

# *Chapter 2(part 2)*

## *Transformations*

*WORKBOOK*

*MCR3U*

$$\textcolor{blue}{g(x)} = \textcolor{black}{af}[\textcolor{red}{k}(x - \textcolor{black}{d})] + \textcolor{black}{c}$$

## **Chapter 2(part 2) Workbook Checklist**

<b>Worksheet</b>	<b>Check</b>
Intro to Transformations - Worksheet	
Transformations of $x^2$ – Worksheet	
Transformations of $\sqrt{x}$ – Worksheet	
Transformations of $\frac{1}{x}$ – Worksheet	
2.7 – Inverse of a Function – Worksheet	
Review	

# W1 - Intro to Transformations

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1) Describe the transformations, in order, that are being done to the function  $f(x)$ .

a)  $g(x) = -4f(x)$

b)  $g(x) = f(3x)$

c)  $g(x) = \frac{1}{2}f(-x)$

d)  $g(x) = -\frac{1}{3}f[\frac{1}{2}(x + 1)]$

e)  $g(x) = 5f[-2(x - 4)]$

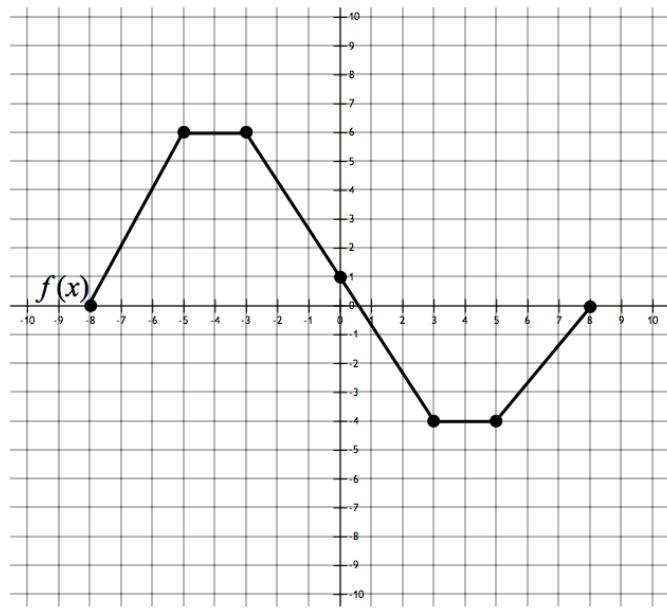
f)  $g(x) = -2f(8x) + 4$

h)  $g(x) = -\frac{1}{4}f[-3(x - 1)] - 5$

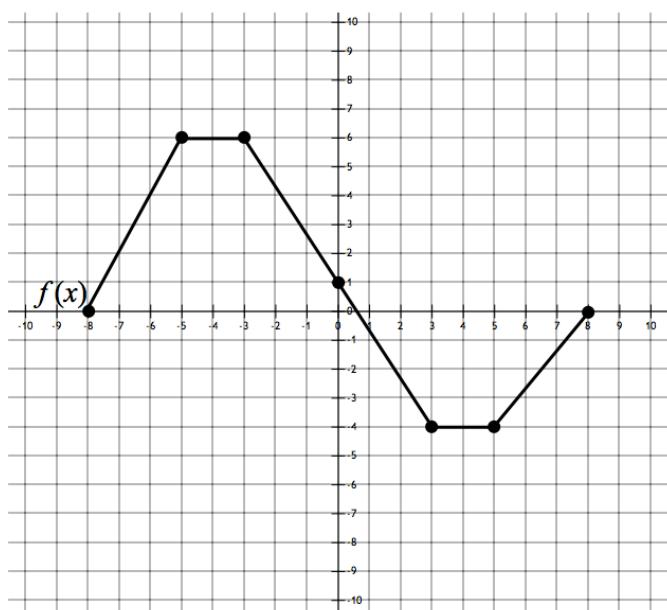
i)  $g(x) = 4f\left[-\frac{1}{2}(x + 2)\right] - 1$

2) For the graph of  $f(x)$  given, sketch the graph of  $g(x)$  after the given transformation.

a)  $g(x) = 2f(x) - 2$



b)  $g(x) = \frac{1}{2}f(x-1) + 1$



## Answers

1) a) vertical reflection over the x-axis and vertical stretch bafo 4 ( $-4y$ )

b) horizontal compression bafo  $\frac{1}{3} \left( \frac{x}{3} \right)$

c) vertical compression bafo  $\frac{1}{2} \left( \frac{y}{2} \right)$ , horizontal relection over the y-axis ( $-x$ )

d) vertical reflection over the x-axis and vertical compression bafo  $\frac{1}{3} \left( \frac{y}{-3} \right)$ , horizontal stretch bafo 2 ( $2x$ ), phase shift left 1 unit ( $x - 1$ )

