

# *Chapter 2(part 1)*

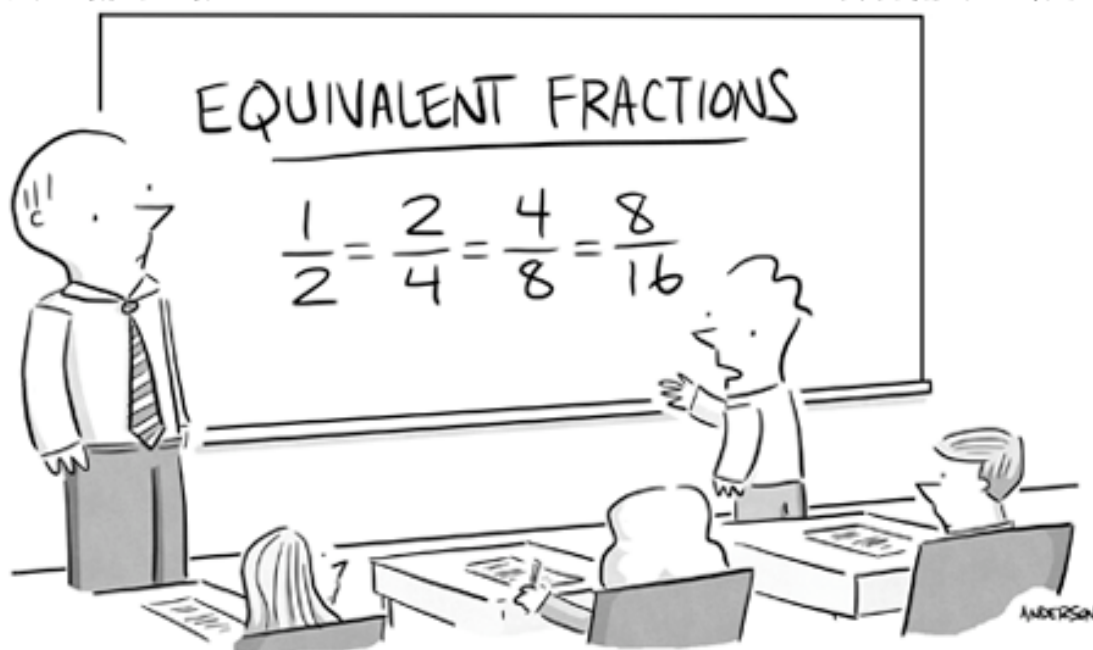
## *Rational Expressions*

### *WORKBOOK*

*MCR3U*

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"I understand they all have the same value, but I have to tell you, the ones on the right feel like more bang for your buck."

## Chapter 2(part 1) Workbook Checklist

Worksheet	Check ✓
Intro to Rational Expressions – Fractions and Exponents Review – Worksheet	
3.3 Rational Exponents – Worksheet	
2.1/2.2 Multiplying and Dividing Rational Expressions – Worksheet	
Extra Practice Multiplying and Dividing Rational Expressions	
2.1/2.2 Adding and Subtracting Rational Expressions – Worksheet	
Rational Expressions Practice	
Review	
Practice Test	

# Intro to Rational Expressions – Fractions and Exponents Review – Worksheet

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1) Add or subtract the following fractions.

a)  $\frac{2}{3} + \frac{3}{4}$

b)  $\frac{3}{5} - \frac{1}{3}$

c)  $\frac{x}{3} + \frac{y}{2}$

d)  $\frac{3x}{5} - \frac{2y}{7}$

2) Simplify (multiplication)

a)  $\left(\frac{2}{3}\right)\left(-\frac{1}{5}\right)$

b)  $\left(\frac{4}{5}\right)\left(\frac{25}{12}\right)$

c)  $\left(\frac{17}{8}\right)\left(\sqrt{\frac{13}{289}}\right)$

d)  $\left(-\frac{5}{8}\right)\left(\sqrt{\frac{75}{128}}\right)$

3) Simplify (multiplication and division)

