Chapter 2(part 2)
Transformations

WORKBOOK

MCR3U

$$g(x) = af[k(x - d)] + c$$

Intro to Transformations – Worksheet SOLUTIONS

MCR3U

Jensen

1) Describe the transformations, in order, that are being done to the function f(x).

a) g(x)= -4f(x)

- vertical reflection over the x-axis $(-y)$
- vertical stretch b.a.f.o 4 $(4y)$

b) g(x)= f(3x)

- horizontal compression b.a.f.o $\frac{1}{3}$ $\left(\frac{x}{3}\right)$

c) $g(x) = \frac{1}{2}f(-x)$

- vertical compression b.a.f.o $\frac{1}{2}$ $\left(\frac{y}{2}\right)$
- horizontal reflection over the y-axis $(-x)$

d) $g(x) = -\frac{1}{3}f[\frac{1}{2}(x+1)]$

- vertical compression bafo $\frac{1}{3}$ $\left(\frac{y}{3}\right)$
- vertical reflection over the x-axis $(-y)$
- horizontal stretch bafo 2. $(2x)$
- phase shift 1 unit left $(x-1)$

e) $g(x) = 5f[-2(x-4)]$

- vertical stretch bafo 5. $(5y)$
- horizontal compression bafo $\frac{1}{2}$ $\left(\frac{x}{2}\right)$
- horizontal reflection across the y-axis. $(-x)$
- phase shift 4 units right $(x+4)$

f) $g(x) = -2f(8x) + 4$

- vertical stretch bafo 2 $(2y)$
- vertical reflection over the x-axis $(-y)$
- horizontal compression bafo $\frac{1}{8}$ $\left(\frac{x}{8}\right)$
- shift up 4 units. $(y+4)$

h) $g(x) = -\frac{1}{4}f[-3(x-1)] - 5$

- vertical compression bafo $\frac{1}{4}$ $\left(\frac{y}{4}\right)$
- vertical reflection over x-axis $(-y)$
- horizontal compression bafo $\frac{1}{3}$ $\left(\frac{x}{3}\right)$
- horizontal reflection over y-axis $(-x)$
- phase shift 1 unit right $(x+1)$
- shift 5 units down $(y-5)$

i) $g(x) = 4f\left[-\frac{1}{2}(x+2)\right] - 1$

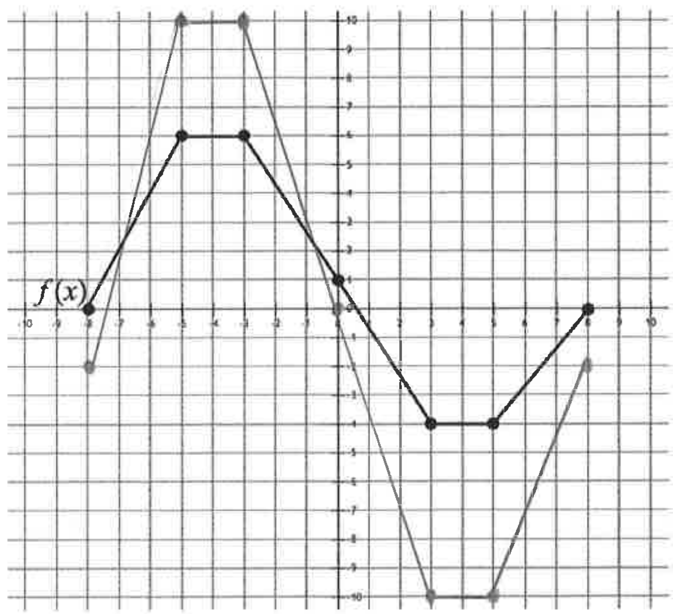
- vertical stretch bafo 4 $(4y)$
- horizontal stretch bafo 2 $(2x)$
- horizontal reflection over y-axis $(-x)$
- phase shift 2 units left $(x-2)$
- shift down 1 unit $(y-1)$

2) For the graph of $f(x)$ given, sketch the graph of $g(x)$ after the given transformation.