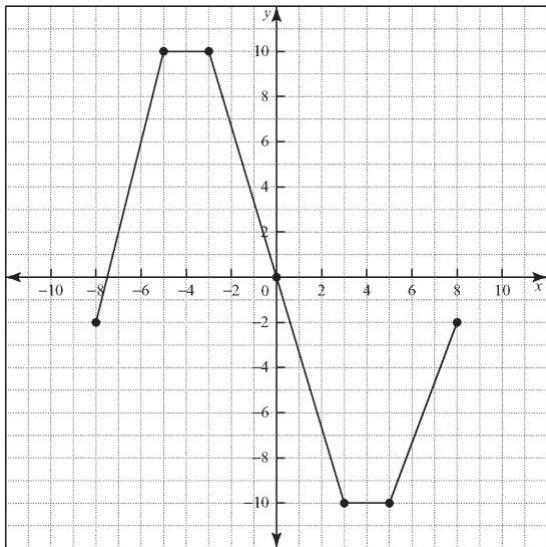
e) vertical stretch bafo 5 ( $5y$ ), horizontal reflection over the y-axis and horizontal compression bafo  $\frac{1}{2} \left( \frac{x}{-2} \right)$ , phase shift right 4 units ( $x + 4$ )

f) vertical reflection over the x-axis and vertical stretch bafo 2 ( $-2y$ ), horizontal compression bafo  $\frac{1}{8} \left( \frac{x}{8} \right)$ , shift up 4 units ( $y + 4$ )

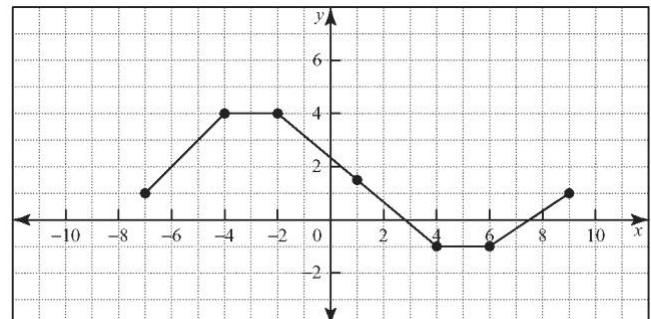
h) vertical reflection over the x-axis and vertical compression bafo  $\frac{1}{4} \left( \frac{y}{-4} \right)$ , horizontal reflection over the y-axis and horizontal compression bafo  $\frac{1}{3} \left( \frac{x}{-3} \right)$ , phase shift right 1 unit ( $x + 1$ ), shift down 5 units ( $y - 5$ )

i) vertical stretch bafo 4 ( $4y$ ), horizontal reflection over the y-axis and horizontal stretch bafo 2 ( $-2x$ ), phase shift left 2 units ( $x - 2$ ), shift down 1 unit ( $y - 1$ )

2) a)



b)



## **W2 - Transformations of Quadratic Functions**

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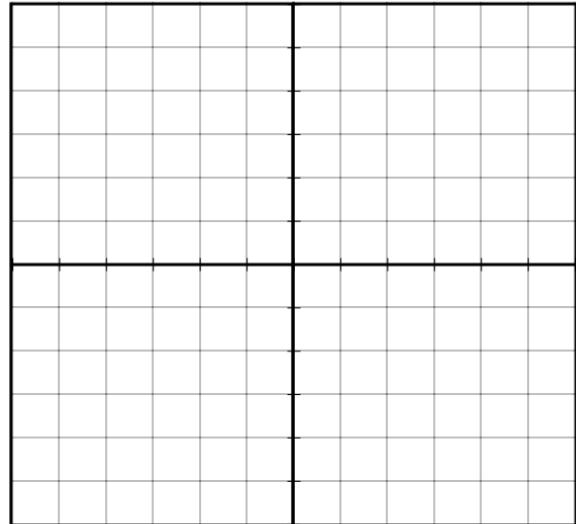
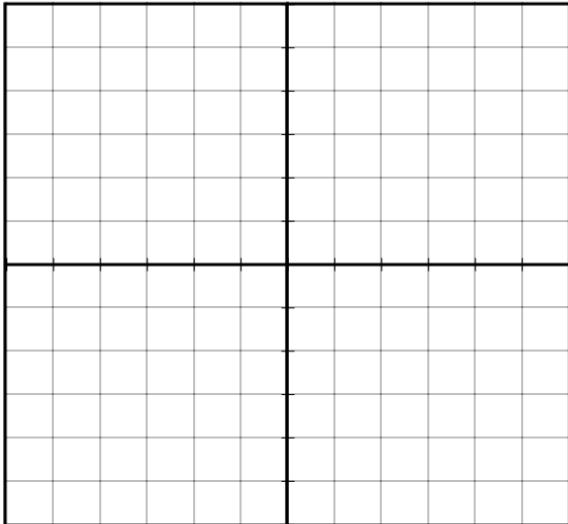


