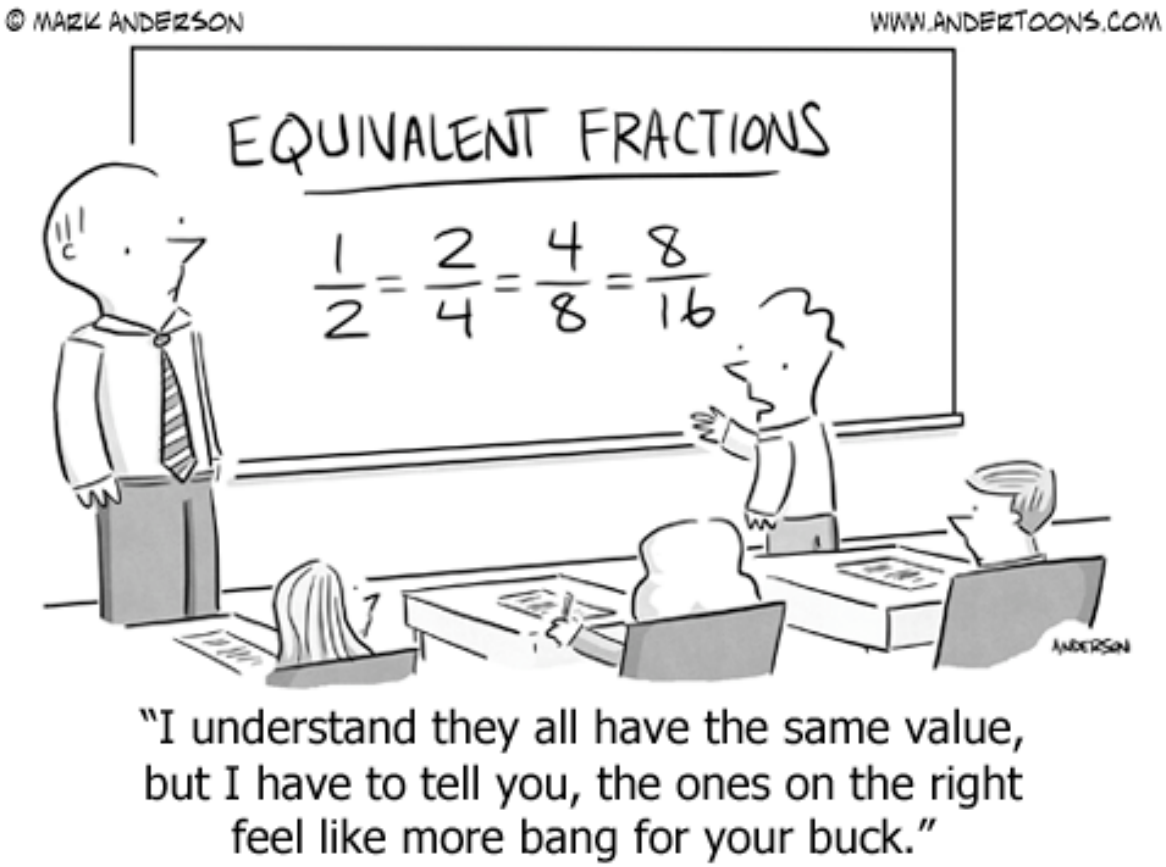
# *Chapter 2(part 1)*
## *Rational Expressions*

## *WORKBOOK*

*MCR3U*



# Chapter 2(part 1) Workbook Checklist

| Worksheet | Check ✓ |
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| Intro to Rational Expressions – Fractions and Exponents Review – Worksheet | |
| 3.3 Rational Exponents – Worksheet | |
| 2.1/2.2 Multiplying and Dividing Rational Expressions – Worksheet | |
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# Intro to Rational Expressions – Fractions and Exponents Review – Worksheet