a)  $\frac{4}{9} \cdot \frac{7}{4}$

b)  $\frac{-11}{5} \cdot \frac{-7}{4}$

c)  $\frac{-17}{10} \div \frac{9}{4}$

d)  $\frac{6}{7} \div \frac{7}{6}$

e)  $\frac{7}{3} \cdot 6$

**4) Simplify (exponents)**

**a)**  $(x^6)(x^4)$

**b)**  $(y^3)(y^7)(y^9)$

**c)**  $m^5 \div m^4$

**d)**  $h^6 \div h^8$

**e)**  $\sqrt{h^6} \div \sqrt{h^8}$

**f)**  $\frac{x^{10}}{x^5}$

**g)**  $(yz^2)^3$

**h)**  $\left[-(x)^2\right]^2$

**i)**  $(x^2)^3(y^3)^2$

**j)**  $\frac{(-x)^2}{(-x)}$

**5) Simplify fractions and exponents**

**a)**  $\frac{7x}{3y} \cdot \frac{12y}{5x^2} + \frac{2}{3}$

**b)**  $\frac{25x^3}{3x} \cdot \frac{7y^2}{5yx^2}$

6) Simplify. Your answer should contain only positive exponents.

a)  $(x^{-2}x^{-3})^4$

b)  $(x^4)^{-3} \cdot 2x^4$

c)  $(n^3)^3 \cdot 2n^{-1}$

d)  $(2v)^2 \cdot 2v^2$

e)  $\frac{2x^2y^4 \cdot 4x^2y^4 \cdot 3x}{3x^{-3}y^2}$

f)  $\frac{2y^3 \cdot 3xy^3}{3x^2y^4}$

g)  $\frac{x^3y^3 \cdot x^3}{4x^2}$

h)  $\frac{3x^2y^2}{2x^{-1} \cdot 4yx^2}$

i)  $\frac{x}{(2x^0)^2}$

j)  $\frac{2m^{-4}}{(2m^{-4})^3}$

## Answers

1. a)  $\frac{17}{12}$     b)  $\frac{4}{15}$     c)  $\frac{2x+3y}{6}$     d)  $\frac{21x-10y}{35}$

2. a)  $-\frac{2}{15}$     b)  $\frac{5}{3}$     c)  $\frac{\sqrt{13}}{8}$     d)  $-\frac{25\sqrt{3}}{64\sqrt{2}}$

3. a)  $(7/9)$     b)  $(77/20)$     c)  $(-34/45)$     d)  $(36/49)$     e) 14

4. a)  $x^{10}$     b)  $y^{19}$     c)  $m$     d)  $\frac{1}{h^2}$     e)  $1/h$     f)  $x^5$     g)  $y^3z^6$     h)  $x^4$     i)  $x^6y^6$     j)  $-x$

5. a)  $\frac{84+10x}{15x}$     b)  $\frac{35y}{3}$

6) a)  $\frac{1}{x^{20}}$     b)  $\frac{2}{x^8}$     c)  $2n^8$     d)  $8v^4$     e)  $8x^8y^6$     f)  $\frac{2y^2}{x}$     g)  $\frac{x^4y^3}{4}$     h)  $\frac{3xy}{8}$     i)  $\frac{x}{4}$     j)  $\frac{m^8}{4}$

### 3.3 Rational Exponents - Worksheet

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1) Evaluate each cube root.

a)  $\sqrt[3]{64}$

b)  $(-1000)^{\frac{1}{3}}$

c)  $\sqrt[3]{\frac{1}{8}}$

d)  $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

2) Evaluate each root.

a)  $81^{\frac{1}{4}}$

b)  $\sqrt[4]{\frac{16}{625}}$

c)  $64^{\frac{1}{6}}$

d)  $\sqrt[5]{-100\,000}$

3) Evaluate.

a)  $8^{\frac{2}{3}}$

b)  $32^{\frac{4}{5}}$

c)  $(-64)^{\frac{5}{3}}$

d)  $\left(\frac{1}{10\,000}\right)^{\frac{3}{4}}$

4) Evaluate.

a)  $16^{-\frac{1}{4}}$

b)  $25^{-\frac{3}{2}}$

c)  $\left(\frac{1}{8}\right)^{-\frac{7}{3}}$

d)  $\left(-\frac{1}{32}\right)^{-\frac{2}{5}}$

e)  $\left(\frac{10\,000}{81}\right)^{-\frac{3}{4}}$

f)  $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$

5) Simplify. Express your answers using only positive exponents.