**1) For each of the following graphs:**

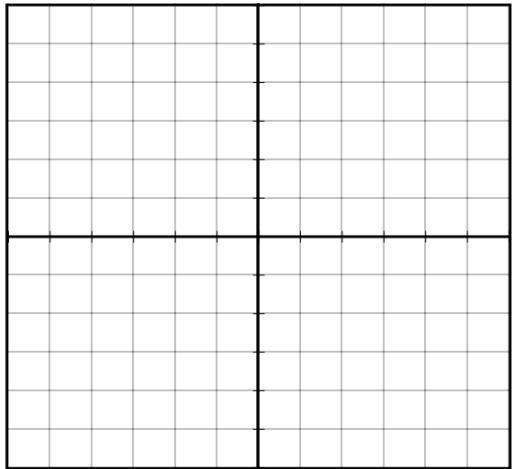
- i) describe the transformations in order ( $a \rightarrow k \rightarrow d \rightarrow c$ )
  - ii) create a table of values for the transformed function
  - iii) graph the transformed function

a)  $y = -x^2 + 2$

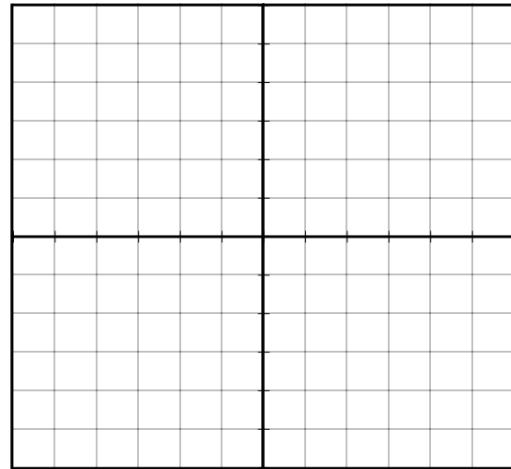
b)  $y = (x - 3)^2 + 1$



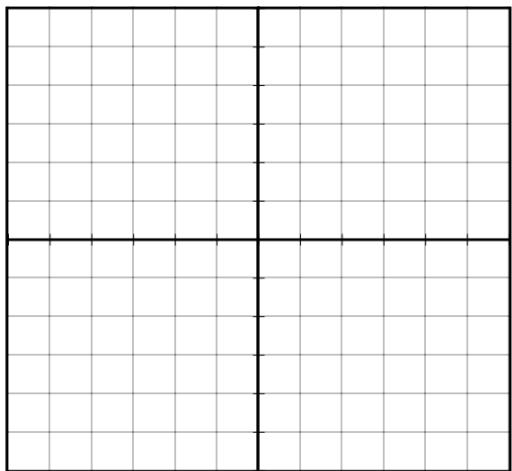
c)  $y = 2x^2 - 5$



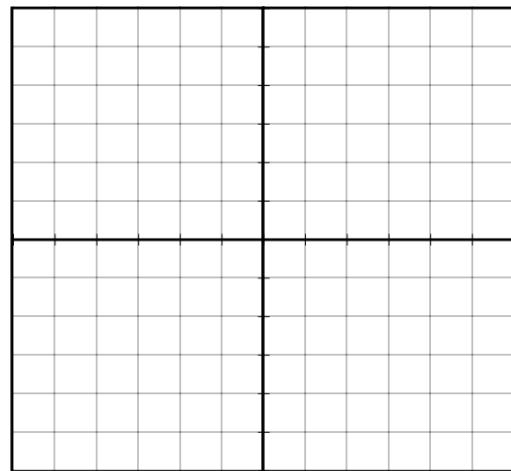
d)  $y = -3(x + 1)^2$



e)  $y = -(x + 2)^2 + 4$



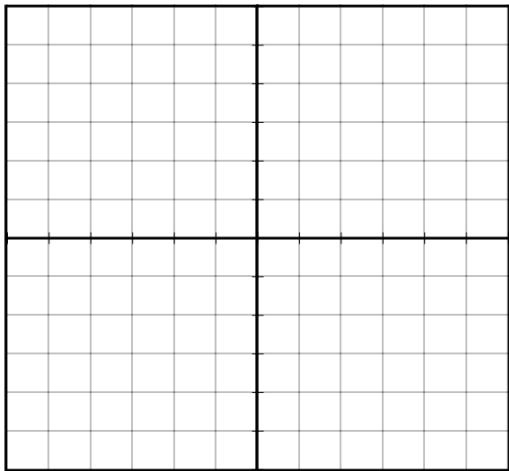
f)  $y = -\frac{1}{2}x^2$



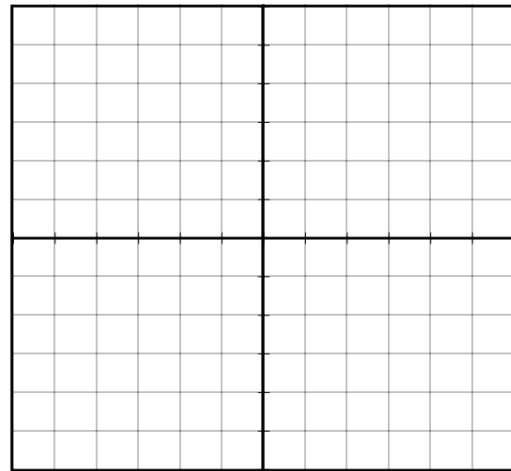