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SOLUTIONS

**1)** Add or subtract the following fractions.

**a)** $\frac{2}{3}+\frac{3}{4}$

$= \frac{8}{12}+\frac{9}{12}$

$= \frac{17}{12}$

**b)** $\frac{3}{5}-\frac{1}{3}$

$= \frac{9}{15}-\frac{5}{15}$

$= \frac{4}{15}$

**c)** $\frac{x}{3}+\frac{y}{2}$

$= \frac{2x}{6}+\frac{3y}{6}$

$= \frac{2x+3y}{6}$

**d)** $\frac{3x}{5}-\frac{2y}{7}$

$= \frac{21x}{35}-\frac{10y}{35}$

$= \frac{21x-10y}{35}$

**2)** Simplify (multiplication)

**a)** $\left(\frac{2}{3}\right)\left(-\frac{1}{5}\right)$

$= \frac{-2}{15}$

**b)** $\left(\frac{\cancel{4}^{1}}{\cancel{5}_{1}}\right)\left(\frac{\cancel{25}^{5}}{\cancel{12}_{3}}\right)$

$= \frac{5}{3}$

**c)** $\left(\frac{17}{8}\right)\left(\sqrt{\frac{13}{289}}\right)$

$= \frac{17}{8}\left(\frac{\sqrt{13}}{17}\right)$

$= \frac{\sqrt{13}}{8}$

**d)** $\left(-\frac{5}{8}\right)\left(\sqrt{\frac{75}{128}}\right)$

$= \left(\frac{-5}{8}\right)\left(\frac{5\sqrt{3}}{8\sqrt{2}}\right)$

$= \frac{-25\sqrt{3}}{64\sqrt{2}}$

**3)** Simplify (multiplication and division)

**a)** $\frac{\cancel{4}}{9}\cdot\frac{7}{\cancel{4}}$

$= \frac{7}{9}$

**b)** $\frac{-11}{5}\cdot\frac{-7}{4}$

$= \frac{77}{20}$

**c)** $\frac{-17}{10}\div\frac{9}{4}$

$= \frac{-17}{5\cancel{10}_{2}}\times\frac{\cancel{4}^{2}}{9}$

$= \frac{-34}{45}$

**d)** $\frac{6}{7}\div\frac{7}{6}$

$= \frac{6}{7}\times\frac{6}{7}$

$= \frac{36}{49}$

**e)** $\frac{7}{\cancel{3}_{1}}\cdot\frac{\cancel{6}^{2}}{1}$

$= 14$

**4)** Simplify (exponents)

**a)** $(x^6)(x^4)$

$= x^{10}$

**b)** $(y^3)(y^7)(y^9)$

$= y^{19}$

**c)** $m^5 \div m^4$

$= m$

**d)** $h^6 \div h^8$

$= h^{-2}$

$= \frac{1}{h^2}$

**e)** $\sqrt{h^6} \div \sqrt{h^8}$

$= (h^6)^{\frac{1}{2}} \div (h^8)^{\frac{1}{2}}$

$= h^3 \div h^4$

$= \frac{1}{h}$

**f)** $\frac{x^{10}}{x^5}$

$= x^5$

**g)** $\left(yz^2\right)^3$

$= y^3 z^6$

**h)** $\left[-(x)^2\right]^2$

$= x^4$

**i)** $\left(x^2\right)^3\left(y^3\right)^2$

$= x^6 y^6$

**j)** $\frac{(-x)^2}{(-x)}$

$= \frac{x^2}{-x^1}$

$= -x$

**5)** Simplify fractions and exponents

**a)** $\frac{7x}{3y} \cdot \frac{12y}{5x^2} + \frac{2}{3}$

$\frac{28xy}{5x^2y} + \frac{2}{3}$

$\frac{28}{5x} + \frac{2}{3}$

$\frac{84}{15x} + \frac{10x}{15x}$

$= \frac{84+10x}{15x}$

**b)** $5\frac{25x^3}{3x} \cdot \frac{7y^2}{5yx^2}$

$= \frac{35x^3y^2}{3x^3y}$

$= \frac{35y}{3}$

**6)** Simplify. Your answer should contain only positive exponents.

a) $(x^{-2}x^{-3})^4$

$= (x^{-5})^4$

$= x^{-20}$

$= \frac{1}{x^{20}}$

**b)** $(x^4)^{-3} \cdot 2x^4$

$= x^{-12} \cdot 2x^4$

$= 2x^{-8}$

$= \frac{2}{x^8}$

**c)** $(n^3)^3 \cdot 2n^{-1}$

$= n^9 \cdot 2n^{-1}$

$= 2n^8$

**d)** $(2v)^2 \cdot 2v^2$

$= 4v^2 \cdot 2v^2$

$= 8v^4$

**e)** $\frac{2x^2y^4 \cdot 4x^2y^4 \cdot 3x}{3x^{-3}y^2}$

$= \frac{24x^5y^8}{3x^{-3}y^2}$

$= 8x^8y^6$

**f)** $\frac{2y^3 \cdot 3xy^3}{3x^2y^4}$

$= \frac{6xy^6}{3x^2y^4}$

$= \frac{2y^2}{x}$

**g)** $\frac{x^3y^3 \cdot x^3}{4x^2}$

$= \frac{x^6y^3}{4x^2}$

$= \frac{x^4y^3}{4}$

**h)** $\frac{3x^2y^2}{2x^{-1} \cdot 4yx^2}$

$= \frac{3x^2y^2}{8xy}$

$= \frac{3xy}{8}$

**i)** $\frac{x}{(2x^0)^2}$

$= \frac{x}{4}$

**j)** $\frac{2m^{-4}}{(2m^{-4})^3}$

$= \frac{2m^{-4}}{8m^{-12}}$

$= \frac{m^8}{4}$

## Answers

**1. a)** $\frac{17}{12}$ **b)** $\frac{4}{15}$ **c)** $\frac{2x+3y}{6}$ **d)** $\frac{21x-10y}{35}$

**2. a)** $-\frac{2}{15}$ **b)** $\frac{5}{3}$ **c)** $\frac{\sqrt{13}}{8}$ **d)** $-\frac{25\sqrt{3}}{64\sqrt{2}}$

**3. a)** (7/9) **b)** (77/20) **c)** (-34/45) **d)** (36/49) **e)** 14

**4. a)** $x^{10}$ **b)** $y^{19}$ **c)** $m$ **d)** ~~$h^2$~~ $\frac{1}{h^2}$ **e)** $1/h$ **f)** $x^5$ **g)** $y^3z^6$ **h)** $x^4$ **i)** $x^6y^6$ **j)** $-x$

**5. a)** $\frac{84+10x}{15x}$ **b)** $\frac{35y}{3}$

**6) a)** $\frac{1}{x^{20}}$ **b)** $\frac{2}{x^8}$ **c)** $2n^8$ **d)** $8v^4$ **e)** $8x^8y^6$ **f)** $\frac{2y^2}{x}$ **g)** $\frac{x^4y^3}{4}$ **h)** $\frac{3xy}{8}$ **i)** $\frac{x}{4}$ **j)** $\frac{m^8}{4}$