a) $g(x) = 2f(x) - 2$

- vertical stretch bafo 2 $(2y)$
- shift down 2 units $(y-2)$

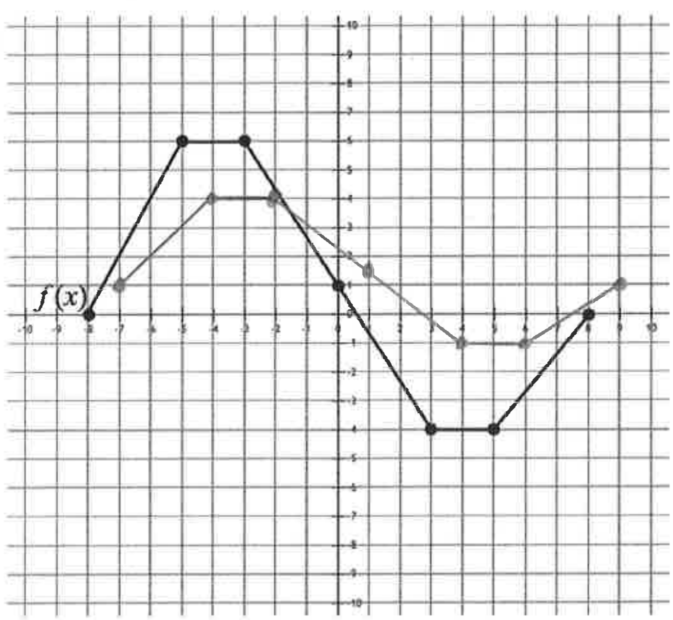
$(x, 2y-2)$



b) $g(x) = \frac{1}{2}f(x-1)+1$

- vertical compression bafo $\frac{1}{2}$ $(\frac{y}{2})$
- phase shift right 1 unit $(x+1)$
- phase shift up 1 unit $(y+1)$

$(x+1, \frac{y}{2}+1)$



Answers

1) a) vertical reflection over the x-axis and vertical stretch bafo 4 $(-4y)$

b) horizontal compression bafo $\frac{1}{3}$ $\left(\frac{x}{3}\right)$

c) vertical compression bafo $\frac{1}{2}$ $\left(\frac{y}{2}\right)$, horizontal relection over the y-axis $(-x)$

d) vertical reflection over the x-axis and vertical compression bafo $\frac{1}{3}$ $\left(\frac{y}{-3}\right)$, horizontal stretch bafo 2 $(2x)$, phase shift left 1 unit $(x-1)$

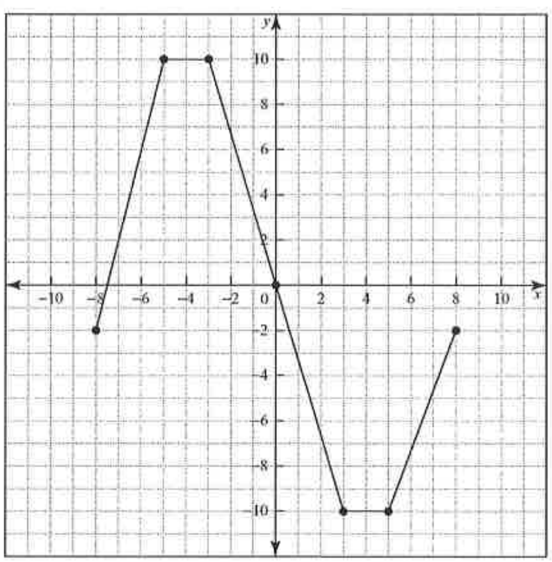
e) vertical stretch bafo 5 $(5y)$, horizontal reflection over the y-axis and horizontal compression bafo $\frac{1}{2}$ $\left(\frac{x}{-2}\right)$, phase shift right 4 units $(x+4)$

f) vertical reflection over the x-axis and vertical stretch bafo 2 $(-2y)$, horizontal compression bafo $\frac{1}{8}$ $\left(\frac{x}{8}\right)$, shift up 4 units $(y+4)$

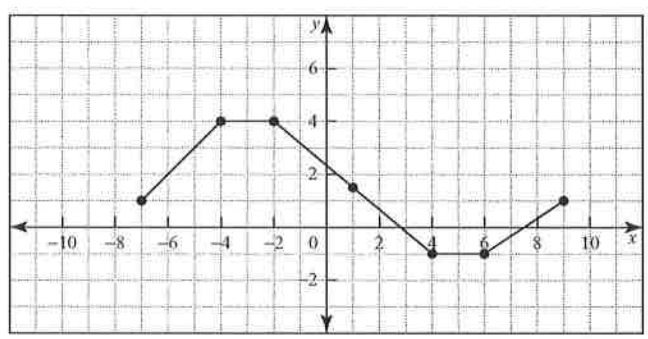
h) vertical reflection over the x-axis and vertical compression bafo $\frac{1}{4}$ $\left(\frac{y}{-4}\right)$, horizontal reflection over the y-axis and horizontal compression bafo $\frac{1}{3}$ $\left(\frac{x}{-3}\right)$, phase shift right 1 unit $(x+1)$, shift down 5 units $(y-5)$

i) vertical stretch bafo 4 $(4y)$, horizontal reflection over the y-axis and horizontal stretch bafo 2 $(-2x)$, hase shift left 2 units $(x-2)$, shift down 1 unit $(y-1)$

