Chapter 1- Functions

Lesson Package

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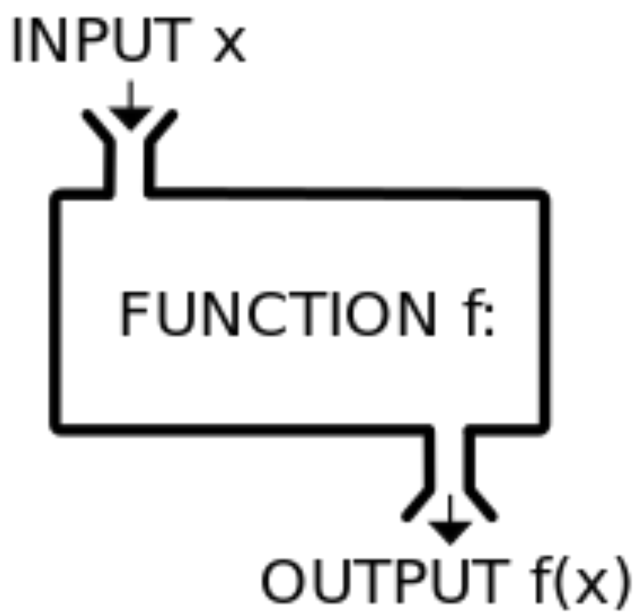

Chapter 1 Outline

Unit Goal: By the end of this unit, you will have an understanding of what a function is and their different representations. You will be able to determine the zeros and the max or min of a quadratic function. You will also be able to simplify expressions involving radicals.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Functions, Domain, and Range	- distinguish a function from a relation - explain meanings of the terms domain and range	A1.1
L2	Function Notation	- explain the meaning of the term function - represent functions using function notation	A1.1, A1.2
L3	Max or Min of a Quadratic	- determine the max or min value of a quadratic function using completing the square and partial factoring	A2.2, A2.3
L4	Radical Expressions	- simplify radical expressions by adding, subtracting, and multiplying	A3.2
L5	Solve Quadratics by Factoring	- Determine the zeros of a quadratic by factoring	A2.1, A2.3
L6	Solve Quadratics using Quaratic Formula	- Determine the zeros of a quadratic using the quadratic formula	A2.1, A2.3
L8	Linear Quadratic Systems	- Solve problems involving the intersection of a linear function and a quadratic function	A2.5

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Max/Min of Quadratic	F		P	
PreTest Review	F/A		P	
Test - Functions	O	A1.1, A1.2, A2.1, A2.2, A2.3, A2.5, A3.2	P	K(21%), T(34%), A(10%), C(34%)

1.1 Functions, Domain, and Range - Lesson
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Section 1: Relation vs. Function

Definitions

Relation –

Functions –

> ***Note:*** *All functions are relations but not all relations are functions. For a relation to be a function, there must be only one 'y' value that corresponds to a given 'x' value.*

Function or Relation Investigation

1) Complete the following tables of values for each relation:

$y = x^2$

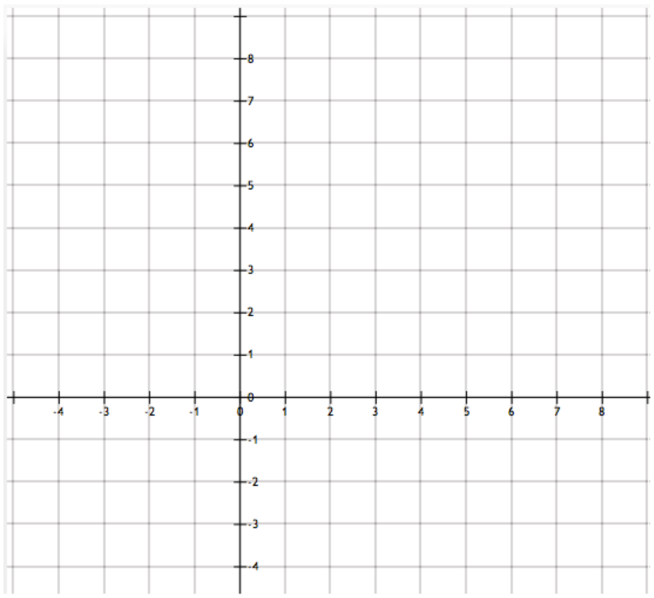
x	y
-3	
-2	
-1	
0	
1	
2	
3	

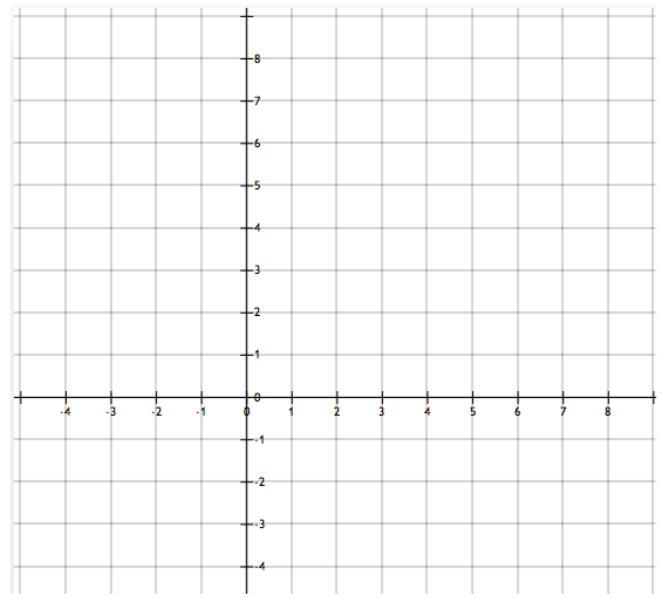
$x = y^2$

x	y
	-3
	-2
	-1
	0
	1
	2
	3

2) Graph both relations

$y = x^2$ $x = y^2$





3) Draw the vertical lines $x = -2,\ x = -1,\ x = 0,\ x = 1$, and $x = 2$ on the graphs above.

