Chapter 2(part 2)
Transformations

Lesson Package

MCR3U

$$g(x) = af[k(x - d)] + c$$

Chapter 2 (part 2) Outline

Unit Goal: Be able to demonstrate an understanding of functions, their representations, and their inverses, and make connections between the algebraic and graphical representations of functions using **transformations.**

Section	Subject	Learning Goals	Curriculum Expectations
L1	Intro to Transformations	- understand the roles of parameters a k, d, and c in functions of the form $f(x) = af[k(x-d)] + c$	A1.8, A1.9
L2	Transformations of x^2	- apply understanding of the roles of parameters a k, d, and c in transformation of the graph of $f(x) = x^2$	A1.8, A1.9
L3	Transformations of $\sqrt{x}$	- apply understanding of the roles of parameters a k, d, and c in transformation of the graph of $f(x) = \sqrt{x}$	A1.8, A1.9
L4	Transformations of $\frac{1}{x}$	- apply understanding of the roles of parameters a k, d, and c in transformation of the graph of $f(x) = \frac{1}{x}$	A1.8, A1.9
L5	Inverse of a Function	- Be able to algebraically determine the equation of the inverse of a function. - Understand the relationship between the domain and range of a function and the domain and range of its inverse.	A1.4, A1.5, A1.6, A1.7

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
PreTest Review	F/A		P	
Test – Transformations of Functions	O	A1.1, A1.2, A2.1, A2.2, A2.3, A2.5, A3.2	P	K(42%), T(10%), A(10%), C(38%)

L1 - Intro to Transformations – Lesson
MCR3U
Jensen

In this chapter you will learn about transformations of functions. There are three main functions that we will use to learn about transformations:

1.

2.

3.

Note: *the equations given for each type of function are considered the base or parent functions of their respective families of functions. All transformations of these functions will be compared to these base functions.*

Before learning about transformations, you must understand what the base functions look like and be able to generate the key points for the graph of each function.

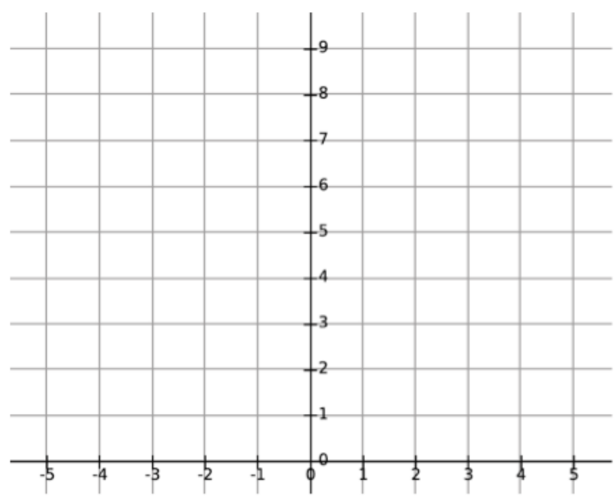
<u>Quadratic Functions</u>

Base Function:

Key Points:

x	y

Graph of Base Function:



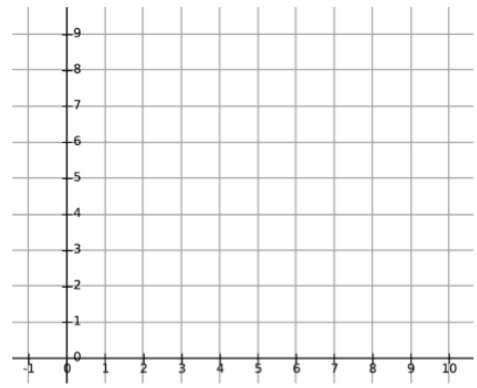
Radical (square root) Functions

Base Function:

Key Points:

x	y

Graph of Base Function:



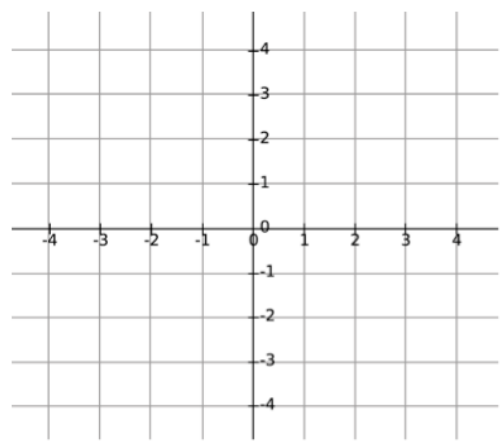
Rational Functions

Base Function:

Key Points:

x	y

Graph of Base Function:



Asymptotes

Asymptote:

The function $f(x) = \frac{1}{x}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq 0$. This is why the vertical line $x = 0$ is an asymptote for this function.

Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.

Transformations of Functions

Transformation:

The general function:

Changes to the *y*-coordinates (vertical changes)

***c*: vertical translation** $g(x) = f(x) + c$

The graph of $g(x) = f(x) + c$ is a vertical translation of the graph of $f(x)$ by c units.

If $c > 0$, the graph shifts **UP**
If $c < 0$, the graph shifts **DOWN**

***a*: vertical stretch/compression** $\boldsymbol{g(x) = a \cdot f(x)}$

The graph of $g(x) = af(x)$ is a vertical stretch or compression of the graph of $f(x)$ by a factor of a.

If $a > 1$ OR $a < -1$, **vertical stretch** by a factor of $\lvert a \rvert$
If -1< $a < 1$, **vertical compression** by a factor of $\lvert a \rvert$
If $a < 0$, **vertical reflection** (reflection over the x-axis)

Note: a vertical stretch or compression means that distance from the x-axis of each point of the parent function changes by a factor of a.