2) a)



b)



Transformations of Quadratic Functions – Worksheet

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SOLUTIONS

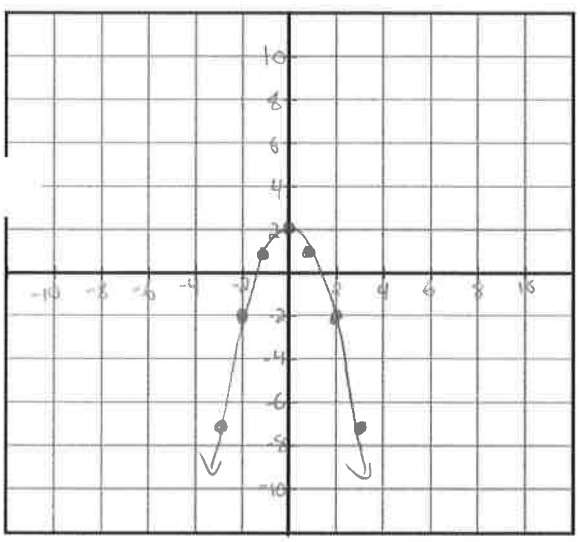
Key points for $y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

1) For each of the following graphs:

i) describe the transformations in order (a → k → d → c)
ii) create a table of values for the transformed function
iii) graph the transformed function

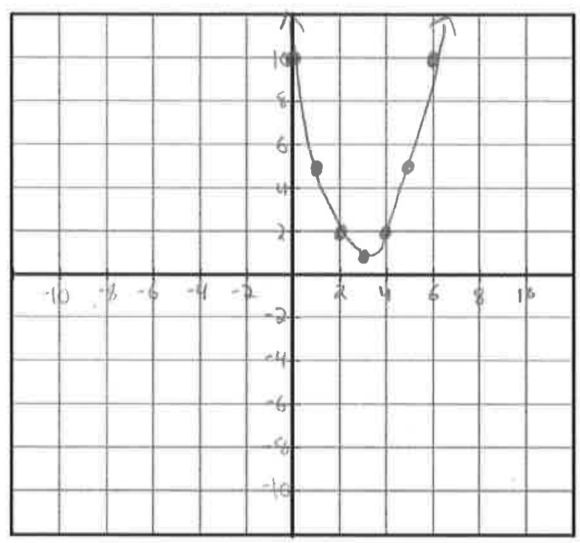
a) $y = -x^2 + 2$



1) vertical reflection (-y)
2) shift up 2 (y+2)

x	$-y+2$
-3	-7
-2	-2
-1	1
0	2
1	1
2	-2
3	-7

b) $y = (x - 3)^2 + 1$

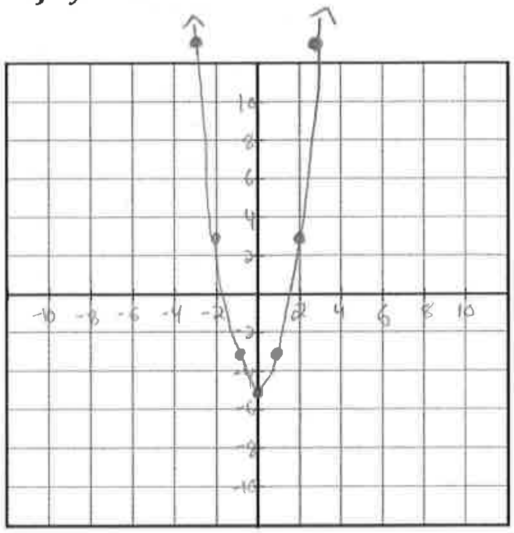


1) shift right 3 units (x+3)
2) shift up 1 unit (y+1)

$x+3$	$y+1$
0	10
1	5
2	2
3	1
4	2
5	5
6	10

(-3,9)
(-2,4)
(-1,1)
(0,0)
(1,1)
(2,4)
(3,9)