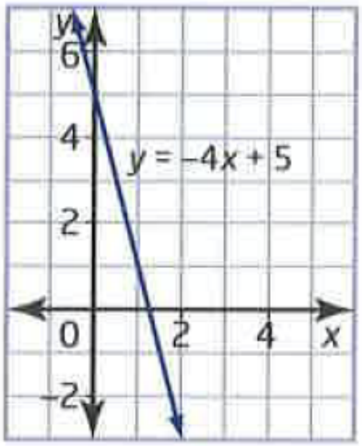
4) Compare how the lines drawn in step 3 intersect each of the relations. Which relation is a function? Explain why.

Section 2: Vertical Line Test

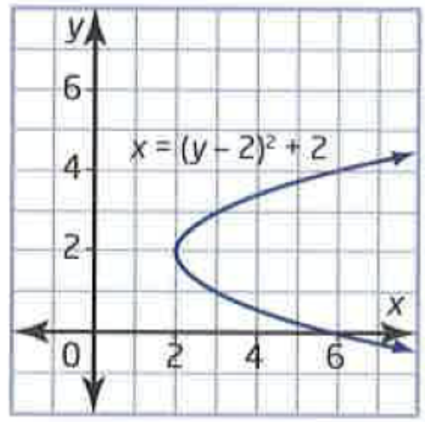
Vertical line test:

Example 1: Use the vertical line test to determine whether each relation is a function or not.

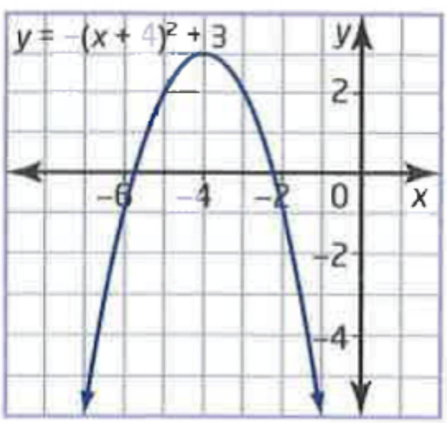
a)



b)



c)



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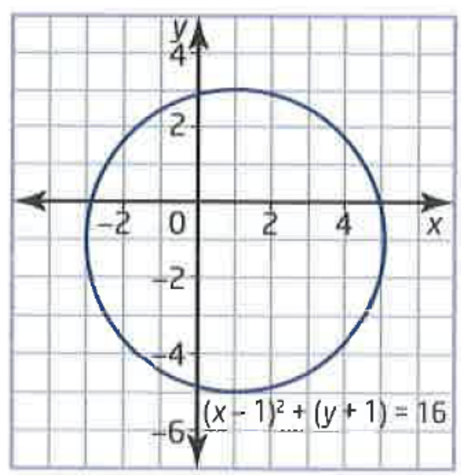


Section 3: Domain and Range

For any relation, the set of values of the independent variable (often the x-values) is called the ____________________ of the relation. The set of the corresponding values of the dependent variable (often the y-values) is called the _______________ of the relation.

Note: *For a function, for each given element of the domain there must be exactly one element in the range.*

Domain:

Range:

General Notation

Real number: *a number in the set of all integers, terminating decimals, repeating decimals, non-terminating decimals, and non repeating decimals. Represented by the symbol* $\mathbb{R}$

Example 2: Determine the domain and range of each relation from the data given.

a) { (-3, 4), (5, -6), (-2, 7), (5, 3), (6, -8) }

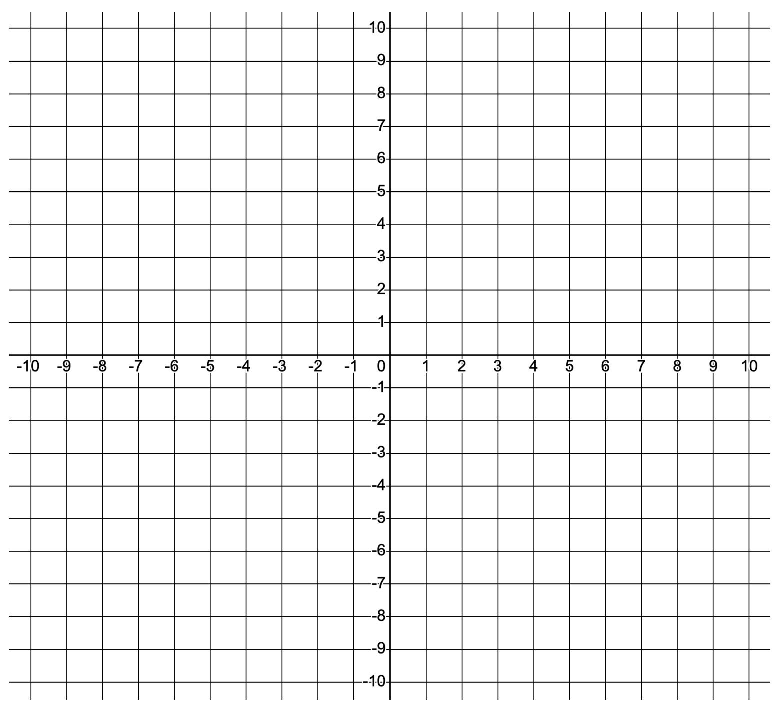
b)