Note: for a vertical reflection, the point (x, y) becomes point $(x, -y)$

Changes to the *x*-coordinates (horizontal changes)

***d:* horizontal translation** $g(x) = f(x - d)$

The graph of $g(x) = f(x - d)$ is a horizontal translation of the graph of $f(x)$ by d units.

If $d > 0$, the graph shifts **RIGHT**
If $d < 0$, the graph shifts **LEFT**

k: horizontal stretch/compression
$\boldsymbol{g(x) = f(kx)}$

The graph of $g(x) = f(kx)$ is a horizontal stretch or compression of the graph of $f(x)$ by a factor of $\frac{1}{k}$

If $k > 1$ OR $k < -1$, **horizontal compression** by a factor of $\frac{1}{\lvert k \rvert}$
If -1< $k < 1$, **horizontal stretch** by a factor of $\frac{1}{\lvert k \rvert}$
If $k < 0$, **horizontal reflection** (reflection over the y-axis)

Note: a vertical stretch or compression means that distance from the y-axis of each point of the parent function changes by a factor of $\frac{1}{k}$.

Note: for a horizontal reflection, the point (x, y) becomes point $(-x, y)$

Order of Transformations:

1. stretches, compressions, reflections

2. translations

Example 1: List the transformations and the order in which they should be done to a function *f*(*x*).

a) $g(x) = -f(x)$

b) $g(x) = 2f\left(\frac{1}{3}x\right)$

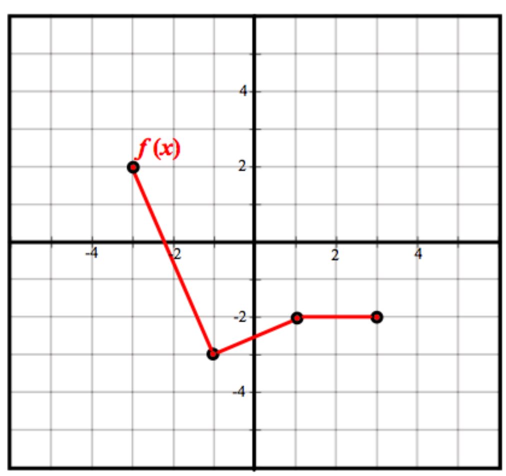
c) $g(x) = 3f(x+2) - 1$

d) $g(x) = \frac{1}{4}f[2(x-1)]$

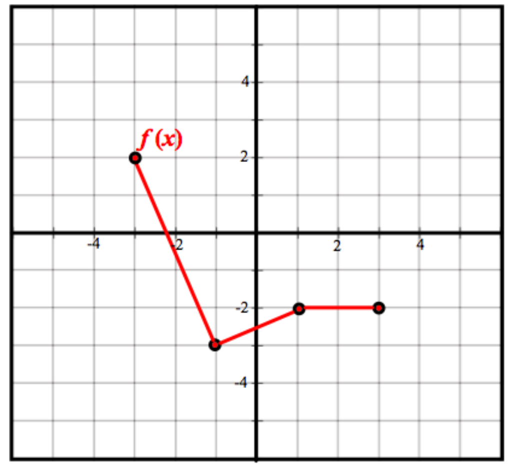
e) $g(x) = -5f\left[-\frac{1}{4}(x+2)\right] + 7$

Example 2: List the transformations and the order in which they should be done to the function *f*(*x*). Use the given graph of *f*(*x*) to sketch the graph of *g*(*x*)

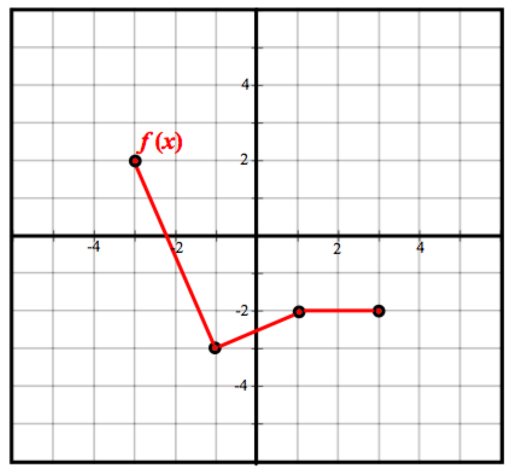
a) $g(x) = f(x+2)$



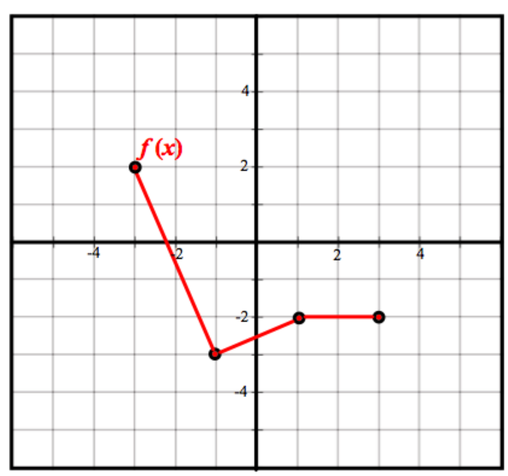
b) $g(x) = - f(x)$



c) $g(x) = f(x) + 3$



d) $g(x) = f(2x) - 1$



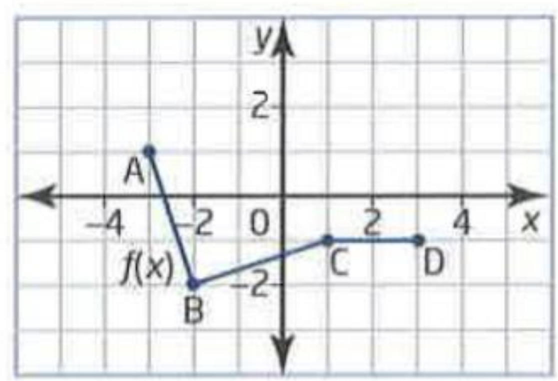
L2 - Transformations of Quadratic Functions – Lesson

