Chapter 2a – Rational Expressions – REVIEW
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SOLUTIONS

Section 1: Negative and Rational Exponents

1) Evaluate. Express as a fraction in lowest terms.

a) 10^{-1}

$= \frac{1}{10}$

b) 4^{-2}

$= \frac{1}{4^2}$

$= \frac{1}{16}$

c) $3^{-2} + 9^{-1}$

$= \frac{1}{9} + \frac{1}{9}$

$= \frac{2}{9}$

d) $5^{-3} + 5^{0}$

$= \frac{1}{125} + 1$

$= \frac{126}{125}$

e) $\left(\frac{1}{5}\right)^{-1}$

$= \left(\frac{5}{1}\right)^{1}$

$= 5$

f) $\left(\frac{3}{4}\right)^{-3}$

$= \left(\frac{4}{3}\right)^{3}$

$= \frac{64}{27}$

2) Simplify. Express your answers using only positive exponents.

a) $(x^{-2})(x^{-1})(x^{0})$

$= x^{-3}$

$= \frac{1}{x^3}$

b) $(3km^2)(2k^{-2}m^{-2})$

$= 6k^{-1}m^{0}$

$= \frac{6}{k}$

c) $w^{-3} \div w^{-2}$

$= w^{-1}$

$= \frac{1}{w}$

d) $\frac{u^{-2}v^{3}}{u^{-3}v^{-2}}$

$= uv^{5}$

e) $(z^{-3})^{-2}$

$= z^{6}$

f) $(2ab^{-1})^{-2}$

$= \frac{1}{(2ab^{-1})^2}$

$= \frac{1}{4a^2b^{-2}}$

$= \frac{b^2}{4a^2}$

3) Simplify. Express your answers using only positive exponents.

a) $(4a^{-2})(-2a^{-3})$

$= -8a^{-5}$

$= \frac{-8}{a^5}$

b) $\frac{(2x^2y)^{-2}(3xy)^{-1}}{(6x^2y^2)^{-2}}$

$= \frac{(6x^2y^2)^2}{(2x^2y)^2(3xy)^1}$

$= \frac{36x^4y^4}{4x^4y^2(3xy)}$

$= \frac{36x^4y^4}{12x^5y^3}$

$= 3x^{-1}y$

$= \frac{3y}{x}$

c) $\left(\frac{1}{4x^2}\right)^{-2}$

$= \left(\frac{4x^2}{1}\right)^2$

$= 16x^4$

d) $\left(\frac{6a^3}{4b^4}\right)^{-2}$

$= \left(\frac{4b^4}{6a^3}\right)^2$

$= \frac{16b^8}{36a^6}$

$= \frac{4b^8}{9a^6}$

4) Evaluate.

a) $\sqrt[3]{64}$

$= 4$

b) $\sqrt[4]{625}$

$= 5$

c) $\sqrt[5]{-3125}$

$= -5$

d) $\left(\frac{1}{64}\right)^{\frac{1}{6}}$

$= \frac{1}{2}$

e) $27^{\frac{2}{3}}$

$= \left(27^{\frac{1}{3}}\right)^2$

$= (3)^2$

$= 9$

f) $(-1000)^{\frac{4}{3}}$

$= \left[(-1000)^{\frac{1}{3}}\right]^4$

$= (-10)^4$

$= 10000$

g) -4^{-3}

$= \frac{-1}{4^3}$

$= \frac{-1}{64}$

h) $\left(\frac{3}{4}\right)^{-2}$

$= \left(\frac{4}{3}\right)^2$

$= \frac{16}{9}$

i) $\left(-\frac{27}{125}\right)^{-\frac{2}{3}}$

$= \left[\left(-\frac{125}{27}\right)^{\frac{1}{3}}\right]^2$

$= \left(\frac{-5}{3}\right)^2$

$= \frac{25}{9}$

5) Simplify. Express answers using only positive exponents.