## 3.3 Rational Exponents – Worksheet

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SOLUTIONS

**1)** Evaluate each cube root.

**a)** $\sqrt[3]{64}$

$= 4$

**b)** $(-1000)^{\frac{1}{3}}$

$= -10$

**c)** $\sqrt[3]{\frac{1}{8}}$

$= \frac{1}{2}$

**d)** $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

$= \frac{2}{3}$

**2)** Evaluate each root.

**a)** $81^{\frac{1}{4}}$

$= 3$

**b)** $\sqrt[4]{\frac{16}{625}}$

$= \frac{2}{5}$

**c)** $64^{\frac{1}{6}}$

$= 2$

**d)** $\sqrt[5]{-100\,000}$

$= -10$

**3)** Evaluate.

**a)** $8^{\frac{2}{3}}$

$= \left(8^{\frac{1}{3}}\right)^2$

$= 4$

**b)** $32^{\frac{4}{5}}$

$= \left(32^{\frac{1}{5}}\right)^4$

$= 16$

**c)** $(-64)^{\frac{5}{3}}$

$= \left[(-64)^{\frac{1}{3}}\right]^5$

$= -1024$

**d)** $\left(\frac{1}{10\,000}\right)^{\frac{3}{4}}$

$= \frac{1}{1000}$

**4)** Evaluate.

**a)** $16^{-\frac{1}{4}}$

$= \frac{1}{16^{\frac{1}{4}}}$

$= \frac{1}{2}$

**b)** $25^{-\frac{3}{2}}$

$= \frac{1}{25^{3/2}}$

$= \frac{1}{125}$

**c)** $\left(\frac{1}{8}\right)^{-\frac{7}{3}}$

$= \left(\frac{8}{1}\right)^{\frac{7}{3}}$

$= 128$

**d)** $\left(-\frac{1}{32}\right)^{-\frac{2}{5}}$

$= \left(-\frac{32}{1}\right)^{\frac{2}{5}}$

$= 4$

**e)** $\left(\frac{10\,000}{81}\right)^{-\frac{3}{4}}$

$= \left(\frac{81}{10000}\right)^{\frac{3}{4}}$

$= \frac{27}{1000}$

**f)** $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$

$= \left(-\frac{27}{8}\right)^{\frac{2}{3}}$

$= \frac{9}{4}$

**5)** Simplify. Express your answers using only positive exponents.

**a)** $x^{\frac{1}{4}} \cdot x^{\frac{1}{4}}$

$= x^{\frac{1}{2}}$

**b)** $\left(m^{\frac{1}{3}}\right)\left(m^{\frac{3}{4}}\right)$

$= m^{4/12} \cdot m^{9/12}$

$= m^{13/12}$

**c)** $\dfrac{w^{\frac{1}{2}}}{w^{\frac{1}{3}}} = \dfrac{w^{3/6}}{w^{2/6}}$

$= w^{1/6}$

**d)** $\dfrac{ab^2}{a^{\frac{1}{2}}b^{\frac{1}{3}}} = \dfrac{a^{2/2}\, b^{6/3}}{a^{1/2}\, b^{1/3}}$

$= a^{1/2} b^{5/3}$

**e)** $\left(y^{\frac{1}{2}}\right)^{\frac{2}{3}}$

$= y^{2/6}$

$= y^{1/3}$

**f)** $\left(u^{\frac{3}{4}}v^{\frac{1}{2}}\right)^{\frac{2}{9}} = u^{6/36} v^{2/18}$

$= u^{1/6} v^{1/9}$

**6)** Simplify. Express your answers using only positive exponents.

**a)** $k^{\frac{3}{4}} \div k^{-\frac{1}{4}}$

$= k^{4/4}$

$= k$

**b)** $\dfrac{p^{-\frac{2}{3}}}{p^{\frac{5}{6}}} = \dfrac{p^{-4/6}}{p^{5/6}}$

$= p^{-9/6}$

$= \dfrac{1}{p^{3/2}}$

**c)** $\left(y^{\frac{2}{3}}\right)^{-3}$

$= y^{-6/3}$

$= y^{-2}$

$= \dfrac{1}{y^2}$

**d)** $\left(w^{-\frac{8}{9}}\right)^{-\frac{3}{4}}$

$= w^{24/36}$

$= w^{2/3}$

**e)** $(8x)^{\frac{2}{3}}(27x)^{-\frac{1}{3}}$

$= \dfrac{4x^{2/3}}{(27x)^{1/3}}$

$= \dfrac{4x^{2/3}}{3x^{1/3}} = \dfrac{4x^{1/3}}{3}$

**f)** $5\left(7y^{-\frac{2}{3}}\right)^{-2}$

$= 5\left(7^{-2}\right)\left(y^{4/3}\right)$

$= \dfrac{5y^{4/3}}{49}$

**7)** The surface area, S, of a sphere can be expressed in terms of its volume, V, using the formula $S(V) = (4\pi)^{\frac{1}{3}}(3V)^{\frac{2}{3}}$. A beach ball has a volume of 24 000 cm$^3$. Find its surface area, to the nearest hundred square centimeters.

$S(24000) = (4\pi)^{1/3}\left[3(24000)\right]^{2/3}$

$= (4\pi)^{1/3}(72000)^{2/3}$

$= 4023.7 \text{ cm}^2$

$\simeq 4000 \text{ cm}^2$

## Answers

1. a) 4 b) $-10$ c) $\frac{1}{2}$ d) $\frac{2}{3}$
2. a) 3 b) $\frac{2}{5}$ c) 2 d) $-10$
3. a) 4 b) 16 c) $-1024$ d) $\frac{1}{1000}$
4. a) $\frac{1}{2}$ b) $\frac{1}{125}$ c) 128
   d) 4 e) $\frac{27}{1000}$ f) $\frac{9}{4}$
5. a) $x^{\frac{1}{2}}$ b) $m^{\frac{13}{12}}$ c) $w^{\frac{1}{6}}$
   d) $a^{\frac{1}{2}}b^{\frac{5}{3}}$ e) $y^{\frac{1}{8}}$ f) $u^{\frac{1}{6}}v^{\frac{1}{9}}$
6. a) $k$ b) $\frac{1}{p^{\frac{3}{2}}}$ c) $\frac{1}{y^2}$
   d) $w^{\frac{2}{3}}$ e) $\frac{4}{3}x^{\frac{1}{3}}$ f) $\frac{5}{49}y^{\frac{1}{3}}$
7. 4000 $\text{cm}^2$

## 2.1/2.2 Multiplying and Dividing Rational Expressions – Worksheet

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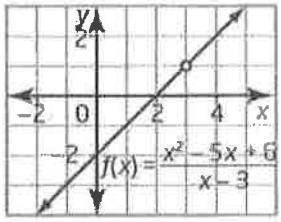
SOLUTIONS

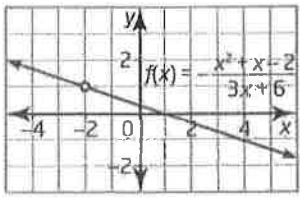
**1)** State the restrictions for each function.