MCR3U
Jensen

DO IT NOW!

a) Complete the table of values for the function $f(x)$ and $g(x)$. Then use the table of values to plot image points and graph the function $g(x)$

$f(x):(x, f(x))$	$g(x):(x, f(x)+4)$
A(-3, 1)	A'(-3, 5)



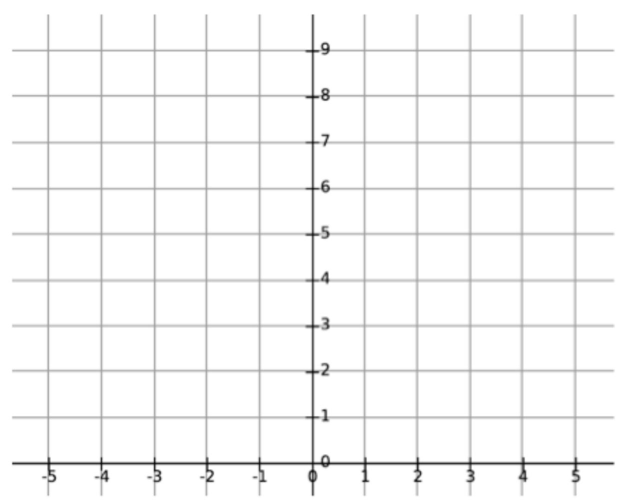
Quadratic Functions

Base Function:

Graph of Base Function:

Key Points:

x	y



Order of Transformations

1. stretches, compressions, reflections

2. translations

Example 1: If $f(x) = x^2$, describe the changes and write the transformed function:

a) $g(x) = 2f(x)$

b) $g(x) = f(2x)$

c) $g(x) = f(x) + 4$

d) $g(x) = f(x + 3)$

e) $g(x) = -f(x)$

f) $g(x) = f(-x)$

Example 2: For each of the following functions, describe the transformations to $f(x) = x^2$ in order and write the transformed equation.

a) $g(x) = -2f[-3(x + 3)] - 1$

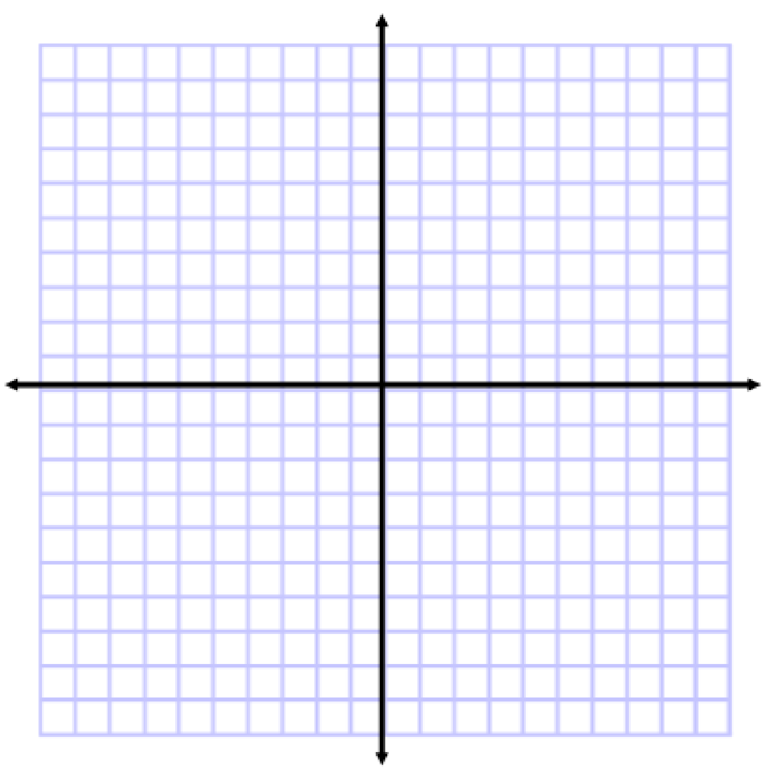
b) $y = \frac{1}{2}f[-3(x - 2)] + 5$

Example 3: for each of the following functions…

i) make a table of values for the parent function
ii) graph the parent function $f(x) = x^2$
iii) describe the transformations
iv) make a table of values of image points
v) graph the transformed function and write it's equation