a) $n^{\frac{1}{2}} \times n^{\frac{1}{3}} \times n^{\frac{1}{4}}$

$= n^{6/12} \cdot n^{4/12} \cdot n^{3/12}$

$= n^{13/12}$

b) $(27y^3)^{\frac{1}{3}} \times \left(\frac{1}{16y^4}\right)^{-\frac{3}{4}}$

$= 3y \times \left(\frac{16y^4}{1}\right)^{\frac{3}{4}}$

$= 3y(8y^3)$

$= 24y^4$

c) $(27x^6)^{\frac{2}{3}} \div (9x^4)^{\frac{1}{2}}$

$= 9x^4 \div 3x^2$

$= 3x^2$

d) $\frac{x^{-\frac{2}{3}}}{x^{-\frac{4}{5}}}$

$= \frac{x^{4/5}}{x^{2/3}}$

$= x^{12/15} \div x^{10/15}$

$= x^{2/15}$

Section 2: Rational Expressions

6) Simplify each expression and state all restrictions on x.

a) $\frac{x+7}{x^2+10+21}$

$= \frac{x+7}{(x+7)(x+3)}$

$= \frac{1}{x+3}$; $x \neq -7, -3$

b) $\frac{x^2-64}{x-8}$

$= \frac{(x-8)(x+8)}{x-8}$

$= x+8$; $x \neq 8$

c) $\frac{x^2-9}{x^2-8x+15}$

$= \frac{(x-3)(x+3)}{(x-3)(x-5)}$

$= \frac{x+3}{x-5}$; $x \neq 3, 5$

7) Simplify each expression and state the restrictions.

a) $\frac{3x^2}{5xy} \times \frac{20xy^3}{12xy}$

$= \frac{60x^3y^3}{60x^2y^2}$

$= xy$; $x \neq 0$, $y \neq 0$

b) $\frac{150a^3b^4}{20a^2b} \div \frac{6b}{8ab^2}$

$= \frac{150a^3b^4}{20a^2b} \times \frac{8ab^2}{6b}$

$= \frac{60a^4b^6}{6a^2b^2}$

$= 10a^2b^4$; $a \neq 0$, $b \neq 0$

c) $\frac{1}{3x}+\frac{5}{2x^2}$

$= \frac{2x(1)}{2x(3x)} + \frac{3(5)}{3(2x^2)}$

$= \frac{2x}{6x^2} + \frac{15}{6x^2}$

$= \frac{2x+15}{6x^2}$; $x \neq 0$

8) Simplify each expression and state restrictions.