**2)** For each function  $g(x)$ :

- i)** describe the transformations from the parent function  $f(x) = x^2$
- ii)** create a table of values of image points for the transformed function
- iii)** graph the transformed function and write its equation

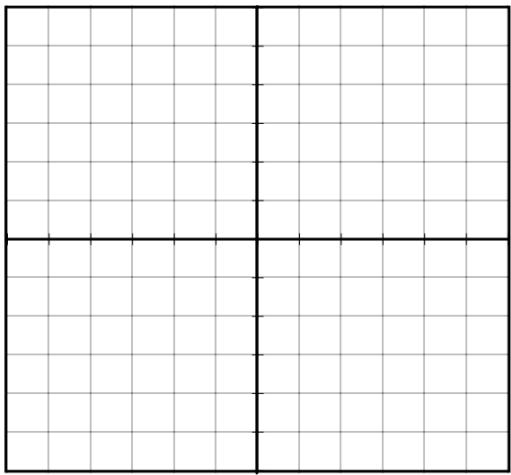
**a)**  $g(x) = -2f\left[\frac{1}{2}(x + 2)\right]$



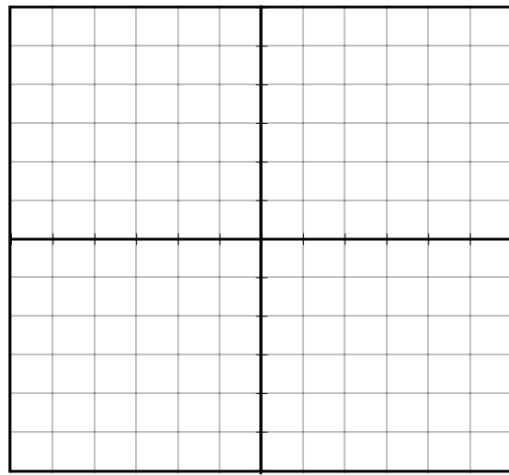
**b)**  $g(x) = 4f(x - 3) - 2$



c)  $y = 2f(x + 4) - 3$



d)  $y = \frac{1}{2}f[-2(x + 2)] - 3$



## W3 - Transformations of $\sqrt{x}$

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Key points of  
 $y = \sqrt{x}$

$x$	$y$

1) State the transformations to the parent function  $f(x) = \sqrt{x}$  in the order that you would do them.