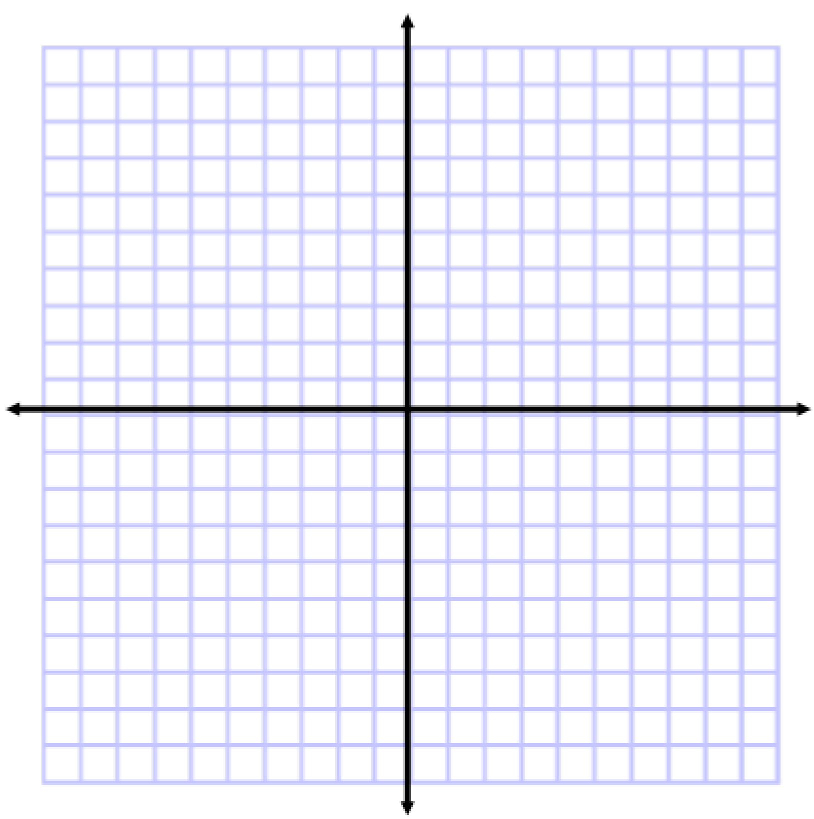
a) $g(x) = -f(2x)$

x	y



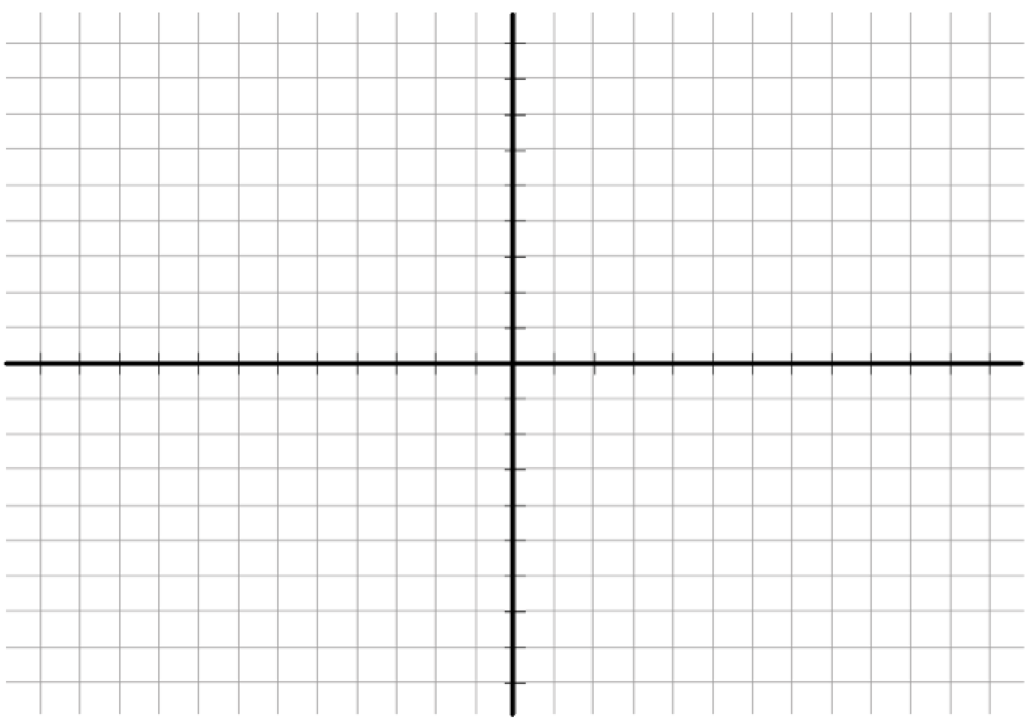
b) $g(x) = f\left[-\frac{1}{2}(x-1)\right]$

x	y



c) $g(x) = -2f[-3(x+3)] - 1$

x	y



L3 - Transformations of $\sqrt{x}$ - Lesson

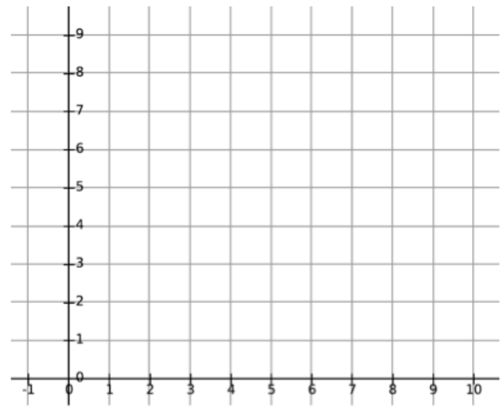
MCR3U
Jensen

Base Function:

Graph of Base Function:

Key Points:

x	y



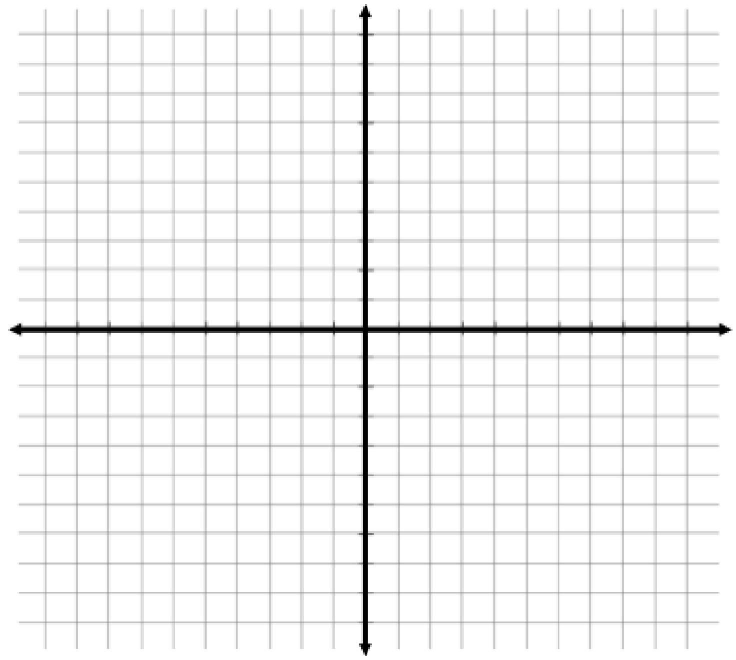
Example 1: Using the parent function $f(x) = \sqrt{x}$, describe the transformations and write the equation of the transformed function $g(x)$.

$$\boldsymbol{g(x) = -2f\left[-\frac{1}{3}(x+6)\right] - 5}$$

Example 2: for each of the following functions…

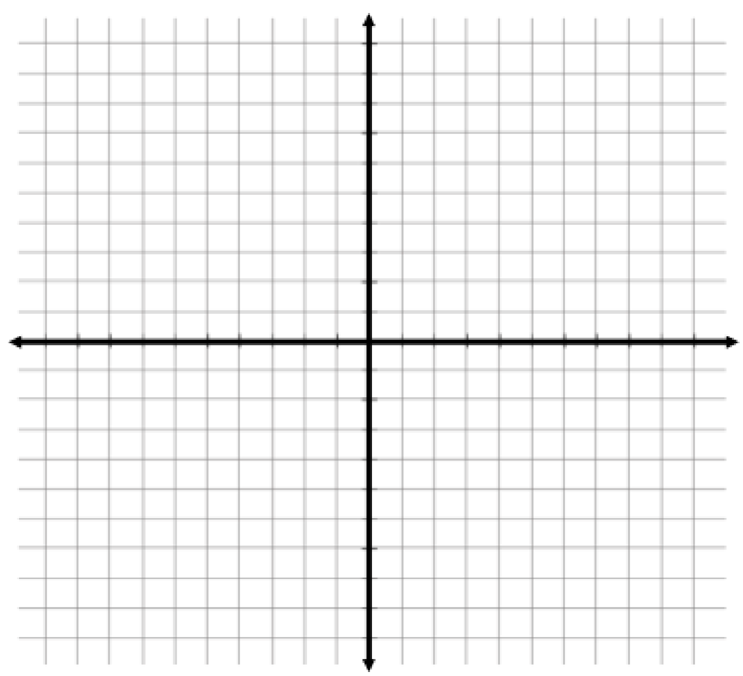
i) make a table of values for the parent function
ii) graph the parent function $f(x) = \sqrt{x}$
iii) describe the transformations
iv) make a table of values of image points
v) graph the transformed function and write its equation