a) $\frac{x^2+7x}{3x+21} \times \frac{x^2+3x+2}{x+2}$

$= \frac{x(x+7)}{3(x+7)} \cdot \frac{(x+2)(x+1)}{(x+2)}$

$= \frac{x(x+1)}{3}$; $x \neq -7, -2$

b) $\frac{x^2+4x-60}{3x+30} \div \frac{x^2-8x+12}{6x-12}$

$= \frac{(x+10)(x-6)}{3(x+10)} \times \frac{6(x-2)}{(x-6)(x-2)}$

$= \frac{6}{3}$

$= 2$; $x \neq -10, 2, 6$

c) $\frac{3}{x^2+7x+10}-\frac{5x}{x^2-4}$

$= \frac{3}{(x+2)(x+5)} - \frac{5x}{(x-2)(x+2)}$

$= \frac{3(x-2)}{(x+2)(x+5)(x-2)} - \frac{5x(x+5)}{(x+2)(x+5)(x-2)}$

$= \frac{3x-6-5x^2-25x}{(x+2)(x+5)(x-2)}$

$= \frac{-5x^2-22x-6}{(x+2)(x+5)(x-2)}$; $x \neq -5, -2, 2$

d) $\frac{-10x}{x^2+18x+32}+\frac{12x}{x^2+6x-160}$

$= \frac{-10x}{(x+16)(x+2)} + \frac{12x}{(x+16)(x-10)}$

$= \frac{-10x(x-10)}{(x+16)(x+2)(x-10)} + \frac{12x(x+2)}{(x+16)(x+2)(x-10)}$

$= \frac{-10x^2+100x+12x^2+24x}{(x+16)(x+2)(x-10)}$

$= \frac{2x^2+124x}{(x+16)(x+2)(x-10)}$

$= \frac{2x(x+62)}{(x+16)(x+2)(x-10)}$; $x \neq -16, -2, 10$

9) Simplify each expression and state any restrictions

a) $\frac{x-8}{x+7} \times \frac{x+15}{x^2+12x-45}$

$= \frac{x-8}{x+7} \cdot \frac{x+15}{(x+15)(x-3)}$

$= \frac{x-8}{(x+7)(x-3)}$; $x \neq -7, 3$

b) $\frac{x^2+12x+20}{x+5} \div \frac{x^2+7x-30}{x+10}$

$= \frac{(x+10)(x+2)}{x+5} \cdot \frac{x+10}{(x+10)(x-3)}$

$= \frac{(x+10)(x+2)}{(x+5)(x-3)}$; $x \neq -10, -5, 3$

c) $\frac{x+3}{x-7} - \frac{x+9}{x-2}$

$= \frac{(x+3)(x-2)}{(x-7)(x-2)} - \frac{(x+9)(x-7)}{(x-7)(x-2)}$

$= \frac{x^2+x-6-(x^2+2x-63)}{(x-7)(x-2)}$

$= \frac{-1x+57}{(x-7)(x-2)}$; $x \neq 2, 7$

d) $\frac{x+8}{x+3} + \frac{x-6}{x^2+9x+18}$

$= \frac{x+8}{x+3} + \frac{x-6}{(x+3)(x+6)}$

$= \frac{(x+6)(x+8)}{(x+3)(x+6)} + \frac{x-6}{(x+3)(x+6)}$

$= \frac{x^2+14x+48+x-6}{(x+3)(x+6)}$

$= \frac{x^2+15x+42}{(x+3)(x+6)}$; $x \neq -3, -6$

e) $\frac{5x+1}{2x-1} - \frac{3x-3}{1-2x}$

$= \frac{-1(5x+1)}{-1(2x-1)} - \frac{3x-3}{1-2x}$

$= \frac{-5x-1}{-2x+1} - \frac{3x-3}{1-2x}$

$= \frac{-5x-1}{1-2x} - \frac{3x-3}{1-2x}$

$= \frac{-5x-1-(3x-3)}{1-2x}$

$= \frac{-8x+2}{1-2x}$

$= \frac{-2(4x-1)}{1-2x}$; $x \neq \frac{1}{2}$

Answers

1) a) $\frac{1}{10}$ **b)** $\frac{1}{16}$ **c)** $\frac{2}{9}$ **d)** $\frac{126}{125}$ **e)** 5 **f)** $\frac{64}{27}$

2) a) $\frac{1}{x^3}$ **b)** $\frac{6}{k}$ **c)** $\frac{1}{w}$ **d)** uv^5 **e)** z^6 **f)** $\frac{b^2}{4a^2}$

3) a) $-\frac{8}{a^5}$ **b)** $\frac{3y}{x}$ **c)** $16x^4$ **d)** $\frac{4b^8}{9a^6}$

4) a) 4 **b)** 5 **c)** -5 **d)** $\frac{1}{2}$ **e)** 9 **f)** $10\,000$ **g)** $-\frac{1}{64}$ **h)** $\frac{16}{9}$ **i)** $\frac{25}{9}$

5) a) $n^{\frac{13}{12}}$ **b)** $24y^4$ **c)** $3x^2$ **d)** $x^{\frac{2}{15}}$

6) a) $\frac{1}{x+3}, x \neq -7, x \neq -3$ **b)** $x+8, x \neq 8$ **c)** $\frac{x+3}{x-5}, x \neq 3, 5$

7) a) $xy, x \neq 0, y \neq 0$ **b)** $10a^2b^4, a \neq 0, b \neq 0$ **c)** $\frac{2x+15}{6x^2}, x \neq 0$ **d)** $\frac{x+2}{(x-4)(x-6)}, x \neq 4, x \neq 6$

8) a) $\frac{x(x+1)}{3}, x \neq -7, -2$ **b)** $2, x \neq -10, 2, 6$ **c)** $\frac{-5x^2-22x-6}{(x+5)(x+2)(x-2)}, x \neq -5, -2, 2$

d) $\frac{2x(x+62)}{(x+16)(x+2)(x-10)}, x \neq -16, -2, 10$

9) a) $\frac{x-8}{(x+7(x-3)}, x \neq -15, -7, 3$ **b)** $\frac{(x+10)(x+2)}{(x+5)(x-3)}, x \neq -10, -5, 3$ **c)** $\frac{-x+57}{(x-7)(x-2)}, x \neq 2, 7$

d) $\frac{x^2+15x+42}{(x+6)(x+3)}, x \neq -6, -3$ **e)** $\frac{8x-2}{2x-1}, x \neq \frac{1}{2}$