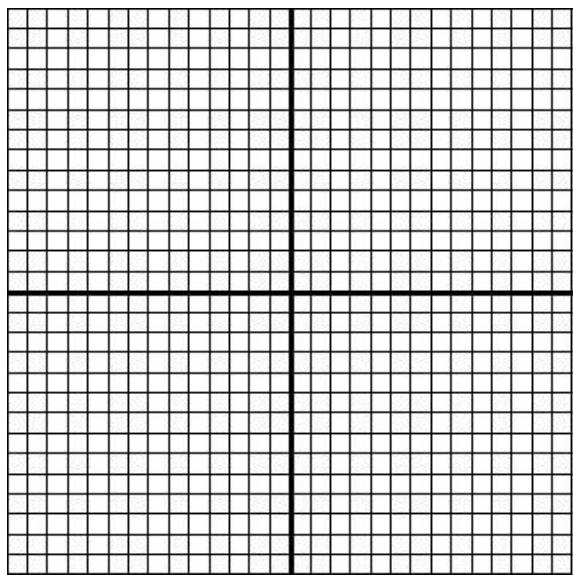
a)  $g(x) = 2\sqrt{x+1} - 3$

b)  $g(x) = 3\sqrt{\frac{1}{2}(x-5)} + 4$

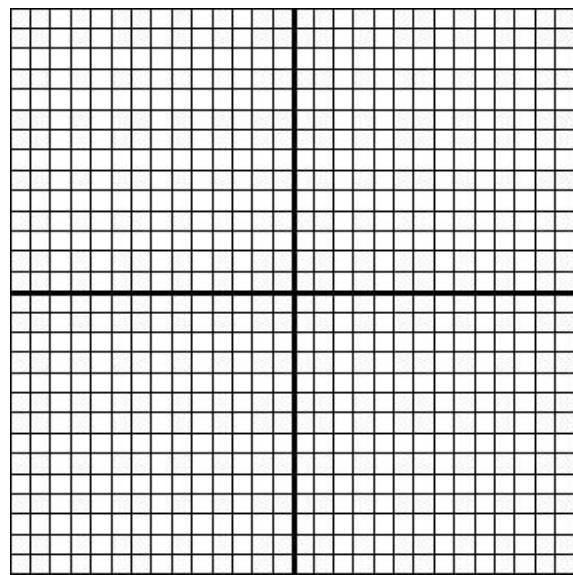
c)  $g(x) = -\frac{1}{2}\sqrt{-3(x)} - 6$

**2)** Graph the parent function,  $f(x) = \sqrt{x}$ . Describe the transformations in order, make a table of values of image points, write the equation of the transformed function and graph it.

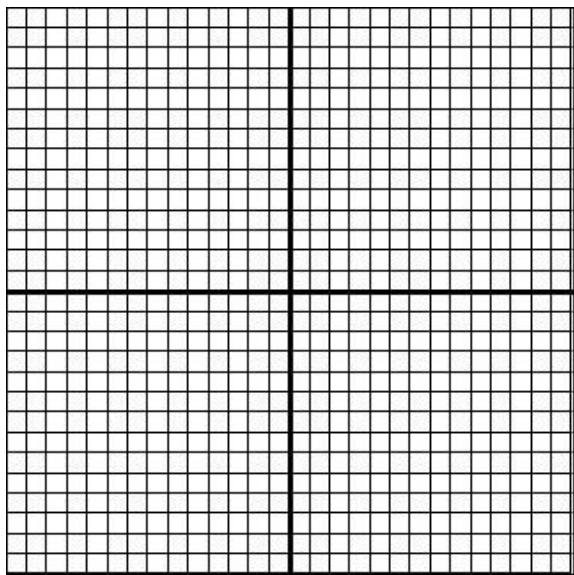
**a)**  $g(x) = f[3(x + 5)]$



**b)**  $g(x) = \frac{1}{4}f(-x)$



c)  $g(x) = -4f[-2(x - 3)] + 1$



3) Use the description to write the transformed function,  $g(x)$ .

- a) The parent function  $f(x) = \sqrt{x}$  is compressed vertically by a factor of  $\frac{1}{3}$  and then translated (shifted) 3 units left.
- b) The parent function  $f(x) = \sqrt{x}$  is reflected over the x-axis, stretch horizontally by a factor of 3 and then translated 1 unit left and 4 units down.

# W4 - Transformations of $\frac{1}{x}$

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Key points of

$$y = \frac{1}{x}$$

$x$	$y$

1) State the transformations to the parent function  $f(x) = \frac{1}{x}$  in the order that you would do them.