Age	$Number$
4	8
5	12
6	5
7	22
8	14
9	9
10	11

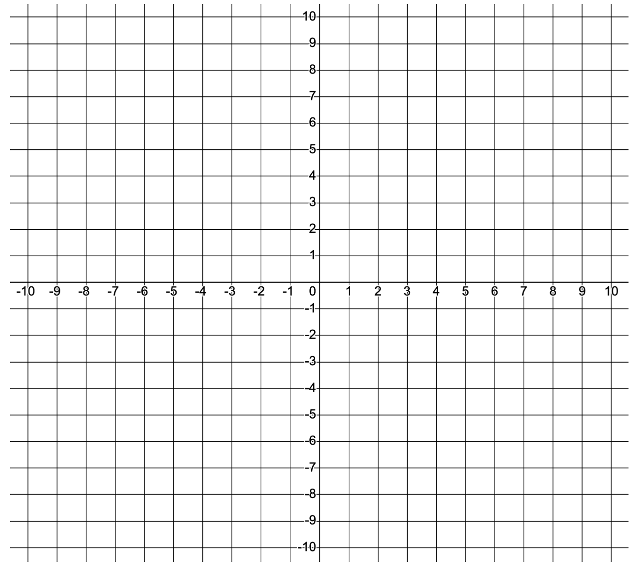
Are each of these relations functions?

Example 3: Determine the domain and range of each relation. Graph the relation first.

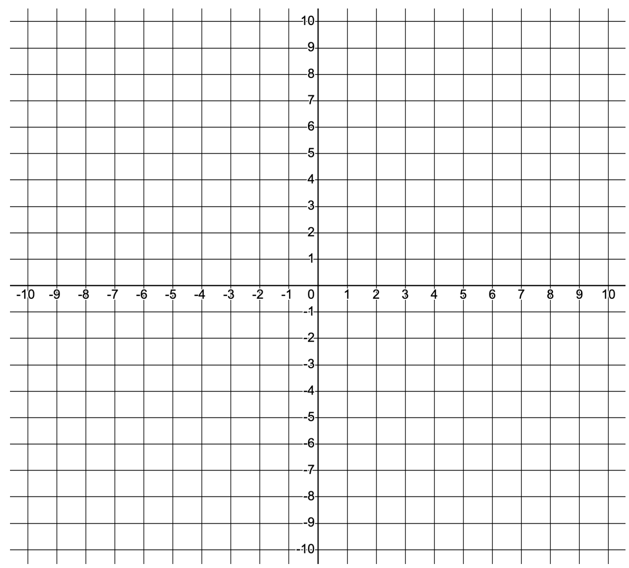
a) $y = 2x - 5$



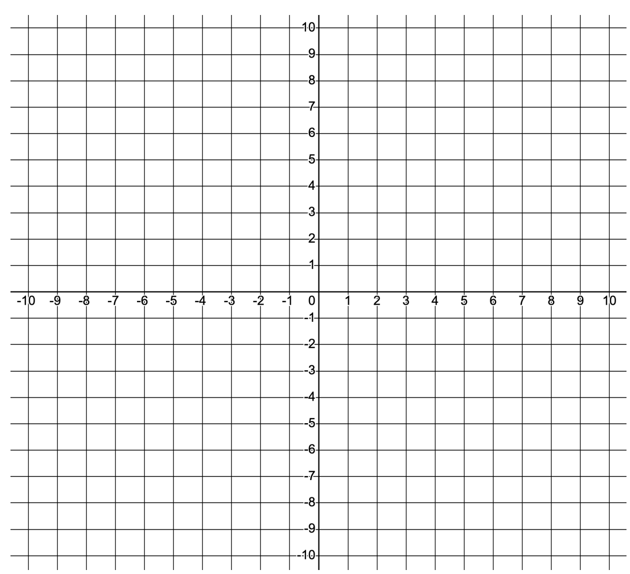
b) $y = (x-1)^2 + 3$



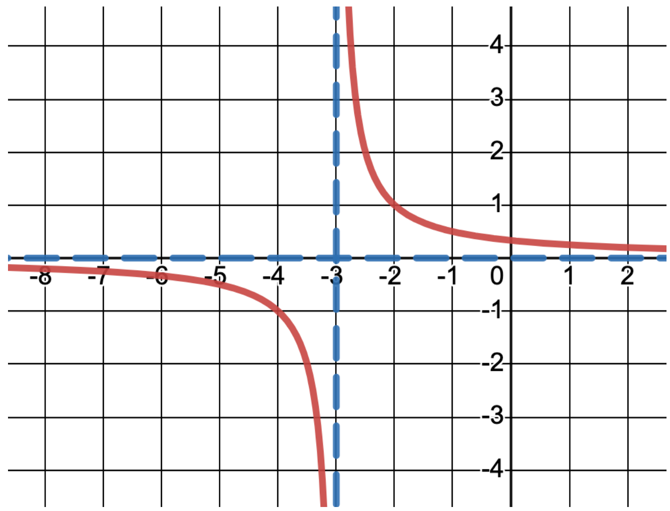
c) $y = \sqrt{x-1} + 3$



d) $x^2 + y^2 = 36$



e) $y = \frac{1}{x+3}$



Asymptotes

Asymptote:

The function $\boldsymbol{y = \frac{1}{x+3}}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq -3$. This is why the vertical line $x = -3$ is an asymptote for this function.

Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.

L2 - 1.2 Functions and Function Notation

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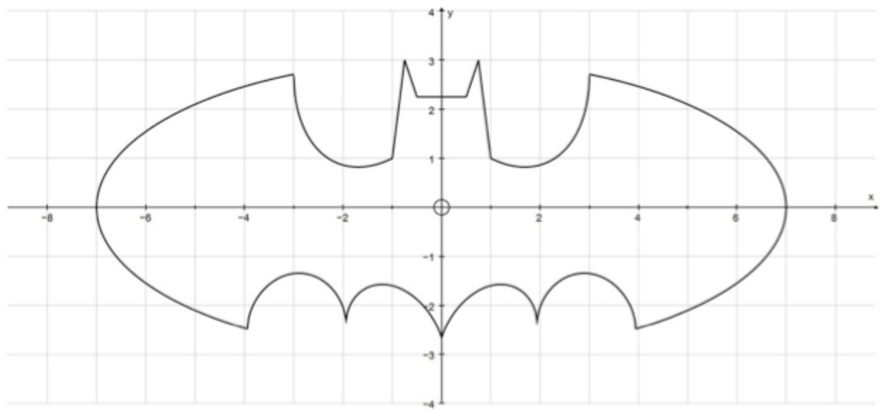
Part 1: Domain & Range Review

a) State the domain and range of the relation shown in the following graph:



b) Is this a function?

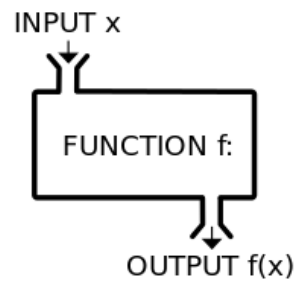
c) What determines if a relation is a function or not?

d) How does the vertical line test help us determine if a relation is a function?

e) What is domain?

f) What is range?

Part 2: Find Values Using Function Notation



What does a function do?

What does function notation look like?

Example 1: For each of the following functions, determine $f(2)$, $f(-5)$, and $f(1/2)$

a) $f(x) = 2x - 4$

b) $f(x) = 3x^2 - x + 7$

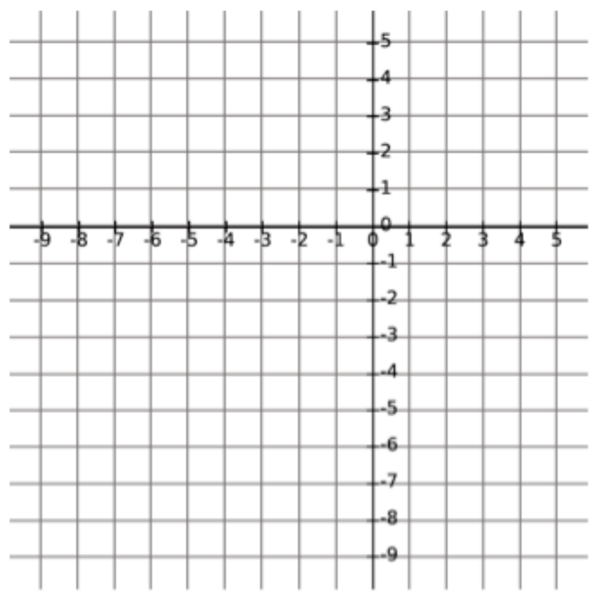
c) $f(x) = 87$

d) $f(x) = \frac{2x}{x^2-3}$

Part 3: Applications of Function Notation

Example 3: For the function $h(t) = -3(t+1)^2 + 5$

i) Graph it and find the domain and range



ii) Find $h(-7)$

Example 4: The temperature of the water at the surface of a lake is 22 degrees Celsius. As Geno scuba dives to the depths of the lake, he finds that the temperature decreases by 1.5 degrees for every 8 meters he descends.

a) Model the water temperature at any depth using function notation.

b) What is the water temperature at a depth of 40 meters?

c) At the bottom of the lake the temperature is 5.5 degrees Celsius. How deep is the lake?