a)  $x^{\frac{1}{4}} \cdot x^{\frac{1}{4}}$

b)  $(m^{\frac{1}{3}})(m^{\frac{3}{4}})$

c)  $\frac{w^{\frac{1}{2}}}{\frac{1}{w^{\frac{1}{3}}}}$

d)  $\frac{ab^{\frac{2}{1}}}{\frac{1}{a^{\frac{2}{2}}b^{\frac{3}{3}}}}$

e)  $(y^{\frac{1}{2}})^{\frac{2}{3}}$

f)  $(u^{\frac{3}{4}}v^{\frac{1}{2}})^{\frac{2}{9}}$

6) Simplify. Express your answers using only positive exponents.

a)  $k^{\frac{3}{4}} \div k^{-\frac{1}{4}}$

b)  $\frac{p^{\frac{2}{3}}}{p^{\frac{5}{6}}}$

c)  $(y^{\frac{2}{3}})^{-3}$

d)  $(w^{-\frac{8}{9}})^{-\frac{3}{4}}$

e)  $(8x)^{\frac{2}{3}}(27x)^{-\frac{1}{3}}$

f)  $5(7y^{-\frac{2}{3}})^{-2}$

7) The surface area,  $S$ , of a sphere can be expressed in terms of its volume,  $V$ , using the formula

$S(V) = (4\pi)^{\frac{1}{3}}(3V)^{\frac{2}{3}}$ . A beach ball has a volume of  $24\,000\text{ cm}^3$ . Find its surface area, to the nearest hundred square centimeters.



## Answers

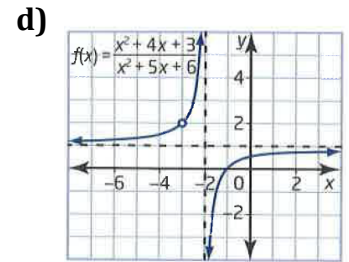
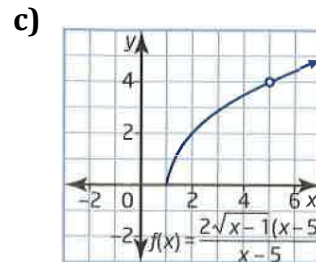
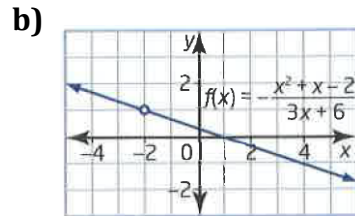
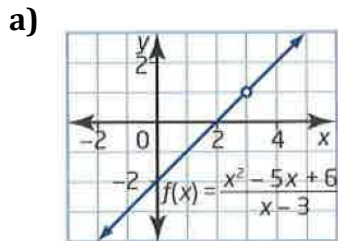
1. a) 4      b)  $-10$       c)  $\frac{1}{2}$       d)  $\frac{2}{3}$
2. a) 3      b)  $\frac{2}{5}$       c) 2      d)  $-10$
3. a) 4      b) 16      c)  $-1024$       d)  $\frac{1}{1000}$
4. a)  $\frac{1}{2}$       b)  $\frac{1}{125}$       c) 128  
d) 4      e)  $\frac{27}{1000}$       f)  $\frac{9}{4}$
5. a)  $x^{\frac{1}{2}}$       b)  $m^{\frac{13}{12}}$       c)  $w^{\frac{1}{6}}$   
d)  $a^{\frac{1}{2}}b^{\frac{5}{3}}$       e)  $y^{\frac{1}{3}}$       f)  $u^{\frac{1}{6}}v^{\frac{1}{9}}$
6. a)  $k$       b)  $\frac{1}{p^{\frac{3}{2}}}$       c)  $\frac{1}{y^2}$   
d)  $w^{\frac{2}{3}}$       e)  $\frac{4}{3}x^{\frac{1}{3}}$       f)  $\frac{5}{49}y^{\frac{4}{3}}$
7.  $4000 \text{ cm}^2$

## 2.1/2.2 Multiplying and Dividing Rational Expressions – Worksheet

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1) State the restrictions for each function.



2) Simplify each expression and state all restrictions on  $x$ .

a)  $\frac{x-8}{x^2-13x+40}$

b)  $\frac{x^2-3x-18}{x^2+x-42}$

c)  $\frac{x+8}{x^2+6x-16}$

3) Simplify and state the restrictions on the variables.

a)  $\frac{14y}{11x} \times \frac{121y}{7x}$

b)  $\frac{15b^3}{4b} \times \frac{20b}{30b^2}$

c)  $\frac{5x}{9y} \div \frac{5x}{18y^2}$

d)  $\frac{26ab}{4a} \div \frac{39a^4b^3}{12b^4}$

4) Simplify and state the restrictions on the variable.