**a)**



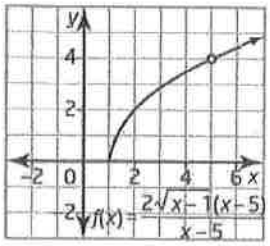

$x \neq 3$

**b)**




$x \neq -2$

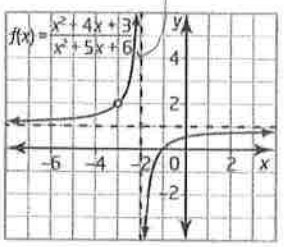
**c)**




$x \neq 5$

$x \geq 1$

**d)**




$x \neq -2, -3$

**2)** Simplify each expression and state all restrictions on $x$.

**a)** $\frac{x-8}{x^2-13x+40}$

$= \frac{x-8}{(x-8)(x-5)}$

$= \frac{1}{x-5}$, $x \neq 5, 8$

**b)** $\frac{x^2-3x-18}{x^2+x-42}$

$= \frac{(x-6)(x+3)}{(x+7)(x-6)}$

$= \frac{x+3}{x+7}$, $x \neq -7, 6$

**c)** $\frac{x+8}{x^2+6x-16}$

$= \frac{x+8}{(x+8)(x-2)}$

$= \frac{1}{x-2}$, $x \neq -8, 2$

**3)** Simplify and state the restrictions on the variables.

**a)** $\frac{14y}{11x} \times \frac{121y}{7x}$

$= \frac{22y^2}{x^2}$, $x \neq 0$

**b)** $\frac{15b^3}{4b} \times \frac{20b}{30b^2}$

$= \frac{5b^4}{2b^3}$

$= \frac{5b}{2}$, $b \neq 0$

**c)** $\frac{5x}{9y} \div \frac{5x}{18y^2}$

$= \frac{5x}{9y} \times \frac{18y^2}{5x}$

$= 2y$, $x \neq 0, y \neq 0$

**d)** $\frac{26ab}{4a} \div \frac{39a^4b^3}{12b^4}$

$= \frac{26ab}{4a} \times \frac{12b^4}{39a^4b^3}$

$= \frac{6ab^5}{3a^5b^3}$

$= \frac{2b^2}{a^4}$, $a \neq 0$, $b \neq 0$

**4)** Simplify and state the restrictions on the variable.

**a)** $\frac{25}{x+10} \times \frac{x+10}{5}$

$= 5$, $x \neq -10$

**b)** $\frac{x+5}{x-3} \times \frac{x-3}{x+7}$

$= \frac{x+5}{x+7}$, $x \neq 3, -7$

**c)** $\frac{x+1}{x} \div \frac{x+1}{2x}$

$= \frac{x+1}{x} \cdot \frac{2x}{x+1}$

$= 2, \; x \neq 0, -1$

**d)** $\frac{x+12}{x+10} \div \frac{x+12}{x-5}$

$= \frac{x+12}{x+10} \cdot \frac{x-5}{x+12}$

$= \frac{x-5}{x+10}, \; x \neq -10, -12, 5$

**5)** Simplify and state the restrictions on the variable.

**a)** $\frac{3x^2}{12x^2+18x} \times \frac{4x+6}{3x+30}$

$= \frac{3x^2}{6x(2x+3)} \cdot \frac{2(2x+3)}{3(x+10)}$

$= \frac{12x^2}{36x(x+10)} \rightarrow = \frac{x}{3(x+10)}, \; x \neq -10, -\frac{3}{2}, 0$