L3 - 1.3 Max or Min of a Quadratic Function
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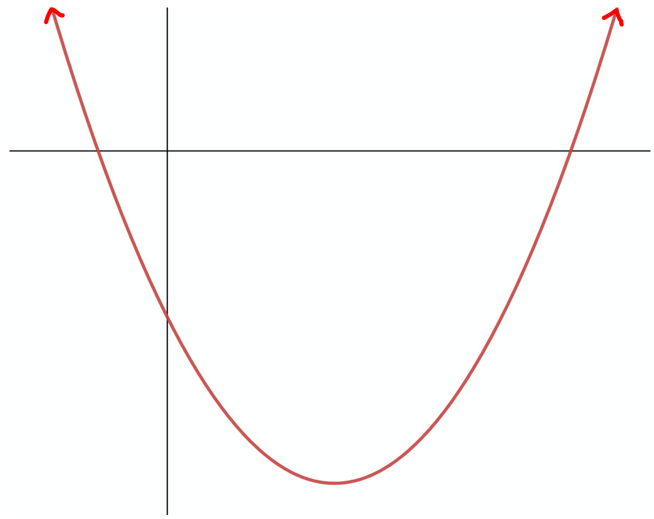
<u>Part 1: Quadratics Review</u>

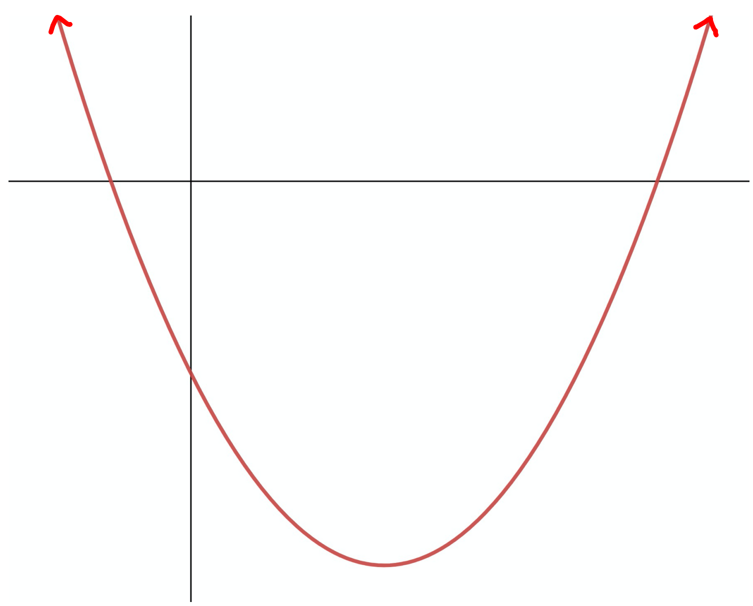
Vertex Form:

$y = a(x - h)^2 + k$



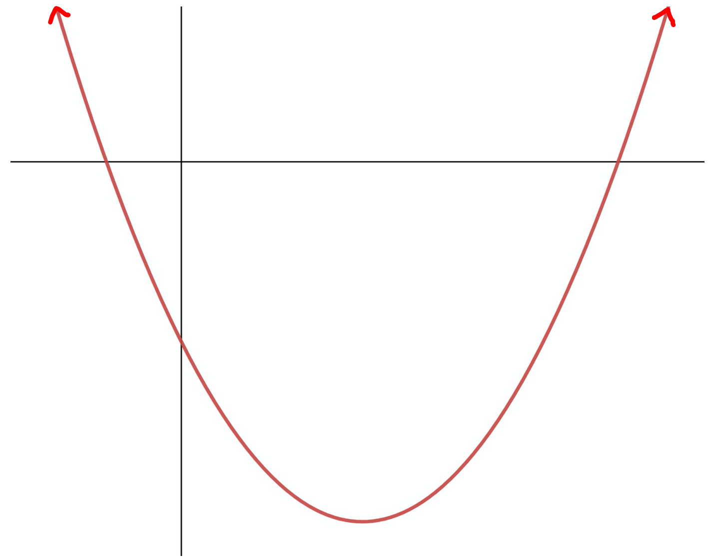
Factored Form:

$y = a(x - r)(x - s)$



Standard Form:

$\boldsymbol{y = ax^2 + bx + c}$



Part 2: Perfect Square Trinomials

Completing the square is a process for changing a standard form quadratic equation into vertex form

$$y = ax^2 + bx + c \rightarrow y = a(x - h)^2 + k$$

Notice that vertex form contains a $(x - h)^2$. A binomial squared can be obtained when factoring a perfect square trinomial:

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

The process of completing the square involves creating this perfect square trinomial within the standard form equation so that it can be factored to create the vertex form equation.

Let's start by analyzing the following perfect square trinomials. Specifically notice how the middle term is 2 times the product of the square roots of the first and last terms.

$x^2 + 10x + 25$ $\qquad\qquad$ $x^2 - 12x + 36$

Example 1: Determine the value of k that would make each quadratic a perfect square trinomial. Then factor the trinomial.

a) $x^2 + 14x + k$

b) $x^2 - 24x + k$

Tip: You can calculate the constant term that makes the quadratic a PST by squaring half of the coefficient of the x term.