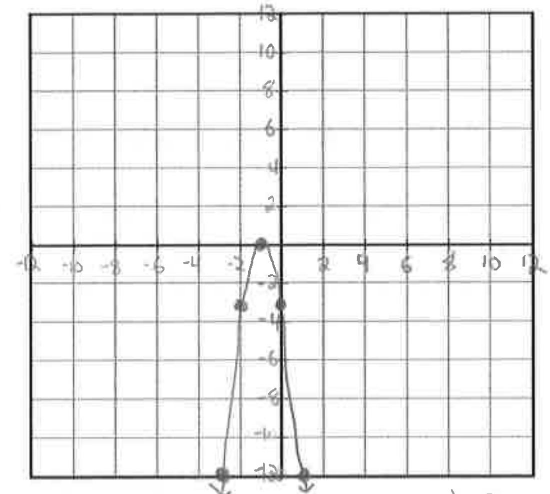
c) $y = 2x^2 - 5$



1) vertical stretch bafo 2 (2y)
2) shift down 5 units (y-5)

x	$2y-5$
-3	13
-2	3
-1	-3
0	-5
1	-3
2	3
3	13

d) $y = -3(x+1)^2$

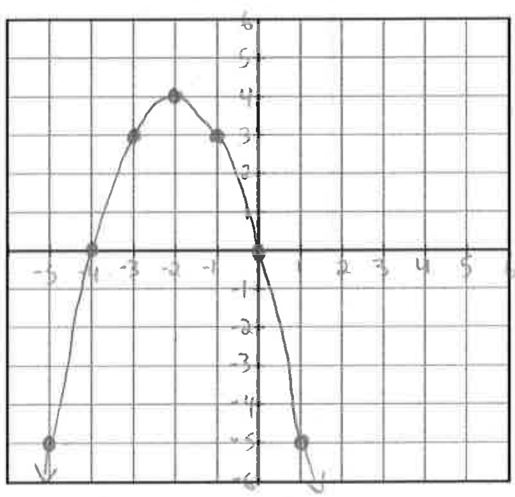


1) vertical stretch by 3 and vertical reflection (-3y)
2) phase shift left 1 unit (x-1)

$x-1$	$-3y$
-4	-27
-3	-12
-2	-3
-1	0
0	-3
1	-12
2	-27

just graph these.

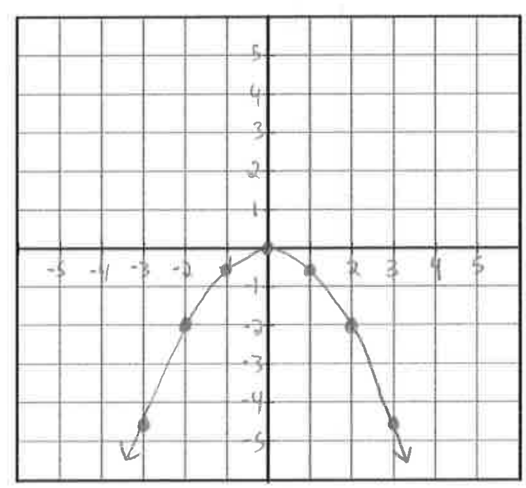
e) $y = -(x+2)^2 + 4$



1) vertical reflection (-y)
2) shift left 2 units (x-2)
3) shift up 4 units (y+4)

$x-2$	$-y+4$
-5	-5
-4	0
-3	3
-2	4
-1	3
0	0
1	-5

f) $y = -\frac{1}{2}x^2$



1) vertical stretch bafo $\frac{1}{2}$ and vertical reflection $\left(\frac{y}{-2}\right)$

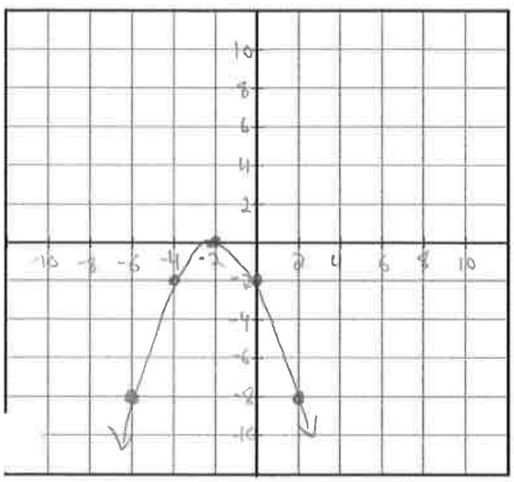
x	$\frac{y}{-2}$
-3	-4.5
-2	-2
-1	-0.5
0	0
1	-0.5
2	-2
3	-4.5

