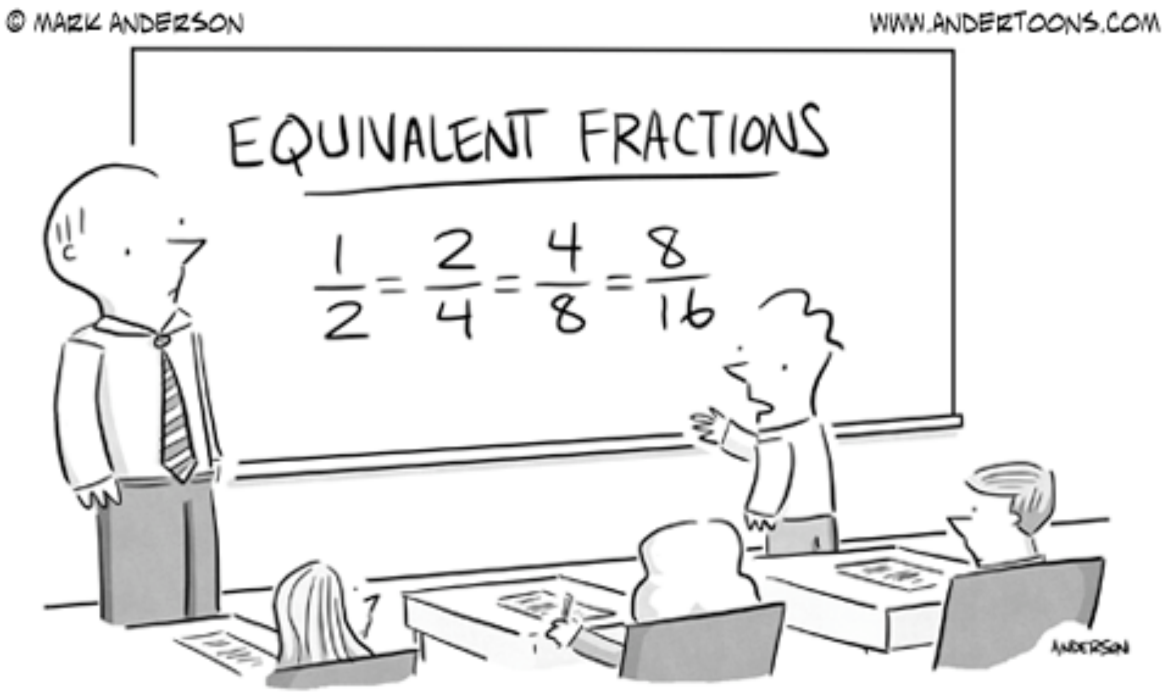
Chapter 2(part 1)
Rational Expressions

Lesson Package

MCR3U



"I understand they all have the same value, but I have to tell you, the ones on the right feel like more bang for your buck."

Unit Goal: By the end of this unit, you will be able to evaluate powers with rational exponents and simplify expressions involving exponents. You will be able to demonstrate an understanding of equivalence as it relates to simplifying polynomial, radical, and rational expressions.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Intro to Rational Expressions	- simplify expressions involving exponents	B1.2, B1.3
L2	Rational Exponents	- simplify and evaluate powers with rational exponents	B1.2, B1.3
L3	Multiply and Divide Rational Expressions	- determine the max or min value of a quadratic function using completing the square and partial factoring	A3.3
L4	Add and Subtract Rational Expressions	- simplify radical expressions by adding, subtracting, and multiplying	A3.3

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Simplify Rational Expressions	F		P	
PreTest Review	F/A		P	
Test – Simplifying Rational Expressions	O	B1.2, B1.3, A3.3	P	K(30%), T(30%), A(10%), C(30%)

Intro to Rational Expressions – Fractions and Exponents Review – Lesson

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Fractions Review

Always find a common denominator when adding or subtracting fractions!