**b)** $\frac{4x+24}{x^2+8x} \times \frac{12x^2}{3x+18}$

$= \frac{4(x+6)}{x(x+8)} \cdot \frac{4 \cdot 3x^2}{3(x+6)}$

$= \frac{16x^2}{x(x+8)} \rightarrow = \frac{16x}{x+8}, \; x \neq -8, -6, 0$

**c)** $\frac{x^2+10x+21}{x+3} \times \frac{x+2}{x^2+9x+14}$

$= \frac{(x+7)(x+3)}{x+3} \cdot \frac{x+2}{(x+2)(x+7)}$

$= 1, \; x \neq -3, -2, -7$

**d)** $\frac{x^2+2x-15}{x^2-9x+18} \times \frac{x-6}{x+5}$

$= \frac{(x+5)(x-3)}{(x-6)(x-3)} \cdot \frac{(x-6)}{(x+5)}$

$= 1, \; x \neq -5, 3, 6$

**6)** Simplify and state the restrictions on the variable

**a)** $\frac{x^2+15x}{4x+24} \div \frac{3x}{3x+18}$

$= \frac{x(x+15)}{4(x+6)} \times \frac{3(x+6)}{3x}$

$= \frac{x+15}{4}, \; x \neq -6, 0$

**b)** $\frac{6x}{8x-72} \div \frac{9x}{2x-18}$

$= \frac{2 \cdot 3x}{8(x-9)} \cdot \frac{2(x-9)}{3 \cdot 3x}$

$= \frac{4}{24}$

$= \frac{1}{6}, \; x \neq 0, 9$

**c)** $\frac{x^2+15x+26}{6x^2} \div \frac{x^2-3x-10}{30x^3}$

$= \frac{(x+2)(x+13)}{6x^2} \times \frac{5 \cdot 6 x^3}{(x-5)(x+2)}$

$= \frac{5x(x+13)}{x-5}$ ; $x \neq -2, 0, 5$

**d)** $\frac{x^2+11x+24}{x^2+2x-3} \div \frac{x-8}{x-1}$

$= \frac{(x+8)(x+3)}{(x+3)(x-1)} \cdot \frac{x-1}{x-8}$

$= \frac{x+8}{x-8}$ ; $x \neq -3, 1, 8$

**7)** Simplify and state the restrictions on the variable

**a)** $\frac{a^2-25}{a+2} \cdot \frac{a^2-4}{a^2-7a+10}$

$= \frac{(a-5)(a+5)}{a+2} \cdot \frac{(a-2)(a+2)}{(a-2)(a-5)}$

$= a+5$ ; $a \neq -2, 2, 5$

**b)** $\frac{y^2-4y-21}{3y^2+6y} \cdot \frac{y^2+8y}{y^2+11y+24}$

$= \frac{(y-7)(y+3)}{3y(y+2)} \cdot \frac{y(y+8)}{(y+8)(y+3)}$

$= \frac{y-7}{3(y+2)}$ ; $y \neq -8, -3, -2, 0$

**c)** $\frac{p^2-2p+1}{p+1} \div \frac{p^2-1}{p+1}$

$= \frac{(p-1)(p-1)}{p+1} \times \frac{p+1}{(p-1)(p+1)}$

$= \frac{p-1}{p+1}$ ; $p \neq -1, 1$

**d)** $\frac{x^2+6x-27}{x^2+11x+18} \div \frac{x-3}{x^2+x-2}$

$= \frac{(x+9)(x-3)}{(x+9)(x+2)} \div \frac{x-3}{(x+2)(x-1)}$

$= \frac{(x+9)(x-3)}{(x+9)(x+2)} \cdot \frac{(x+2)(x-1)}{x-3}$

$= x-1$ ; $x \neq -9, -2, 1, 3$

**Answers**

**1) a)** $x \neq 3$ **b)** $x \neq -2$ **c)** $x \geq 1,\ x \neq 5$ **d)** $x \neq -3,\ x \neq -2$

**2) a)** $\frac{1}{x-5}, x \neq 5,\ x \neq 8$ **b)** $\frac{x+3}{x+7}, x \neq -7,\ x \neq 6$ **c)** $\frac{1}{x-2}, x \neq -8,\ x \neq 2$

**3) a)** $\frac{22y^2}{x^2},\ x \neq 0$ **b)** $\frac{5b}{2},\ b \neq 0$ **c)** $2y,\ x \neq 0,\ y \neq 0$ **d)** $\frac{2b^2}{a^4},\ a \neq 0,\ b \neq 0$

**4) a)** $5, x \neq -10$ **b)** $\frac{x+5}{x+7},\ x \neq -7,\ x \neq 3$ **c)** $2,\ x \neq -1, x \neq 0$ **d)** $\frac{x-5}{x+10},\ x \neq -12, x \neq -10, x \neq 5$

**5) a)** $\frac{x}{3(x+10)}, x \neq -10, -\frac{3}{2}, 0$ **b)** $\frac{16x}{x+8}, x \neq -8, -6, 0$ **c)** $1, x \neq -7, -3, -2$ **d)** $1, x \neq -5,\ 3,\ 6$

**6) a)** $\frac{x+15}{4},\ x \neq -6, 0$ **b)** $\frac{1}{6},\ x \neq 0,\ 9$ **c)** $\frac{5x(x+13)}{x-5},\ x \neq -2, 0, 5$ **d)** $\frac{x+8}{x-8}, x \neq -3, 1, 8$

**7) a)** $a + 5, a \neq 2, -2, 5$ **b)** $\frac{y-7}{3(y+2)}, y \neq -8, -3, -2, 0$ **c)** $\frac{p-1}{p+1}, p \neq -1, 1$ **d)** $x - 1, x \neq -9, -2, 1, 3$