Note: *this only works when the coefficient of x^2 is 1.*

Part 3: Completing the Square

Completing the Square Steps

$$ax^2 + bx + c \rightarrow a(x-h)^2 + k$$

1) Put brackets around the first 2 terms
2) Factor out the constant in front of the x^2 term
3) Look at the last term in the brackets, divide it by 2 and then square it
4) Add AND subtract that term behind the last term in the brackets
5) Move the negative term outside the brackets by multiplying it by the $'a'$ value
6) Simplify the terms outside the brackets
7) Factor the perfect square trinomial

$$a^2 + 2ab + b^2 = (a+b)^2$$

Example 2: Rewrite each quadratic in vertex form by completing the square. Then state the vertex, whether it is a max or min point, and the axis of symmetry.

a) $y = x^2 + 8x + 5$

b) $y = 2x^2 - 12x + 11$

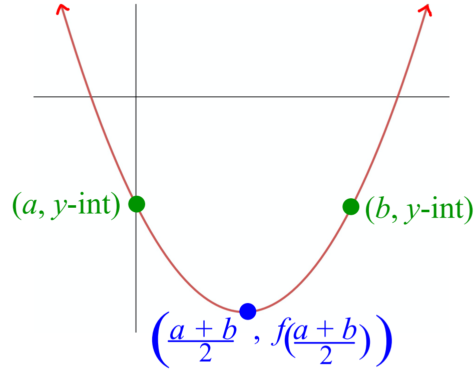
c) $y = -3x^2 + 9x - 13$

d) $y = -\frac{2}{3}x^2 + 8x + 5$

Part 4: Partial Factoring (another method to find the vertex)

Partial Factoring Steps

1) Set the quadratic equal to the y-intercept
2) Solve the equation for x
3) Find the x-value of the vertex by averaging your answers from the previous step
4) Substitute the x-value of the vertex into the original equation and solve for y-value



Example 3: Use partial factoring to find the vertex. Then state if it is a max or min.

a) $y = x^2 + 2x - 6$

b) $y = 4x^2 - 12x + 3$

c) $y = -3x^2 + 9x - 2$

Example 4: Maximizing Revenue

Rachel and Ken are knitting scarves to sell at the craft show. They were planning to sell the scarves for $10 each, the same as last year when they sold 40 scarves. However, they know that if they adjust the price, they might be able to make mor profit. They have been told that for every 50-cent increase in the price, they can expect to sell four fewer scarves. What selling price will maximize their revenue and what will the revenue be?

L4 - 1.4-Working with Radicals
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Investigation

a) Complete the following table:

A	B
$\sqrt{4} \times \sqrt{4} =$	$\sqrt{4 \times 4} =$
$\sqrt{81} \times \sqrt{81} =$	$\sqrt{81 \times 81} =$
$\sqrt{225} \times \sqrt{225} =$	$\sqrt{225 \times 225} =$
$\sqrt{5} \times \sqrt{5} =$	$\sqrt{5 \times 5} =$
$\sqrt{31} \times \sqrt{31} =$	$\sqrt{31 \times 31} =$
$\sqrt{12} \times \sqrt{9} =$	$\sqrt{12 \times 9} =$
$\sqrt{23} \times \sqrt{121} =$	$\sqrt{23 \times 121} =$

b) What do you notice about the results in each row?

c) Make a general conclusion about an equivalent expression for $\sqrt{a} \times \sqrt{b}$

Definitions

Radicand:

Entire Radical:

Mixed Radical:

<u>More About Radicals</u>

Some numbers cannot be expressed as fractions. These are called _________________ numbers. One type of irrational number is of the form $\sqrt{n}$ where n is not a perfect square. These numbers are sometimes called _________________.

An approximate value can be found for these irrational numbers using a calculator but it is better to work with an exact value. Answers should be left in radical form when an EXACT answer is needed. Sometimes entire radicals can be simplified by removing perfect square factors. The resulting expression is called a ______________________________.

PERFECT SQUARES

$1^2 = 1$	$11^2 = 121$
$2^2 = 4$	$12^2 = 144$
$3^2 = 9$	$13^2 = 169$
$4^2 = 16$	$14^2 = 196$
$5^2 = 25$	$15^2 = 225$
$6^2 = 36$	$16^2 = 256$
$7^2 = 49$	$17^2 = 289$
$8^2 = 64$	$18^2 = 324$
$9^2 = 81$	$19^2 = 361$
$10^2 = 100$	$20^2 = 400$

Example 1: Express each radical as a mixed radical in simplest form.

Hint: *remove perfect square factors and then simplify*

a) $\sqrt{50}$ **b)** $\sqrt{27}$ **c)** $\sqrt{180}$

Adding and Subtracting Radicals

Adding and subtracting radicals works in the same way as adding and subtracting polynomials. You can only add ___________ terms or, in this case, _____________ _____________________.

Example:

$2\sqrt{3} + 5\sqrt{7}$ cannot be added because they do not have the same radical.

However, $3\sqrt{5} + 6\sqrt{5}$ have a common radical, so they can be added. $3\sqrt{5} + 6\sqrt{5} = 9\sqrt{5}$

Note, the radical stays the same when adding and subtracting expressions with like radicals.

Example 2: Simplify the following

a) $9\sqrt{7} - 3\sqrt{7}$

b) $4\sqrt{3} - 2\sqrt{27}$

c) $5\sqrt{8} + 3\sqrt{18}$

d) $\frac{1}{4}\sqrt{28} - \frac{3}{4}\sqrt{63} + \frac{2}{3}\sqrt{50}$

Multiplying Radicals

Example 3: Simplify fully

a) $(2\sqrt{3})(3\sqrt{6})$

Multiply the coefficients together and then multiply the radicands together. Then simplify!

b) $2\sqrt{3}(4+5\sqrt{3})$

Don't forget the distributive property:

$a(x+y) = ax + ay$

c) $-7\sqrt{2}(6\sqrt{8}-11)$

d) $(\sqrt{3}+5)(2-\sqrt{3})$

Don't forget FOIL. Each term in the first binomial must be multiplied by each term in the second binomial.

e) $(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})$

There is a shortcut! This is a difference of squares.