a) $g(x) = \frac{1}{2}f(x) + 1$



x	y

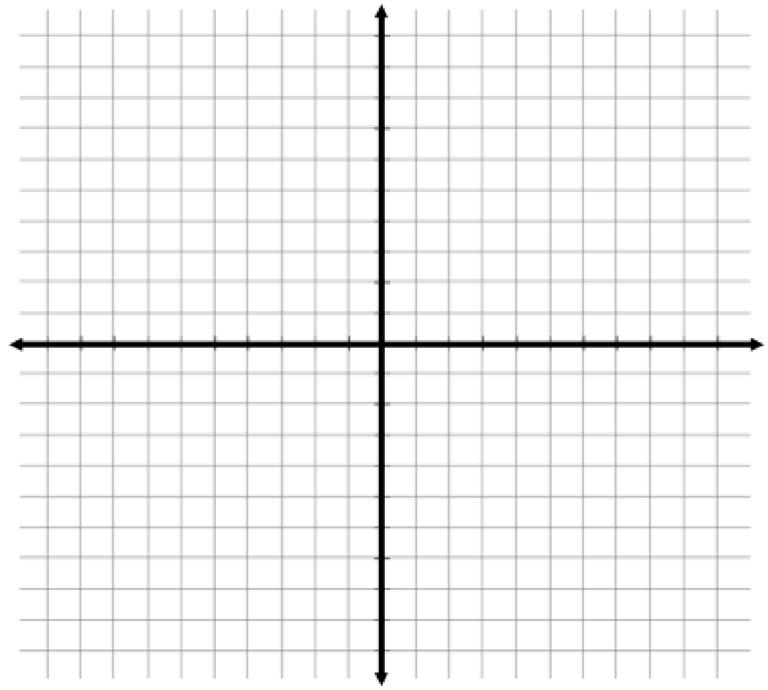
b) $g(x) = -f[2(x-3)]$



x	y

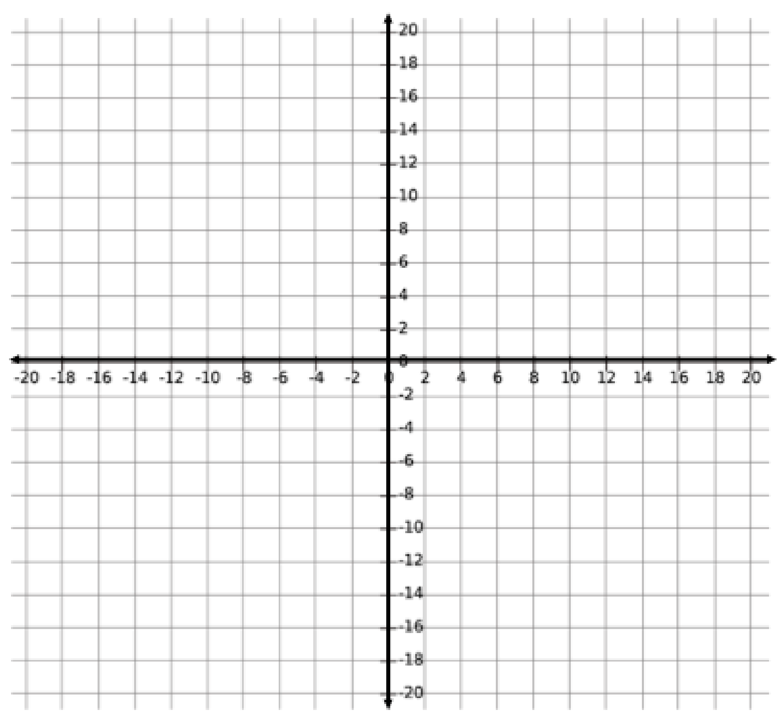
c) $g(x) = -2f(x+3) - 1$

x	y



d) $g(x) = 3f\left(-\frac{1}{2}x + 2\right) + 1$

x	y



L4 - Transformations of $\frac{1}{x}$ - Lesson

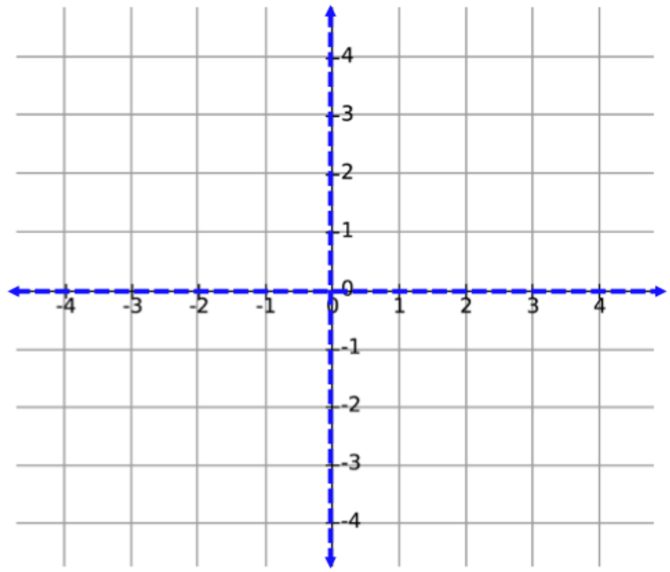
MCR3U
Jensen

Base Function:

Graph of Base Function:

Key Points:

x	y



<u>Asymptotes</u>

Asymptote:

The function $f(x) = \frac{1}{x}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq 0$. This is why the vertical line $x = 0$ is an asymptote for this function.

Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.

Example 1: Describe the combination of transformations that must be applied to the base function $f(x) = \frac{1}{x}$ to obtain the transformed function. Then, write the corresponding equations.

a) $g(x) = 4f(x - 3) + 0.5$

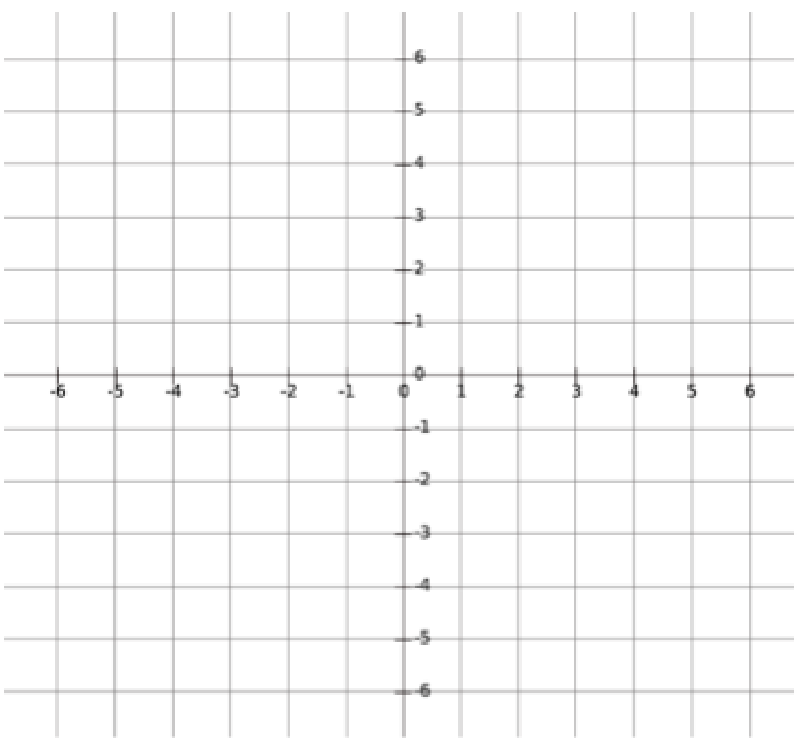
b) $g(x) = f[-2(x + 0.5)] - 1$

Example 2: for each of the following functions…

i) make a table of values for the parent function $f(x) = \frac{1}{x}$
ii) describe the transformations
iii) make a table of values of image points
iv) graph the transformed function and write it's equation