a)  $g(x) = \frac{2}{3(x-1)}$

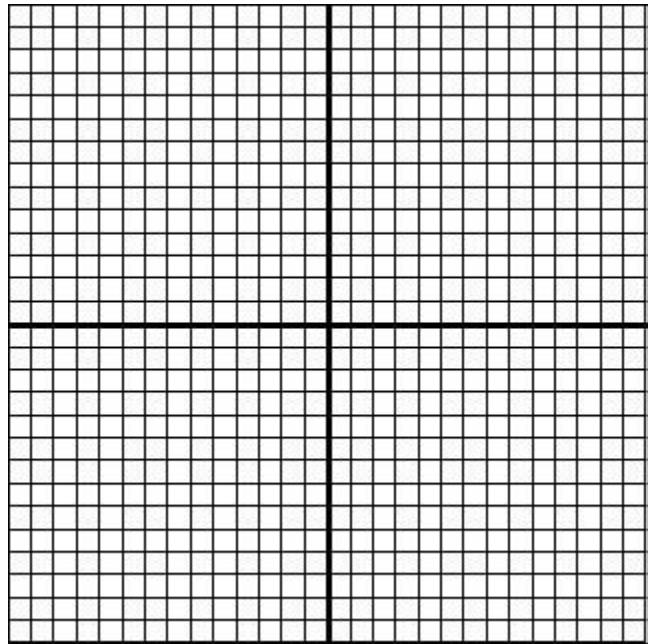
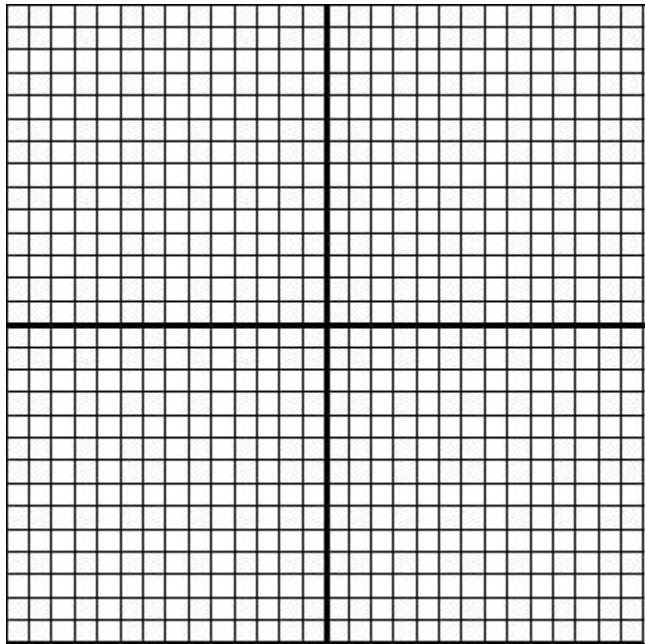
b)  $g(x) = \frac{-1}{x+2} - 1$

c)  $g(x) = \frac{1}{\frac{1}{2}(x+1)} - 0.5$

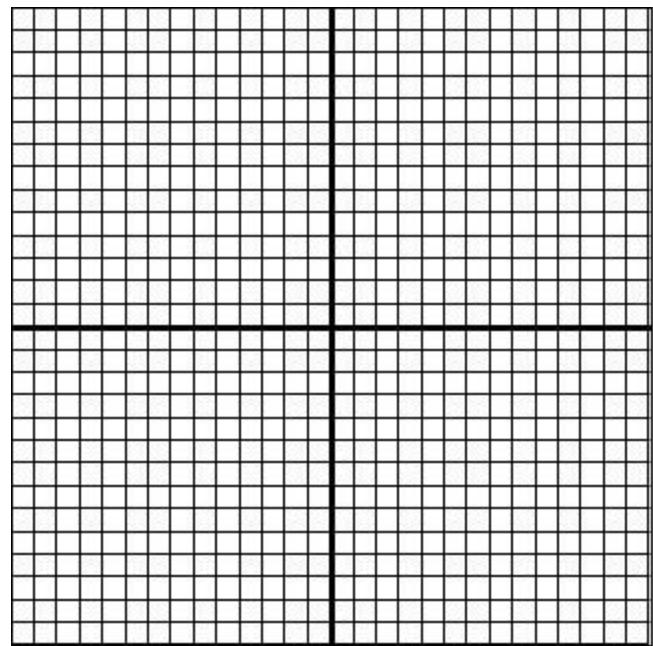
2) Describe the transformations to the parent function  $f(x) = \frac{1}{x}$  in order, make a table of values of image points, write the equation of the transformed function and graph it.

a)  $g(x) = f\left[\frac{1}{2}(x + 1)\right]$

b)  $g(x) = 2f(-x)$



c)  $g(x) = -f[-2(x - 0.5)] + 1$



3) Use the description to write the transformed function,  $g(x)$ .

a) The parent function,  $f(x) = \frac{1}{x}$ , is compressed vertically by a factor of  $\frac{1}{3}$  and then translated (shifted) 3 units left.

b) The parent function,  $f(x) = \frac{1}{x}$ , is reflected over the x-axis, stretch horizontally by a factor of 3 and then translated 1 unit left and 4 units down.