a)  $\frac{25}{x+10} \times \frac{x+10}{5}$

b)  $\frac{x+5}{x-3} \times \frac{x-3}{x+7}$

$$\text{c) } \frac{x+1}{x} \div \frac{x+1}{2x}$$

$$\text{d) } \frac{x+12}{x+10} \div \frac{x+12}{x-5}$$

5) Simplify and state the restrictions on the variable.

$$\text{a) } \frac{3x^2}{12x^2+18x} \times \frac{4x+6}{3x+30}$$

$$\text{b) } \frac{4x+24}{x^2+8x} \times \frac{12x^2}{3x+18}$$

$$\text{c) } \frac{x^2+10x+21}{x+3} \times \frac{x+2}{x^2+9x+14}$$

$$\text{d) } \frac{x^2+2x-15}{x^2-9x+18} \times \frac{x-6}{x+5}$$

6) Simplify and state the restrictions on the variable

$$\text{a) } \frac{x^2+15x}{4x+24} \div \frac{3x}{3x+18}$$

$$\text{b) } \frac{6x}{8x-72} \div \frac{9x}{2x-18}$$

$$\text{c) } \frac{x^2+15x+26}{6x^2} \div \frac{x^2-3x-10}{30x^3}$$

$$\text{d) } \frac{x^2+11x+24}{x^2+2x-3} \div \frac{x-8}{x-1}$$

7) Simplify and state the restrictions on the variable

$$\text{a) } \frac{a^2-25}{a+2} \cdot \frac{a^2-4}{a^2-7a+10}$$

$$\text{b) } \frac{y^2-4y-21}{3y^2+6y} \cdot \frac{y^2+8y}{y^2+11y+24}$$

$$\text{c) } \frac{p^2-2p+1}{p+1} \div \frac{p^2-1}{p+1}$$

$$\text{d) } \frac{x^2+6x-27}{x^2+11x+18} \div \frac{x-3}{x^2+x-2}$$

## Answers

1) a)  $x \neq 3$  b)  $x \neq -2$  c)  $x \geq 1, x \neq 5$  d)  $x \neq -3, x \neq -2$

2) a)  $\frac{1}{x-5}, x \neq 5, x \neq 8$  b)  $\frac{x+3}{x+7}, x \neq -7, x \neq 6$  c)  $\frac{1}{x-2}, x \neq -8, x \neq 2$

3) a)  $\frac{22y^2}{x^2}, x \neq 0$  b)  $\frac{5b}{2}, b \neq 0$  c)  $2y, x \neq 0, y \neq 0$  d)  $\frac{2b^2}{a^4}, a \neq 0, b \neq 0$

4) a)  $5, x \neq -10$  b)  $\frac{x+5}{x+7}, x \neq -7, x \neq 3$  c)  $2, x \neq -1, x \neq 0$  d)  $\frac{x-5}{x+10}, x \neq -12, x \neq -10, x \neq 5$

5) a)  $\frac{x}{3(x+10)}, x \neq -10, -\frac{3}{2}, 0$  b)  $\frac{16x}{x+8}, x \neq -8, -6, 0$  c)  $1, x \neq -7, -3, -2$  d)  $1, x \neq -5, 3, 6$

6) a)  $\frac{x+15}{4}, x \neq -6, 0$  b)  $\frac{1}{6}, x \neq 0, 9$  c)  $\frac{5x(x+13)}{x-5}, x \neq -2, 0, 5$  d)  $\frac{x+8}{x-8}, x \neq -3, 1, 8$

7) a)  $a + 5, a \neq 2, -2, 5$  b)  $\frac{y-7}{3(y+2)}, y \neq -8, -3, -2, 0$  c)  $\frac{p-1}{p+1}, p \neq -1, 1$  d)  $x - 1, x \neq -9, -2, 1, 3$

## Extra Practice Multiplying and Dividing Rational Expressions

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1) Multiply the following rational expressions. Simplify and state restrictions.

a)  $\frac{x+4}{x} \cdot \frac{x^2}{x^2+5x+4}$

b)  $\frac{x^2+10x+16}{5x-10} \cdot \frac{x-2}{x^2+9x+8}$

c)  $\frac{2x^2-10x}{x^2-9x+20} \cdot \frac{x^2-8x+16}{4x^2}$

d)  $\frac{2x+4}{x+4} \cdot \frac{5x^2+21x+4}{10x+2}$

2) Divide the following rational expressions. Simplify and state restrictions.