## 2.1/2.2 Adding and Subtracting Rational Expressions – Worksheet

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SOLUTIONS

**1)** Simplify and state any restrictions

**a)** $\frac{x+1}{18}+\frac{x-1}{45}$

$= \frac{5(x+1)}{90} + \frac{2(x-1)}{90}$

$= \frac{5x+5+2x-2}{90}$

$= \frac{7x+3}{90}$

**b)** $\frac{2}{3x}-\frac{1}{4x}$

$= \frac{8}{12x} - \frac{3}{12x}$

$= \frac{5}{12x}$, $x \neq 0$

**c)** $\frac{3}{ab}+\frac{5}{4b}$

$= \frac{12}{4ab} + \frac{5a}{4ab}$

$= \frac{12+5a}{4ab}$, $a \neq 0, b \neq 0$

**d)** $\frac{2+a}{a^2b}+\frac{4-a}{3ab^2}$

$= \frac{3b(2+a)}{3a^2b^2} + \frac{a(4-a)}{3a^2b^2}$

$= \frac{6b+3ab+4a-a^2}{3a^2b^2}$, $a \neq 0, b \neq 0$

**2)** Simplify and state the restrictions.

**a)** $\frac{1}{x-6}-\frac{1}{x+6}$

$= \frac{1(x+6)}{(x-6)(x+6)} - \frac{(x-6)}{(x-6)(x+6)}$

$= \frac{x+6-x+6}{(x-6)(x+6)}$

$= \frac{12}{(x-6)(x+6)}$, $x \neq -6, 6$

**b)** $\frac{12}{x+8}+\frac{3}{x-9}$

$= \frac{12(x-9)}{(x+8)(x-9)} + \frac{3(x+8)}{(x+8)(x-9)}$

$= \frac{12x-108+3x+24}{(x+8)(x-9)}$

$= \frac{15x-84}{(x+8)(x-9)}$, $x \neq -8, 9$

**c)** $\frac{x+10}{x-6} - \frac{x-3}{x+4}$

$= \frac{(x+4)(x+10)}{(x+4)(x-6)} - \frac{(x-6)(x-3)}{(x+4)(x-6)}$

$= \frac{x^2+14x+40-(x^2-9x+18)}{(x+4)(x-6)}$

$= \frac{23x+22}{(x+4)(x-6)}$ ; $x \neq -4, 6$

**d)** $\frac{x+5}{x+1} + \frac{x+2}{x-2}$

$\frac{(x+5)(x-2)}{(x+1)(x-2)} + \frac{(x+1)(x+2)}{(x+1)(x-2)}$

$= \frac{x^2+3x-10}{(x+1)(x-2)} + \frac{x^2+3x+2}{(x+1)(x-2)}$

$= \frac{2x^2+6x-8}{(x+1)(x-2)}$

$= \frac{2(x^2+3x-4)}{(x+1)(x-2)}$

$= \frac{2(x+4)(x-1)}{(x+1)(x-2)}$ ; $x \neq -1, 2$

**3)** Simplify and state the restrictions.

**a)** $\frac{x}{x^2-9x+8} + \frac{2}{x-8}$

$= \frac{x}{(x-8)(x-1)} + \frac{2}{x-8}$

$= \frac{x}{(x-8)(x-1)} + \frac{2(x-1)}{(x-8)(x-1)}$

$= \frac{x+2x-2}{(x-8)(x-1)}$

$= \frac{3x-2}{(x-8)(x-1)}$ ; $x \neq 8, 1$

**b)** $\frac{x+3}{x+5} + \frac{x+2}{x^2+3x-10}$

$= \frac{x+3}{x+5} + \frac{x+2}{(x+5)(x-2)}$

$= \frac{(x+3)(x-2)}{(x+5)(x-2)} + \frac{x+2}{(x+5)(x-2)}$

$= \frac{x^2+x-6+x+2}{(x+5)(x-2)}$

$= \frac{x^2+2x-4}{(x+5)(x-2)}$ ; $x \neq -5, 2$

**c)** $\frac{x}{x^2+3x+2} - \frac{3x-2}{x^2+8x+7}$

$= \frac{x}{(x+2)(x+1)} - \frac{3x-2}{(x+7)(x+1)}$

$= \frac{x(x+7)}{(x+2)(x+1)(x+7)} - \frac{(3x-2)(x+2)}{(x+2)(x+1)(x+7)}$

$= \frac{x^2+7x-(3x^2+4x-4)}{(x+2)(x+1)(x+7)}$

$= \frac{-2x^2+3x+4}{(x+2)(x+1)(x+7)}$ ; $x \neq -2, -1, -7$

**d)** $\frac{x+4}{x^2-121} - \frac{2x-1}{x^2+8x-33}$

$= \frac{x+4}{(x-11)(x+11)} - \frac{2x-1}{(x+11)(x-3)}$

$= \frac{(x+4)(x-3)}{(x-11)(x+11)(x-3)} - \frac{(2x-1)(x-11)}{(x-11)(x+11)(x-3)}$

$= \frac{x^2+x-12-(2x^2-23x+11)}{(x-11)(x+11)(x-3)}$

$= \frac{-1x^2+24x-23}{(x-11)(x+11)(x-3)}$

see bottom of next page...

**4)** Binomial expressions can differ by a factor of -1. Factor -1 from one of the denominators to identify the common denominator. Then, simplify each expression and state the restrictions.

a) $\frac{1}{x-2} - \frac{1}{2-x}$

$= \frac{-1(1)}{-1(x-2)} - \frac{1}{2-x}$

$= \frac{-1}{-x+2} - \frac{1}{2-x}$

$= \frac{-1}{2-x} - \frac{1}{2-x}$

$= \frac{-2}{2-x}$ , $x \neq 2$

**b)** $\frac{2b+3}{4b-1} + \frac{b+6}{1-4b}$

$= \frac{2b+3}{4b-1} + \frac{-1(b+6)}{-1(1-4b)}$

$= \frac{2b+3}{4b-1} + \frac{-b-6}{-1+4b}$

$= \frac{2b+3}{4b-1} + \frac{-b-6}{4b-1}$

$= \frac{2b+3-b-6}{4b-1}$

$= \frac{b-3}{4b-1}$ , $b \neq \frac{1}{4}$

3d) $= \frac{-1(x^2-24x+23)}{(x-11)(x+11)(x-3)}$

$= \frac{-1(x-23)(x-1)}{(x-11)(x+11)(x-3)}$ , $x \neq -11, 3, 11$

**Answers**

**1) a)** $\frac{7x+3}{90}$, no restrictions **b)** $\frac{5}{12x}, x \neq 0$ **c)** $\frac{12+5a}{4ab}, a \neq 0, b \neq 0$ **d)** $\frac{6b+3ab+4a-a^2}{3a^2b^2}, a \neq 0, b \neq 0$

**2) a)** $\frac{12}{(x-6)(x+6)}, x \neq -6, 6$ **b)** $\frac{15x-84}{(x+8)(x-9)}, x \neq -8, 9$ **c)** $\frac{23x+22}{(x-6)(x+4)}, x \neq -4, x \neq 6$ **d)** $\frac{2(x+4)(x-1)}{(x+1)(x-2)}, x \neq -1, 2$

**3) a)** $\frac{3x-2}{(x-1)(x-8)}, x \neq 1, 8$ **b)** $\frac{x^2+2x-4}{(x+5)(x-2)}, x \neq -5, 2$ **c)** $\frac{-2x^2+3x+4}{(x+1)(x+2)(x+7)}, x \neq -7, -2, -1$ **d)** $\frac{-(x-23)(x-1)}{(x+11)(x-11)(x-3)}, x \neq -11, 3, 11$