## W5 - Inverse of a Function

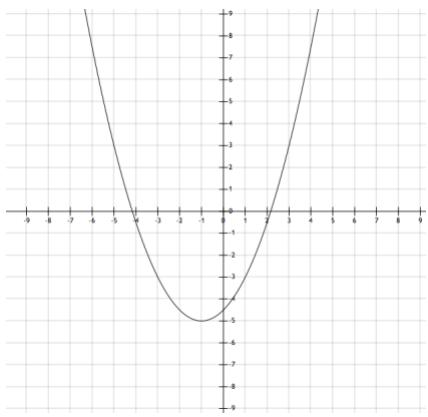
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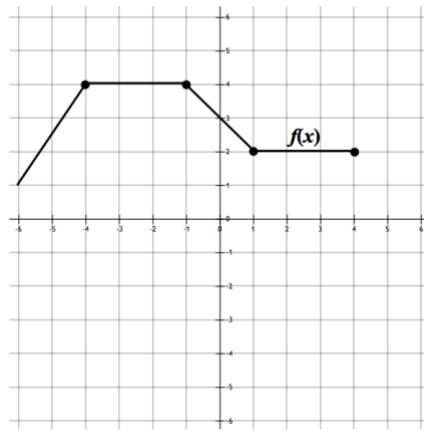


1) Sketch the graph of the inverse of each function. Is the inverse of  $f(x)$  a function? Explain.

a)



b)



2) Determine the equation of the inverse of each function.

a)  $f(x) = 2x$

b)  $f(x) = 6x - 5$

c)  $f(x) = \frac{2x+4}{5}$

3) Determine the equation of the inverse of each function

a)  $f(x) = x^2 + 6$

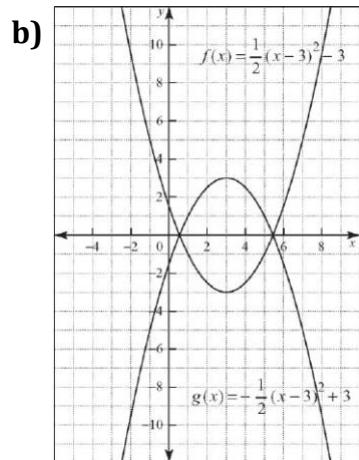
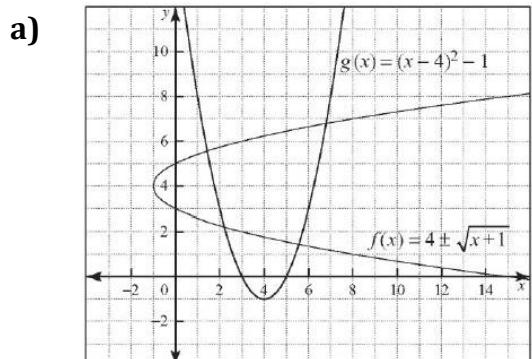
b)  $f(x) = (x + 8)^2$

4) For each quadratic function, complete the square and then determine the equation of the inverse.

a)  $f(x) = x^2 + 6x + 15$

b)  $f(x) = 2x^2 + 24x - 3$

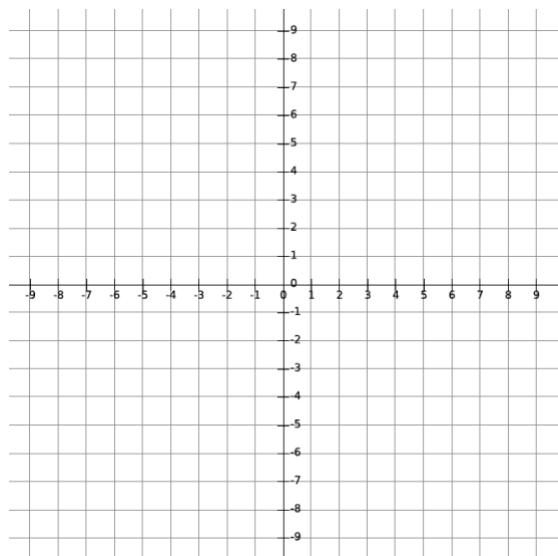
5) Determine if the two relations shown are inverses of each other. Justify your conclusion.



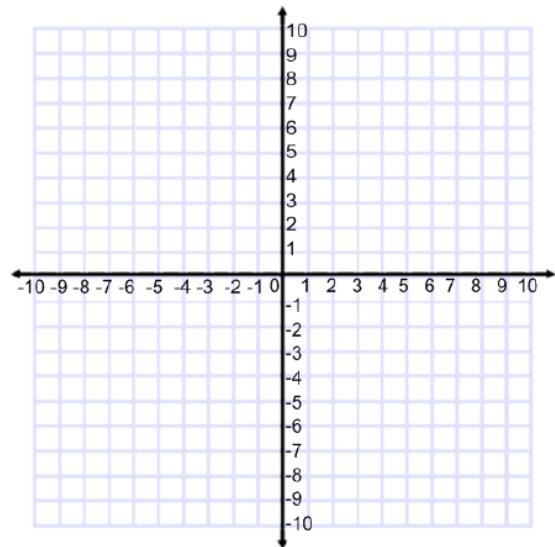
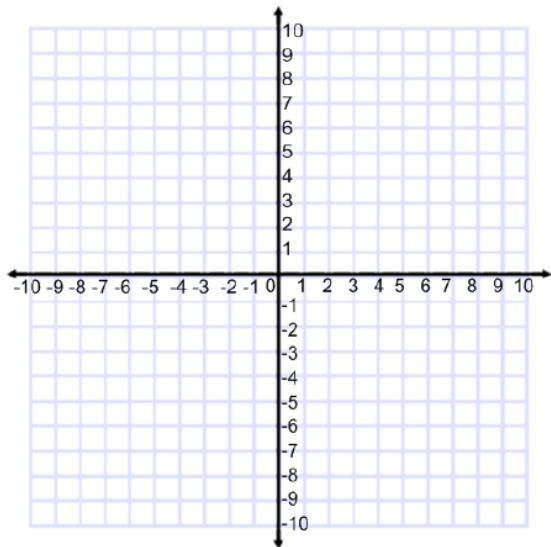
6) For the function  $f(x) = -5x + 6$