$(a+b)(a-b) = a^2 - b^2$

L5-1.5-Solving Quadratic by Factoring

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Review of Radicals

Simplify each of the following:

1) $(\sqrt{2}+3\sqrt{3})(5\sqrt{3}-10)$

2) $\dfrac{2-\sqrt{80}}{4}$

3) $4\sqrt{10}\left(3+2\sqrt{2}\right)$

How to Solve Quadratics

Solving a quadratic means to find the x-intercepts (or roots).

To solve a quadratic equation:

1) It must be set to equal 0. Before factoring, it must be in the form $ax^2+bx+c=0$
2) Factor the left side of the equation
3) Set each factor to equal zero and solve for 'x'.

zero product rule: if two factors have a product of zero; one or both of the factors must equal zero.

Example 1: Solve the following quadratics by factoring

a) $0 = x^2 - 15x + 56$

When factoring ax²+bx+c=0 when 'a' is 1 or can be factored out

Steps to follow:

1) Check if there is a common factor that can be divided out
2) Look at the 'c' value and the 'b' value
3) Determine what factors multiply to give 'c' and add to give 'b'
4) put those factors into (x+r)(x+s) for 'r' and 's'.
5) make sure nothing else can be factored

b) $-6 = x^2 - 5x$

c) $0 = 2x^2 - 8x - 42$

Example 2: Solve by factoring

a) $8x^2 + 2x - 15 = 0$

Steps for factoring $ax^2 + bx + c$ when $a \neq 1$

1) Check for any common factors that can be factored out
2) Replace the middle term bx with two terms whose coefficients have a sum of b and a product of $a \times c$
3) Group pairs of terms and remove a common factor from each pair
4) Remove the common binomial factor

b) $2x^2 - 11x = -15$

Example 3: For the quadratic $y = 2x^2 - 4x - 16$

a) Find the roots of the quadratic by factoring

b) Find the axis of symmetry (average of x-intercepts)

c) Find the coordinates of the vertex and state if it is a max or min value

L6 – 1.5 - Solve Using the Q.F.

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Part 1: DO IT NOW!

a) Do you remember the quadratic formula?

b) Use the quadratic formula to find the x-intercepts of:

Don't forget that to solve a quadratic, it must be set equal to zero because at an x-intercept, the y-coordinate will be zero.

$$\mathbf{0 = 2x^2 + 7x - 4}$$

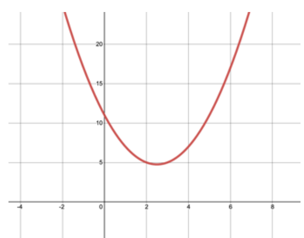
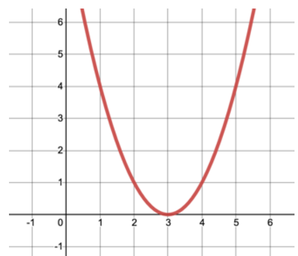
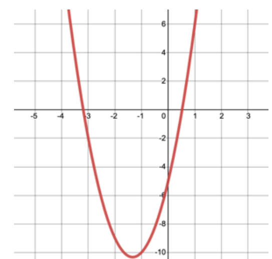
Part 2: Discriminant Review

Do all parabolas have two x-intercepts?

What are the three different scenarios?

The way to determine how many x-intercepts a parabola might have is by evaluating the $b^2 - 4ac$ part of the quadratic formula (called the "**discriminant**")

Discriminant: the value under the square root

# of solutions	0	1	2
Discriminant	$b^2 - 4ac < 0$	$b^2 - 4ac = 0$	$b^2 - 4ac > 0$
Example Equation	$0 = x^2 - 5x + 11$ $x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(11)}}{2(1)}$ $x = \frac{5 \pm \sqrt{-19}}{2}$ No real solutions Notice $b^2 - 4ac = -19$ which is less than zero	$0 = x^2 - 6x + 9$ $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$ $x = \frac{6 \pm \sqrt{0}}{2(1)}$ $x = 3$ Notice $b^2 - 4ac = 0$	$0 = 3x^2 + 8x - 5$ $x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(-5)}}{2(3)}$ $x = \frac{-8 \pm \sqrt{124}}{6}$ $x = -3.19, 0.52$ Notice $b^2 - 4ac = 124$ which is greater than zero
Example Graph	 Notice the graph never crosses the x-axis	 Notice the vertex is ON the x-axis	 Notice the graph crosses through the x-axis twice

Another piece of useful information about the discriminant:

If $b^2 - 4ac$ is a ______________________ number, you will get rational solutions which means the quadratic is factorable. If you aren't sure if a quadratic is factorable, just check to see if $b^2 - 4ac$ is a perfect square number (0,1,4,9,16,25,36, ...)