Adding and Subtracting Fractions

a) $\frac{1}{2}+\frac{1}{8}$

b) $\frac{2x}{3y}-\frac{y}{2}$

Multiplying and Dividing Fractions

You do NOT need a common denominator when multiplying or dividing fractions!

a) $\frac{3}{2}\cdot\frac{4}{5}$

b) $\frac{2}{3}\div\frac{4}{3}$

Rule: We can NEVER have a fraction with a denominator of 0. Why?

Rule: Cross multiplication of fractions only happens when…

Rule: We can cancel out ONLY when multiplying fractions

Rule: We can NOT cancel out when adding or subtracting fractions

Name	Rule	Examples
Adding and Subtracting Monomials	COMBINE LIKE TERMS! (do not change common variables and exponents)	$3x^2y + 2x^2y =$
Product Rule	$x^a \cdot x^b =$	$(-2x^2y)(3x^3y^2) =$
Quotient Rule	$\frac{x^a}{x^b} =$	$\frac{28x^5}{42x} =$
Power of a Power Rule	$(x^a)^b =$	$(-2x^3)^2 =$
Negative Exponent Rule	$x^{-a} =$	$\frac{4x^2}{8x^5} =$
Exponent of Zero	x^0	$87^0 =$

Simplify the following rational expressions using exponent laws.

a) $\frac{12k^2m^8}{4k^5m^5}$

b) $\frac{5c^3d \cdot 4c^2d^2}{(2c^2d)^2}$

c) $\frac{(3xy)^3}{9x^4y^4}$

d) $\frac{(2z^3)^{-2}}{w^5z^2}$

e) $\frac{(x^{-4})^5x^3}{3x^{-1}}$

Combining fractions and exponents

Ex. $\frac{3x^3}{2x^2} + \frac{4y^4}{3y}$

3.3 Rational Exponents – Lesson
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Intro to Rational Exponents (fraction exponents):

$$\sqrt{x}$$

Powers with a rational exponent of the form $\frac{1}{n}$

A power involving a rational exponent with numerator 1 and denominator *n* can be interpreted as the *n*th root of the base:

Example 1: Evaluate each of the following

a) $8^{\frac{1}{3}}$

b) $\sqrt[5]{-32}$

c) $-16^{\frac{1}{4}}$

d) $\sqrt[4]{\frac{16}{81}}$

e) $(-27)^{-\frac{1}{3}}$

Powers with a rational exponent of the form $\frac{m}{n}$

You can evaluate a power involving a rational exponent with numerator *m* and denominator *n* by taking the *n*th root of the base raised to the exponent *m*:

Example 2: Simplify each of the following powers

a) $\sqrt[5]{y^2}$

b) $\sqrt[3]{x}$

c) $\sqrt{a^{-3}b^{\frac{4}{3}}}$

d) $\sqrt[4]{x^3y^2}$

e) $\frac{\sqrt[3]{x^2y}\cdot y^2}{x^3}$

Example 3: Evaluate each of the following

a) $8^{\frac{2}{3}}$

b) $81^{\frac{5}{4}}$

c) $\left(\frac{49}{81}\right)^{-\frac{3}{2}}$

Apply Exponent Rules

Example 4: Simplify and express answer using only positive exponents

a) $\dfrac{\left(x^{\frac{2}{3}}\right)\left(x^{\frac{2}{3}}\right)}{\left(x^{\frac{1}{3}}\right)}$

b) $\left(y^{\frac{1}{4}}\right)^2 \times \left(y^{-\frac{1}{3}}\right)^2$

c) $\left(5x^{\frac{1}{2}}\right)^2 \times 4x^{-\frac{1}{2}}$

d) $\dfrac{(m^{-2})^3\sqrt{m^4}}{m\sqrt{pq^{-3}}}$

e) $\dfrac{(x^2)^{-4} \cdot \sqrt[5]{y^3}}{y\sqrt{x^{-2}y}}$

2.1/2.2 Multiplying and Dividing Rational Expressions – Lesson
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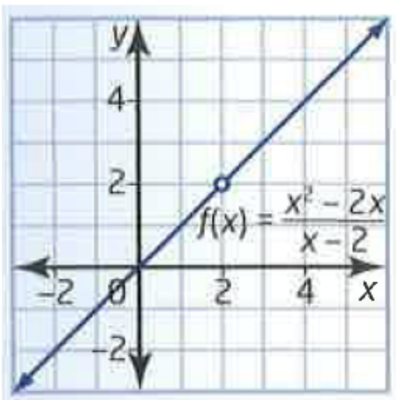
What is a Rational Expression?