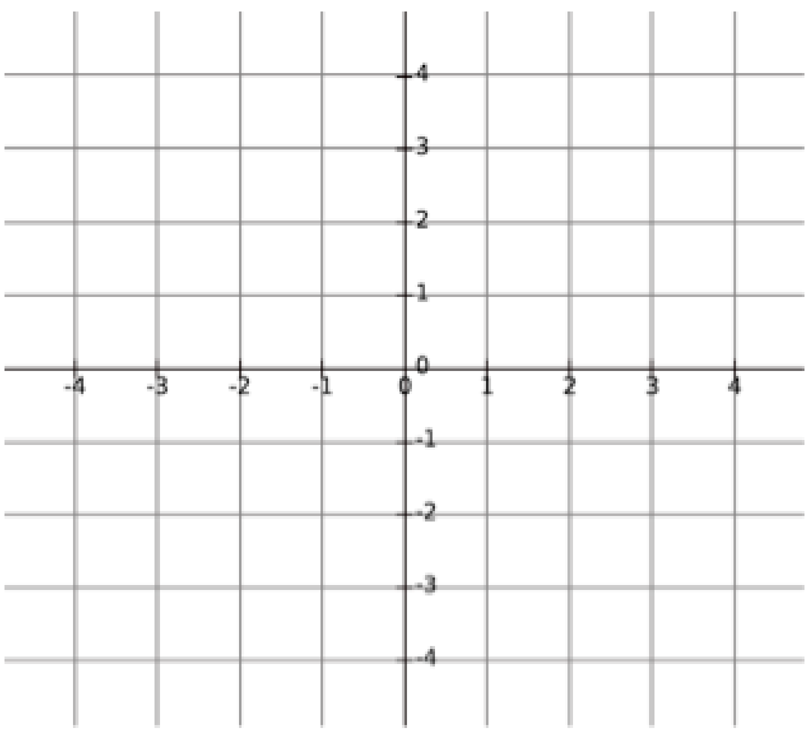
a) $g(x) = 2f(x - 1) + 2$

x	y



b) $g(x) = -f[2(x+0.5)] - 1$

x	y



L5 - 2.6 Inverse of a Function – Lesson

MCR3U
Jensen

Inverse of a function:

· The inverse of a function f is denoted as f^{-1}
· The function and its inverse have the property that if f(a) = b, then $f^{-1}(b) = a$
· So if $f(5) = 13$, then $f^{-1}(13) = 5$
· More simply put: The inverse of a function has all the same points as the original function, except that the x's and y's have been reversed.

It is important to note that $f^{-1}(x)$ is read as "the inverse of f at x". The -1 does not behave like an exponent.

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

To draw an inverse, all you need to do is swap the x and y coordinates of each point.

<u>Finding Inverses Numerically</u>

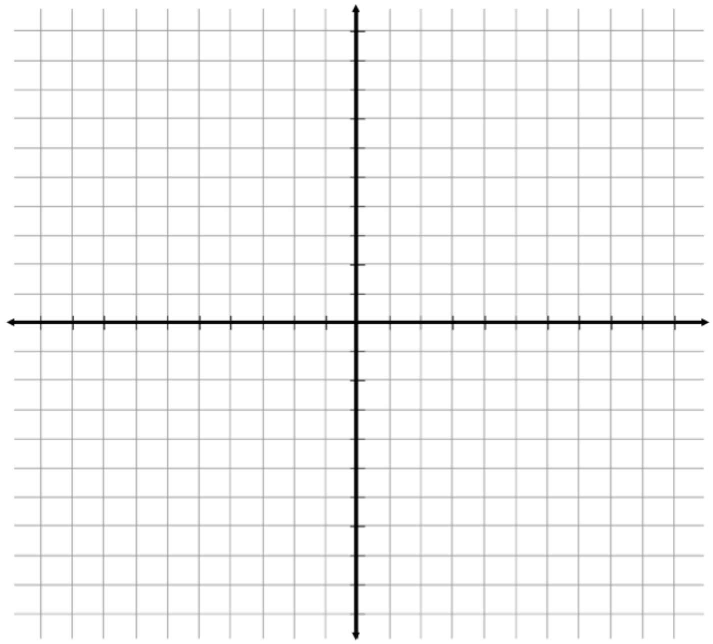
Example 1: The table shows ordered pairs belonging to a function $f(x)$. Determine $f^{-1}(x)$, then state the domain and range of $f(x)$ and its inverse.

$f(x)$	$f^{-1}(x)$
(-5, 0)	
(-4, 2)	
(-3, 5)	
(-2, 6)	
(0, 7)	

Example 2:

a) Graph the function $f(x) = x^2$ and its inverse $f^{-1}(x)$.