**4) a)** $\frac{2}{x-2}, x \neq 2$ **b)** $\frac{b-3}{4b-1}, b \neq \frac{1}{4}$

**Chapter 2a – Rational Expressions – REVIEW**
MCR3U
Jensen

SOLUTIONS

## Section 1: Negative and Rational Exponents

**1)** Evaluate. Express as a fraction in lowest terms.

**a)** $10^{-1}$

$= \frac{1}{10}$

**b)** $4^{-2}$

$= \frac{1}{4^2}$

$= \frac{1}{16}$

**c)** $3^{-2} + 9^{-1}$

$= \frac{1}{9} + \frac{1}{9}$

$= \frac{2}{9}$

**d)** $5^{-3} + 5^{0}$

$= \frac{1}{125} + 1$

$= \frac{126}{125}$

**e)** $\left(\frac{1}{5}\right)^{-1}$

$= \left(\frac{5}{1}\right)^{1}$

$= 5$

**f)** $\left(\frac{3}{4}\right)^{-3}$

$= \left(\frac{4}{3}\right)^{3}$

$= \frac{64}{27}$

**2)** Simplify. Express your answers using only positive exponents.

**a)** $(x^{-2})(x^{-1})(x^{0})$

$= x^{-3}$

$= \frac{1}{x^3}$

**b)** $(3km^2)(2k^{-2}m^{-2})$

$= 6k^{-1}m^{0}$

$= \frac{6}{k}$

**c)** $w^{-3} \div w^{-2}$

$= w^{-1}$

$= \frac{1}{w}$

**d)** $\frac{u^{-2}v^{3}}{u^{-3}v^{-2}}$

$= uv^{5}$

**e)** $(z^{-3})^{-2}$

$= z^{6}$

$= \frac{1}{z^6}$

**f)** $(2ab^{-1})^{-2}$

$= \frac{1}{(2ab^{-1})^2}$

$= \frac{1}{4a^2b^{-2}}$

$= \frac{b^2}{4a^2}$

**3)** Simplify. Express your answers using only positive exponents.

**a)** $(4a^{-2})(-2a^{-3})$

$= -8a^{-5}$

$= \frac{-8}{a^5}$

**b)** $\frac{(2x^2y)^{-2}(3xy)^{-1}}{(6x^2y^2)^{-2}}$

$= \frac{(6x^2y^2)^2}{(2x^2y)^2(3xy)^1}$

$= \frac{36x^4y^4}{4x^4y^2(3xy)}$

$= \frac{36x^4y^4}{12x^5y^3}$

$= 3x^{-1}y$

$= \frac{3y}{x}$

**c)** $\left(\frac{1}{4x^2}\right)^{-2}$

$= \left(\frac{4x^2}{1}\right)^2$

$= 16x^4$

**d)** $\left(\frac{6a^3}{4b^4}\right)^{-2}$

$= \left(\frac{4b^4}{6a^3}\right)^2$

$= \frac{16b^8}{36a^6}$

$= \frac{4b^8}{9a^6}$

**4)** Evaluate.

**a)** $\sqrt[3]{64}$

$= 4$

**b)** $\sqrt[4]{625}$

$= 5$

**c)** $\sqrt[5]{-3125}$

$= -5$

**d)** $\left(\frac{1}{64}\right)^{\frac{1}{6}}$

$= \frac{1}{2}$

**e)** $27^{\frac{2}{3}}$

$= \left(27^{\frac{1}{3}}\right)^2$

$= (3)^2$

$= 9$

**f)** $(-1000)^{\frac{4}{3}}$

$= \left[(-1000)^{\frac{1}{3}}\right]^4$

$= (-10)^4$

$= 10000$

**g)** $-4^{-3}$

$= \frac{-1}{4^3}$

$= \frac{-1}{64}$

**h)** $\left(\frac{3}{4}\right)^{-2}$

$= \left(\frac{4}{3}\right)^2$

$= \frac{16}{9}$

**i)** $\left(-\frac{27}{125}\right)^{-\frac{2}{3}}$

$= \left[\left(-\frac{125}{27}\right)^{\frac{1}{3}}\right]^2$

$= \left(\frac{-5}{3}\right)^2$

$= \frac{25}{9}$

**5)** Simplify. Express answers using only positive exponents.

**a)** $n^{\frac{1}{2}} \times n^{\frac{1}{3}} \times n^{\frac{1}{4}}$

$= \frac{1}{n^9}$

**b)** $(27y^3)^{\frac{1}{3}} \times \left(\frac{1}{16y^4}\right)^{-\frac{3}{4}}$

$= 3y \times \left(\frac{16y^4}{1}\right)^{\frac{3}{4}}$

$= 3y(8y^3)$

$= 24y^4$

**c)** $(27x^6)^{\frac{2}{3}} \div (9x^4)^{\frac{1}{2}}$

$= 9x^4 \div 3x^2$

$= 3x^2$

**d)** $\frac{x^{-\frac{2}{3}}}{x^{-\frac{4}{5}}}$

$= \frac{x^{4/5}}{x^{2/3}}$

$= \frac{x^{12/15}}{x^{10/15}}$

$= x^{2/15}$

## Section 2: Rational Expressions

**6)** Simplify each expression and state all restrictions on $x$.

**a)** $\frac{x+7}{x^2+10+21}$

$= \frac{x+7}{(x+7)(x+3)}$

$= \frac{1}{x+3}$ ; $x \neq -7, -3$

**b)** $\frac{x^2-64}{x-8}$

$= \frac{(x-8)(x+8)}{x-8}$

$= x+8$ ; $x \neq 8$

**c)** $\frac{x^2-9}{x^2-8x+15}$

$= \frac{(x-3)(x+3)}{(x-3)(x-5)}$

$= \frac{x+3}{x-5}$ ; $x \neq 3, 5$

**7)** Simplify each expression and state the restrictions.

**a)** $\frac{3x^2}{5xy} \times \frac{20xy^3}{12xy}$

$= \frac{60x^3y^3}{60x^2y^2}$

$= xy$ ; $x \neq 0$, $y \neq 0$

**b)** $\frac{150a^3b^4}{20a^2b} \div \frac{6b}{8ab^2}$

$= \frac{150a^3b^4}{20a^2b} \times \frac{8ab^2}{6b}$

$= \frac{60a^4b^6}{6a^2b^2}$

$= 10a^2b^4$ ; $a \neq 0$, $b \neq 0$