a)  $\frac{x^2-5x+6}{5} \div \frac{x-3}{15}$

b)  $\frac{x^2-5x+6}{8x^2+24x} \div \frac{x-2}{4x+12}$

c)  $\frac{x^2-2x+1}{x+1} \div \frac{x^2-1}{x+1}$

d)  $\frac{x^2+7x+12}{x^2+3x-10} \div \frac{x^2-x-20}{x^2-25}$

## Answers

1) a)  $\frac{x}{x+1}$ ;  $x \neq -4, -1, 0$  b)  $\frac{x+2}{5(x+1)}$ ;  $x \neq -8, -1, 2$  c)  $\frac{x-4}{2x}$ ;  $x \neq 0, 4, 5$  d)  $x + 2$ ;  $x \neq -4, -\frac{1}{5}$

2) a)  $3x - 6$ ;  $x \neq 3$  b)  $\frac{x-3}{2x}$ ;  $x \neq -3, 0, 2$  c)  $\frac{x-1}{x+1}$ ;  $x \neq -1, 1$  d)  $\frac{x+3}{x-2}$ ;  $x \neq -5, -4, 2, 5$

## 2.1/2.2 Adding and Subtracting Rational Expressions - Worksheet

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1) Simplify and state any restrictions

a)  $\frac{x+1}{18} + \frac{x-1}{45}$

b)  $\frac{2}{3x} - \frac{1}{4x}$

c)  $\frac{3}{ab} + \frac{5}{4b}$

d)  $\frac{2+a}{a^2b} + \frac{4-a}{3ab^2}$

2) Simplify and state the restrictions.

a)  $\frac{1}{x-6} - \frac{1}{x+6}$

b)  $\frac{12}{x+8} + \frac{3}{x-9}$



$$\text{c) } \frac{x+10}{x-6} - \frac{x-3}{x+4}$$

$$\text{d) } \frac{x+5}{x+1} + \frac{x+2}{x-2}$$

3) Simplify and state the restrictions.

$$\text{a) } \frac{x}{x^2-9x+8} + \frac{2}{x-8}$$

$$\text{b) } \frac{x+3}{x+5} + \frac{x+2}{x^2+3x-10}$$

$$\text{c) } \frac{x}{x^2+3x+2} - \frac{3x-2}{x^2+8x+7}$$

$$\text{d) } \frac{x+4}{x^2-121} - \frac{2x-1}{x^2+8x-33}$$

4) Binomial expressions can differ by a factor of -1. Factor -1 from one of the denominators to identify the common denominator. Then, simplify each expression and state the restrictions.

a)  $\frac{1}{x-2} - \frac{1}{2-x}$

b)  $\frac{2b+3}{4b-1} + \frac{b+6}{1-4b}$

### Answers

1) a)  $\frac{7x+3}{90}$ , no restrictions b)  $\frac{5}{12x}$ ,  $x \neq 0$  c)  $\frac{12+5a}{4ab}$ ,  $a \neq 0, b \neq 0$  d)  $\frac{6b+3ab+4a-a^2}{3a^2b^2}$ ,  $a \neq 0, b \neq 0$

2) a)  $\frac{12}{(x-6)(x+6)}$ ,  $x \neq -6, 6$  b)  $\frac{15x-84}{(x+8)(x-9)}$ ,  $x \neq -8, 9$  c)  $\frac{23x+22}{(x-6)(x+4)}$ ,  $x \neq -4, x \neq 6$  d)  $\frac{2(x+4)(x-1)}{(x+1)(x-2)}$ ,  $x \neq -1, 2$

3) a)  $\frac{3x-2}{(x-1)(x-8)}$ ,  $x \neq 1, 8$  b)  $\frac{x^2+2x-4}{(x+5)(x-2)}$ ,  $x \neq -5, 2$  c)  $\frac{-2x^2+3x+4}{(x+1)(x+2)(x+7)}$ ,  $x \neq -7, -2, -1$  d)  $\frac{-(x-23)(x-1)}{(x+11)(x-11)(x-3)}$ ,  $x \neq -11, 3, 11$

4) a)  $\frac{2}{x-2}$ ,  $x \neq 2$  b)  $\frac{b-3}{4b-1}$ ,  $b \neq \frac{1}{4}$

## Rational Expressions Practice

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1. Simplify and state the restrictions on the variable.