$f(x)$	$f^{-1}(x)$

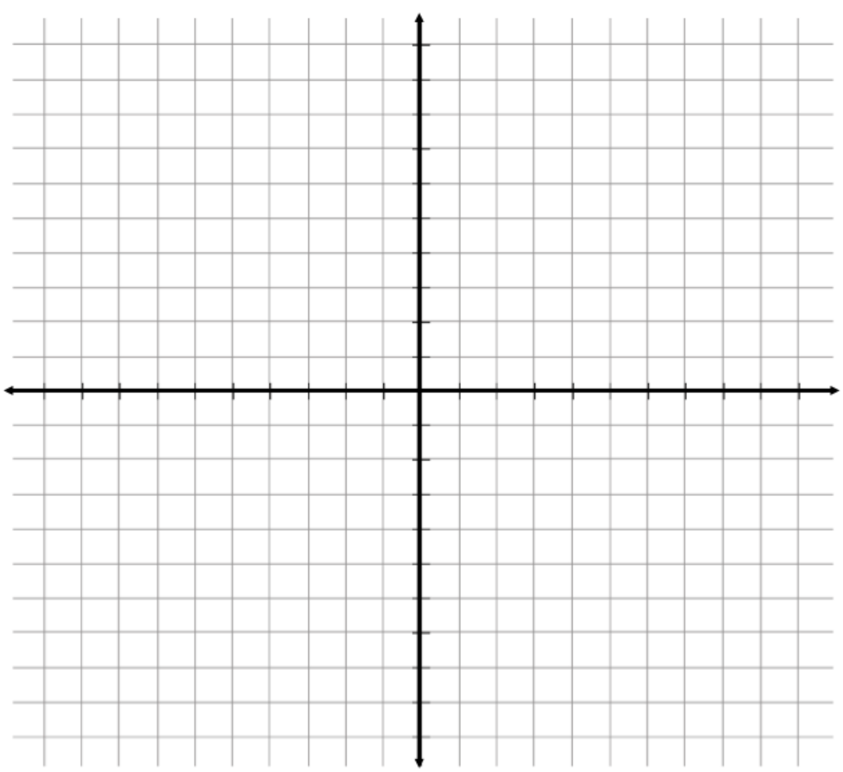


b) State the domain and range of both functions

Example 3: Sketch the graph of $g(x) = -2\sqrt{\left(-\frac{1}{2}x\right)} + 3$, then graph $g^{-1}(x)$.

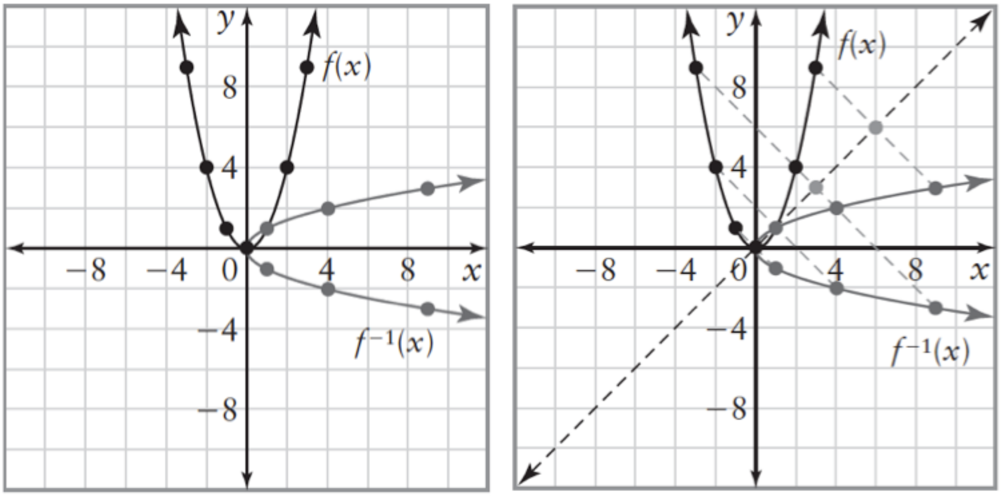
x	***y***

x	***y***

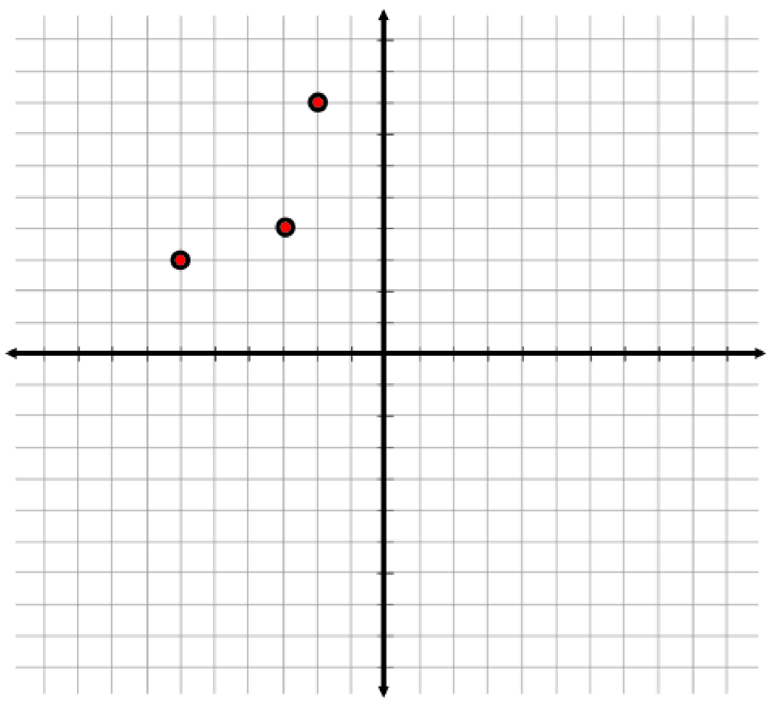


Finding Inverses by Graphing

The graph of $f^{-1}(x)$ is the graph of $f(x)$ reflected in the line $y = x$. This is true for all functions and their inverses. If you find the midpoint of each pair of points from example 2 and connect them you can prove this theorem.



Example 4: Sketch the inverse of $f(x)$



Finding Inverses Algebraically

Algebraic Method for finding the inverse:

1. Replace $f(x)$ with "y"
2. Switch the x and y variables
3. Isolate for y
4. replace y with $f^{-1}(x)$

a) $g(x) = \frac{3x}{4}$

b) $h(x) = 4x + 3$

c) $f(x) = x^2 - 1$

d) $h(x) = \frac{4x+3}{5}$

e) $f(x) = 2x^2 + 16x + 29$

> **Note:** for algebraic inverses of quadratic functions, before interchanging x and y's you must re-write in vertex form.

f) $r(x) = \sqrt{x} + 2$