solve for *x* by factoring or using the QF (the solution will tell you for what value of *x* the functions have the same *y* value)

4. Plug this value of *x* back in to either of the original functions to solve for *y*.

Possible solutions for a linear-quadratic system:

<table>
<tr><th>2 intersections</th><th>1 intersection</th><th>0 intersections</th></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>Secant: A line that intersects a curve at two distinct points.

discriminant > 0</td><td>Tangent line: A line that touches a curve at one point and has the slope of the curve at that point.

discriminant = 0</td><td></td></tr>
</table>

Example 1

a) How many points of intersection are there for the following system of equations?

$$f(x) = \frac{1}{2}x^2 + 2x - 8 \qquad g(x) = 4x - 10$$

set $f(x) = g(x)$

$\frac{1}{2}x^2 + 2x - 8 = 4x - 10$ (set equal to each other)

$\frac{1}{2}x^2 + 2x - 4x - 8 + 10 = 0$ (set equal to zero)

$\frac{1}{2}x^2 - 2x + 2 = 0$

(common factor)

$\frac{1}{2}(x^2 - 4x + 4) = 0$

$x^2 - 4x + 4 = 0$

$b^2 - 4ac = (-4)^2 - 4(1)(4)$ (check discriminant)

$= 0$

$\therefore$ 1 solution

b) Solve the linear-quadratic system (give exact answers)

$x^2 - 4x + 4 = 0$

solve by factoring.
(hint: it is a perfect square trinomial)

$(x-2)^2 = 0$

$x = 2$

Plug $x=2$ back in to either original equation

(linear is usually easier)

$g(x) = 4x - 10$

$g(2) = 4(2) - 10$

$g(2) = -2$

$\therefore$ The POI is $(2, -2)$

Example 2

Solve the following linear quadratic system

$y = 3x^2 + 21x - 5$

$y = 10x - 1$

$3x^2 + 21x - 5 = 10x - 1$

$3x^2 + 11x - 4 = 0$

$\frac{4}{1}$ $\cancel{\frac{12}{3}}$ -12 11 $\frac{-1}{3}$

$(x+4)(3x-1) = 0$

$x+4=0$ $\quad$ $3x-1=0$

$x_1 = -4$ $\quad$ $x_2 = \frac{1}{3}$

POI #1

$y = 10(-4) - 1$

$y = -41$

$(-4, -41)$

POI #2

$y = 10\left(\frac{1}{3}\right) - 1$

$y = \frac{10}{3} - \frac{3}{3}$

$y = \frac{7}{3}$

$\left(\frac{1}{3}, \frac{7}{3}\right)$

Part 4: Application

Example 3: If a line with slope 4 has one point of intersection with the quadratic function $y = \frac{1}{2}x^2 + 2x - 8$, what is the y-intercept of the line? Write the equation of the line in slope y-intercept form.

$y = 4x + k$

Recall: equation of a line is $y = mx + k$ where k is the y-intercept and m is the slope.

Recall: *for a lin-quad system to have 1 solution, the discriminant must be zero.*

$4x + k = \frac{1}{2}x^2 + 2x - 8$

$0 = \frac{1}{2}x^2 - 2x - 8 - k$

Then $a = \frac{1}{2}$ $b = -2$ and $c = -8 - k$

$b^2 - 4ac = 0$

$(-2)^2 - 4(\frac{1}{2})(-8-k) = 0$

$4 - 2(-8-k) = 0$

$4 + 16 + 2k = 0$

$2k = -20$

$k = -10$

∴ The equation of the line must be $y = 4x - 10$