a) determine  $f^{-1}(x)$

b) Graph  $f(x)$  and its inverse



7) Use transformations to graph the function  $f(x) = 2(x - 2)^2 + 1$ . Find the inverse function  $f^{-1}(x)$  and graph it by reflecting  $f(x)$  over the line  $y = x$  (switch x and y co-ordinates)



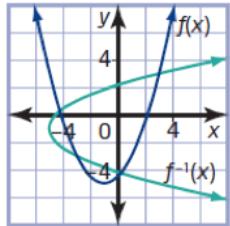
8) Determine the equation of the inverse for the given functions and state the domain and range.

a)  $f(x) = \sqrt{x + 3}$

b)  $f(x) = \frac{3}{x-2} + 2$

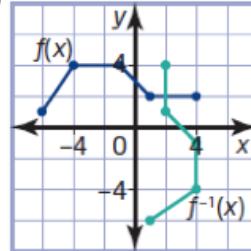
## Answers

**1) a)**



the inverse is NOT a function

**b)**



inverse is NOT a function

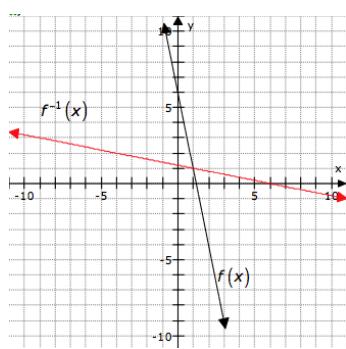
**2) a)**  $f^{-1}(x) = \frac{x}{2}$    **b)**  $f^{-1}(x) = \frac{x+5}{6}$    **c)**  $f^{-1}(x) = \frac{5x-4}{2}$

**3) a)**  $f^{-1}(x) = \pm\sqrt{x-6}$    **b)**  $f^{-1}(x) = \pm\sqrt{x} - 8$

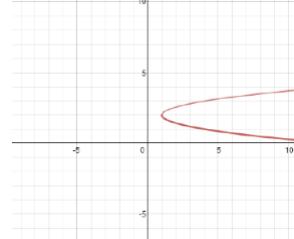
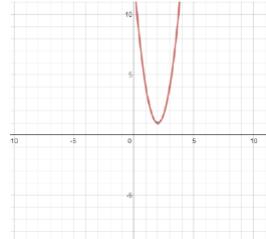
**4) a)**  $f^{-1}(x) = \pm\sqrt{x-6} - 3$    **b)**  $f^{-1}(x) = \pm\sqrt{\frac{x+75}{2}} - 6$

**5) a)** yes   **b)** no

**6) a)**  $f^{-1}(x) = \frac{-x+6}{5}$    **b)**



**7)**  $f^{-1}(x) = 2 \pm \sqrt{\frac{x-1}{3}}$



**8) a)**  $f^{-1}(x) = x^2 - 3$ ; Domain for  $f(x)$ :  $\{X \in \mathbb{R} | x \geq -3\}$ , Range for  $f(x)$ :  $\{Y \in \mathbb{R} | y \geq 0\}$   
 Domain for  $f^{-1}(x)$ :  $\{X \in \mathbb{R} | x \geq 0\}$ , Range for  $f(x)$ :  $\{Y \in \mathbb{R} | y \geq -3\}$

**b)**  $f^{-1}(x) = \frac{3}{x-2} + 2$ ; Domain for  $f(x)$ :  $\{X \in \mathbb{R} | x \neq 2\}$ , Range for  $f(x)$ :  $\{Y \in \mathbb{R} | y \neq 2\}$   
 Domain for  $f^{-1}(x)$ :  $\{X \in \mathbb{R} | x \neq 2\}$ , Range for  $f(x)$ :  $\{Y \in \mathbb{R} | y \neq 2\}$