Part 3: Solve a Quadratic with 2 Roots

Exact answer: *as a radical or fraction. Exact answers do not have decimals.*

Example 1: Find the exact solutions of $3x^2 - 10x + 5 = 0$

Example 2: Find the exact solutions of $-2x^2 + 8x - 5 = 0$

Part 4: Solving a Quadratic with 1 Root

Note: *when a quadratic only has 1 solution, the x-intercept is also the vertex*

Example 3: Find the exact roots of $4x^2 + 24x + 36 = 0$

Part 5: Solving Quadratics with 0 Roots

2 Scenarios causing 0 roots:

i) vertex is above the x-axis and opens up

ii) vertex is below the x-axis and opens down

Example 4: Find the x-intercepts of $8x^2 - 11x + 5 = 0$

Part 6: Use the Discriminant to Determine the Number of Roots

Example 5: For each of the following quadratics, use the discriminant to state the number of roots it will have.

a) $2x^2 + 5x - 5 = 0$

b) $3x^2 - 7x + 5 = 0$

c) $-4x^2 + 12x - 9 = 0$

Part 7: Application

Example 6: A ball is thrown and the equation below model it's path:

$$\boldsymbol{h = -0.25d^2 + 2d + 1.5}$$

'h' is the height in meters above the ground and 'd' is the horizontal distance in meters from the person who threw the ball.

a) At what height was the ball thrown from?

b) How far has the ball travelled horizontally when it lands on the ground?

L7-1.7-Solve Linear-Quadratic Systems
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Review of Solving Linear Systems

Solve the following linear system using the method of substitution:

$y = 3x + 7$

$y = 2x - 5$

Recall: *solving a linear system means to find the point of intersection (POI)*

Method of Substitution: *solving a linear system by substituting for one variable from one equation into the other equation.*

Steps to Solving A Linear-Quadratic System

1. Set equations equal to each-other

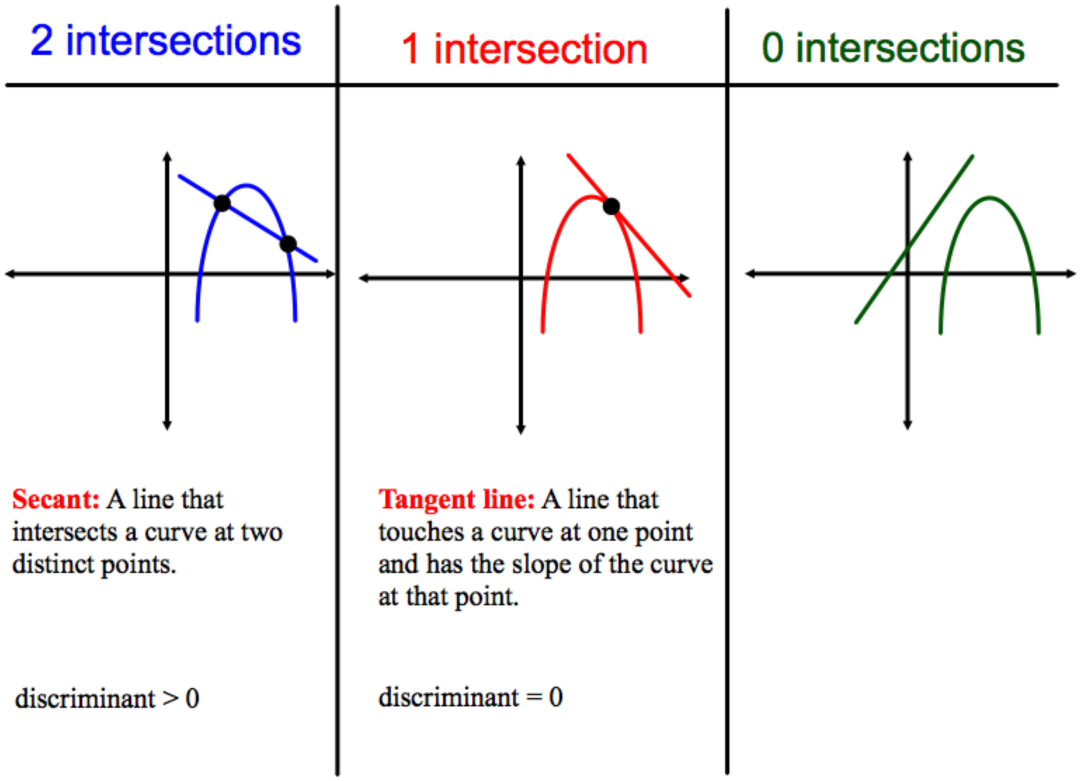
Line = Parabola

2. Rearrange to set the equation equal to zero

3. Solve for x by factoring or using the QF (the solution will tell you for what value of x the functions have the same y value)

4. Plug this value of x back in to either of the original functions to solve for y.

<u>Possible solutions for a linear-quadratic system:</u>



Example 1:

a) How many points of intersection are there for the following system of equations?

$f(x) = \frac{1}{2}x^2 + 2x - 8$

$g(x) = 4x - 10$

b) Solve the linear-quadratic system (give exact answers)

Example 2: Solve the following linear-quadratic system

$y = 3x^2 + 21x - 5$
$y = 10x - 1$

Part 4: Application

Example 3: If a line with slope 4 has one point of intersection with the quadratic function $y = \frac{1}{2}x^2 + 2x - 8$, what is the y-intercept of the line? Write the equation of the line in slope y-intercept form.

Recall: equation of a line is $y = mx + b$ where b is the y-intercept and m is the slope.

Recall: *for a lin-quad system to have 1 solution, the discriminant must be zero.*