a)  $\frac{2x^3}{3x^2 - 4x} \times \frac{6x - 8}{2x - 2}$

b)  $\frac{2x^2 - 3x}{x^2} \times \frac{x^2 - 3x - 4}{2x^2 - x - 3}$

c)  $\frac{2x^2 - 13x + 20}{x} \times \frac{x^2 + 4x - 5}{x^2 - 5x + 4}$

d)  $\frac{x^2 - 1}{x^2 - 4} \times \frac{x^2 - 3x + 4}{x^2 + 5x + 4}$

2. Simplify and state the restrictions on the variable.

a)  $\frac{x^2 - 4}{x + 3} \div \frac{x^2 - x - 6}{x^2 + x - 6}$

b)  $\frac{x^2 + 2x - 8}{x^2 + x - 6} \div \frac{x^2 + 3x - 4}{x^2 - 9}$

c)  $\frac{x^2 + x}{3x^2 - 14x + 8} \div \frac{x^2 + 3x}{3x^2 - 10x - 8}$

d)  $\frac{x^3 + 4x^2 + 3x}{2x^2 - 5x - 12} \div \frac{x^2 + 3x}{2x^2 + 9x + 9}$

3. Simplify and state the restrictions on the variable.

a)  $\frac{4}{x^2 - x - 6} + \frac{2}{x^2 - 2x - 3}$

b)  $\frac{2x}{x^2 + x - 12} - \frac{x}{x^2 - 9}$

c)  $\frac{3x - 1}{2x^2 - 7x - 4} + \frac{2x - 3}{2x^2 + 3x + 1}$

d)  $\frac{2x - 1}{x^2 + 3x + 2} - \frac{2x + 2}{x^2 + 5x + 6}$

4. Remove a common factor of  $-1$  from one of the denominators and then simplify the expressions. Be sure to state any restrictions for the variable.

a)  $\frac{4}{2x - 1} + \frac{2}{1 - 2x}$

b)  $\frac{3x}{x - 7} - \frac{x}{7 - x}$

c)  $\frac{6x - 1}{3x - 4} + \frac{4x - 1}{4 - 3x}$

d)  $\frac{5x + 2}{5x - 2} - \frac{3x}{2 - 5x}$

### ANSWERS

1. a)  $\frac{2x^2}{x - 1}, x \neq 0, x \neq 1, x \neq \frac{4}{3}$

b)  $\frac{x - 4}{x}, x \neq -1, x \neq 0, x \neq \frac{3}{2}$

c)  $\frac{(2x - 5)(x + 5)}{x}, x \neq 0, x \neq 1, x \neq 4$

d)  $\frac{(x - 1)(x^2 - 3x + 4)}{(x - 2)(x + 2)(x + 4)}, x \neq -4, x \neq -2, x \neq -1, x \neq 2$

2. a)  $\frac{(x - 2)^2}{x - 3}, x \neq -3, x \neq -2, x \neq 2, x \neq 3$

b)  $\frac{x - 3}{x - 1}, x \neq -4, x \neq -3, x \neq 1, x \neq 2, x \neq 3$

c)  $\frac{(x + 1)(3x + 2)}{(x + 3)(3x - 2)}, x \neq -3, x \neq -\frac{2}{3}, x \neq 0, x \neq \frac{2}{3}, x \neq 4$

d)  $\frac{(x + 1)(x + 3)}{(x - 4)}, x \neq -3, x \neq -\frac{3}{2}, x \neq 0, x \neq 4$

3. a)  $\frac{2(3x + 4)}{(x - 3)(x + 2)(x + 1)}, x \neq -2, x \neq -1, x \neq 3$

b)  $\frac{x(x + 2)}{(x - 3)(x + 3)(x + 4)}, x \neq -4, x \neq -3, x \neq 3$

c)  $\frac{5x^2 - 9x + 11}{(2x + 1)(x - 4)(x + 1)}, x \neq -\frac{1}{2}, x \neq -1, x \neq 4$

d)  $\frac{x - 5}{(x + 1)(x + 2)(x + 3)}, x \neq -3, x \neq -2, x \neq -1$

4. a)  $\frac{2}{2x - 1}, x \neq \frac{1}{2}$

b)  $\frac{4x}{x - 7}, x \neq 7$

c)  $\frac{2x}{3x - 4}, x \neq \frac{4}{3}$

d)  $\frac{2(4x + 1)}{5x - 2}, x \neq \frac{2}{5}$