Rational Expression:

Example of a graph of a rational expression:



The open circle is used to represent a hole in the graph. This corresponds to any restrictions on the variable (denominator can't be 0).

Stating Restrictions

Note: rational expressions must be checked for restrictions by determining where the denominator is equal to ___________. These restrictions must be stated when the expression is simplified.

Example 1: State the restrictions for the following rational expressions

a) $\frac{x+2}{x-2}$ **b)** $\frac{x+2}{(x-3)(x+4)}$ **c)** $\frac{5}{x(x+3)}$

Rule: We can cancel out ONLY when multiplying fractions

Rule: We can **NOT** cancel out when adding or subtracting fractions

Simplifying Rational Expressions

Example 2: Simplifying each expression and determine any restrictions on the variable.

> ***Note:*** *factor where possible and then state restrictions before cancelling factors.*

a) $\frac{3x^2}{yx}$

b) $\frac{x-3}{x^2+3x-18}$

c) $\frac{x^2+10x+21}{x+3}$

d) $\frac{x+1}{x^2+3x+2}$

e) $\frac{x^2-9}{x^2+7x+12}$

f) $\frac{6x^2-7x-5}{3x^2+x-10}$

Multiplying Rational Expressions

1. factor where possible
2. cancel common factors
3. multiply numerators and denominators
4. state restrictions (throughout process)

a) $\frac{4x^2}{3x} \cdot \frac{12x^3}{2x}$

b) $\frac{4x+24}{x^2+8x} \cdot \frac{12x^2}{3x+18}$

c) $\frac{a^2+2a}{3a} \cdot \frac{20a^2}{5a^2+10a}$

d) $\frac{x+1}{2x} \cdot \frac{3x}{x^2+4x+3}$

e) $\frac{5x^2-13x+8}{x-7} \cdot \frac{1}{5x-8}$

Dividing Rational Expressions

a) $\frac{10ab^2}{4a} \div \frac{15a^2}{12b^2}$

1. flip second fraction and change to multiplication
2. factor where possible
3. cancel common factors
4. multiply numerators and denominators
5. state restrictions (throughout process)

b) $\frac{a^2+2a}{3a} \div \frac{5a^2+10a}{20a^2}$

c) $\frac{2x^2-8x}{x^2-3x-10} \div \frac{4x^2}{x^2-9x+20}$

2.2 Adding and Subtracting Rational Expressions – Lesson
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DO IT NOW

a) $\frac{1}{6}+\frac{1}{5}$

> ***Note:*** *the product of the denominators will give a common denominator (but not always the lowest common denominator)*

b) Simplify and state restrictions

$\frac{x^2-1}{x^2-4}\times\frac{x^2+3x-4}{x^2+5x+4}$

c) $\frac{2x}{3y}+\frac{y}{6x}$

Add and Subtract Rational Expressions With Monomial Denominators

a) $\frac{1}{5x}+\frac{1}{2x}$

b) $\frac{ab^2+2}{2ab^2}-\frac{b+2}{2b}$

Add and Subtract Rational Expressions with Polynomial Denominators

a) $\frac{x+5}{x-3}+\frac{x-7}{x+2}$

1. get a common denominator **2.** expand numerators **3.** add/subtract fractions **4.** simplify where possible **5.** state restrictions (throughout process)

b) $\frac{x+9}{x^2+2x-48}-\frac{x-9}{x^2-x-30}$

c) $\frac{x-2}{x+2}+\frac{x+10}{x^2+6x+8}$

d) $\frac{2x}{x-1}-\frac{x+2}{x^2+3x-4}$

e) $\frac{a+1}{5-2a}-\frac{a-4}{2a-5}$