**c)** $\frac{1}{3x}+\frac{5}{2x^2}$

$= \frac{2x(1)}{2x(3x)} + \frac{3(5)}{3(2x^2)}$

$= \frac{2x}{6x^2} + \frac{15}{6x^2}$

$= \frac{2x+15}{6x^2}$ ; $x \neq 0$

**8)** Simplify each expression and state restrictions.

**a)** $\frac{x^2+7x}{3x+21} \times \frac{x^2+3x+2}{x+2}$

$= \frac{x(x+7)}{3(x+7)} \cdot \frac{(x+2)(x+1)}{(x+2)}$

$= \frac{x(x+1)}{3}$ ; $x \neq -7, -2$

**b)** $\frac{x^2+4x-60}{3x+30} \div \frac{x^2-8x+12}{6x-12}$

$= \frac{(x+10)(x-6)}{3(x+10)} \times \frac{6(x-2)}{(x-6)(x-2)}$

$= \frac{6}{3}$

$= 2$ ; $x \neq -10, 2, 6$

**c)** $\frac{3}{x^2+7x+10}-\frac{5x}{x^2-4}$

$= \frac{3}{(x+2)(x+5)} - \frac{5x}{(x-2)(x+2)}$

$= \frac{3(x-2)}{(x+2)(x+5)(x-2)} - \frac{5x(x+5)}{(x+2)(x+5)(x-2)}$

$= \frac{3x-6-5x^2-25x}{(x+2)(x+5)(x-2)}$

$= \frac{-5x^2-22x-6}{(x+2)(x+5)(x-2)}$ ; $x \neq -5, -2, 2$

**d)** $\frac{-10x}{x^2+18x+32}+\frac{12x}{x^2+6x-160}$

$= \frac{-10x}{(x+16)(x+2)} + \frac{12x}{(x+16)(x-10)}$

$= \frac{-10x(x-10)}{(x+16)(x+2)(x-10)} + \frac{12x(x+2)}{(x+16)(x+2)(x-10)}$

$= \frac{-10x^2+100x+12x^2+24x}{(x+16)(x+2)(x-10)}$

$= \frac{2x^2+124x}{(x+16)(x+2)(x-10)}$

$= \frac{2x(x+62)}{(x+16)(x+2)(x-10)}$ ; $x \neq -16, -2, 10$

**9)** Simplify each expression and state any restrictions

**a)** $\frac{x-8}{x+7} \times \frac{x+15}{x^2+12x-45}$

$= \frac{x-8}{x+7} \cdot \frac{x+15}{(x+15)(x-3)}$

$= \frac{x-8}{(x+7)(x-3)}$ ; $x \neq -7, 3$

**b)** $\frac{x^2+12x+20}{x+5} \div \frac{x^2+7x-30}{x+10}$

$= \frac{(x+10)(x+2)}{x+5} \cdot \frac{x+10}{(x+10)(x-3)}$

$= \frac{(x+10)(x+2)}{(x+5)(x-3)}$ ; $x \neq -10, -5, 3$

**c)** $\frac{x+3}{x-7} - \frac{x+9}{x-2}$

$= \frac{(x+3)(x-2)}{(x-7)(x-2)} - \frac{(x+9)(x-7)}{(x-7)(x-2)}$

$= \frac{x^2+x-6-(x^2+2x-63)}{(x-7)(x-2)}$

$= \frac{-1x+57}{(x-7)(x-2)}$ ; $x \neq 2, 7$

**d)** $\frac{x+8}{x+3} + \frac{x-6}{x^2+9x+18}$

$= \frac{x+8}{x+3} + \frac{x-6}{(x+3)(x+6)}$

$= \frac{(x+6)(x+8)}{(x+3)(x+6)} + \frac{x-6}{(x+3)(x+6)}$

$= \frac{x^2+14x+48+x-6}{(x+3)(x+6)}$

$= \frac{x^2+15x+42}{(x+3)(x+6)}$ ; $x \neq -3, -6$

**e)** $\frac{5x+1}{2x-1} - \frac{3x-3}{1-2x}$

$= \frac{-1(5x+1)}{-1(2x-1)} - \frac{3x-3}{1-2x}$

$= \frac{-5x-1}{-2x+1} - \frac{3x-3}{1-2x}$

$= \frac{-5x-1}{1-2x} - \frac{3x-3}{1-2x}$

$= \frac{-5x-1-(3x-3)}{1-2x}$

$= \frac{-8x+2}{1-2x}$

$= \frac{-2(4x-1)}{1-2x}$ ; $x \neq \frac{1}{2}$

**Answers**

**1) a)** $\frac{1}{10}$ **b)** $\frac{1}{16}$ **c)** $\frac{2}{9}$ **d)** $\frac{126}{125}$ **e)** $5$ **f)** $\frac{64}{27}$

**2) a)** $\frac{1}{x^3}$ **b)** $\frac{6}{k}$ **c)** $\frac{1}{w}$ **d)** $uv^5$ **e)** $z^6$ **f)** $\frac{b^2}{4a^2}$

**3) a)** $-\frac{8}{a^5}$ **b)** $\frac{3y}{x}$ **c)** $16x^4$ **d)** $\frac{4b^8}{9a^6}$

**4) a)** $4$ **b)** $5$ **c)** $-5$ **d)** $\frac{1}{2}$ **e)** $9$ **f)** $10\,000$ **g)** $-\frac{1}{64}$ **h)** $\frac{16}{9}$ **i)** $\frac{25}{9}$

**5) a)** $n^{\frac{13}{12}}$ **b)** $24y^4$ **c)** $3x^2$ **d)** $x^{\frac{2}{15}}$

**6) a)** $\frac{1}{x+3}, x \neq -7, x \neq -3$ **b)** $x+8, x \neq 8$ **c)** $\frac{x+3}{x-5}, x \neq 3, 5$

**7) a)** $xy, x \neq 0, y \neq 0$ **b)** $10a^2b^4, a \neq 0, b \neq 0$ **c)** $\frac{2x+15}{6x^2}, x \neq 0$ **d)** $\frac{x+2}{(x-4)(x-6)}, x \neq 4, x \neq 6$

**8) a)** $\frac{x(x+1)}{3}, x \neq -7, -2$ **b)** $2, x \neq -10, 2, 6$ **c)** $\frac{-5x^2-22x-6}{(x+5)(x+2)(x-2)}, x \neq -5, -2, 2$

**d)** $\frac{2x(x+62)}{(x+16)(x+2)(x-10)}, x \neq -16, -2, 10$

**9) a)** $\frac{x-8}{(x+7(x-3)}, x \neq -15, -7, 3$ **b)** $\frac{(x+10)(x+2)}{(x+5)(x-3)}, x \neq -10, -5, 3$ **c)** $\frac{-x+57}{(x-7)(x-2)}, x \neq 2, 7$

**d)** $\frac{x^2+15x+42}{(x+6)(x+3)}, x \neq -6, -3$ **e)** $\frac{8x-2}{2x-1}, x \neq \frac{1}{2}$