$(-3,9)$
$(-2,4)$
$(-1,1)$
$(0,0)$
$(1,1)$
$(2,4)$
$(3,9)$

2) For each function $g(x)$:

- **i)** describe the transformations from the parent function $f(x) = x^2$
- **ii)** create a table of values of image points for the transformed function
- **iii)** graph the transformed function and write its equation

a) $g(x) = -2f\left[\frac{1}{2}(x+2)\right]$



1) vertical stretch bafo 2; vertical reflection $(-2y)$
2) horizontal stretch bafo 2 $(2x)$
3) shift left 2 units $(x-2)$

$2x-2$	$-2y$
-8	-18
-6	-8
-4	-2
-2	0
0	-2
2	-8
4	-18

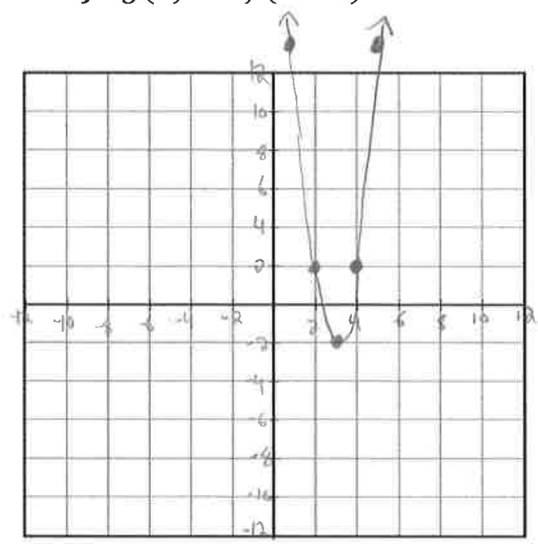
graph these

$g(x) = -2\left[\frac{1}{2}(x+2)\right]^2$

$g(x) = -2\left(\frac{1}{4}\right)(x+2)^2$

$g(x) = -\frac{1}{2}(x+2)^2$

b) $g(x) = 4f(x-3) - 2$

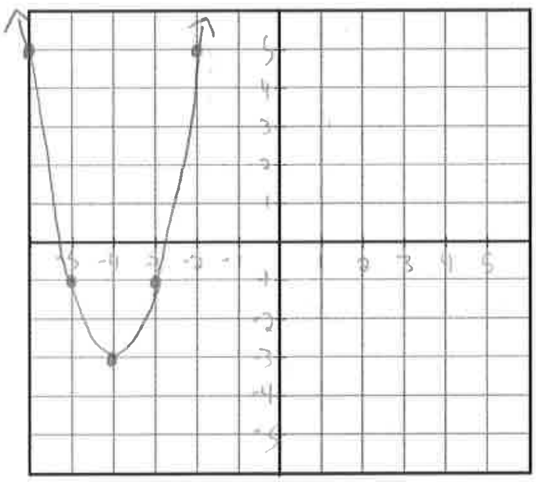


1) vertical stretch bafo 4 $(4y)$
2) shift right 3 units $(x+3)$
3) shift down 2 units $(y-2)$

$x+3$	$4y-2$
0	34
1	14
2	2
3	-2
4	2
5	14
6	34

$g(x) = 4(x-3)^2 - 2$

c) $y = 2f(x+4) - 3$



1) vertical stretch bafo 2 $(2y)$

2) shift left 4 units $(x-4)$

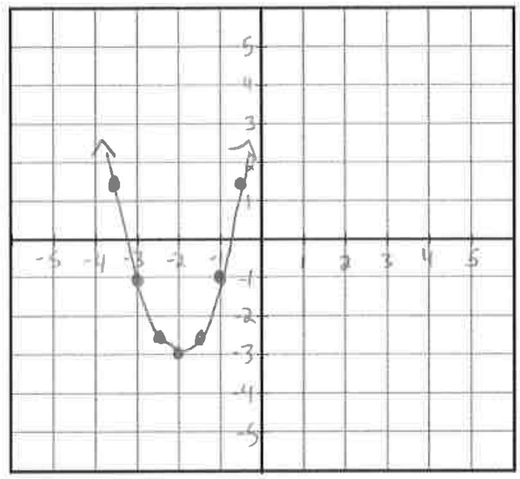
3) shift down 3 units $(y-3)$

	$x-4$	$2y-3$
(-3,9)	-7	15
(-2,4)	-6	5
(-1,1)	-5	-1
(0,0)	-4	-3
(1,1)	-3	-1
(2,4)	-2	5
(3,9)	-1	15

graph these

$y = 2(x+4)^2 - 3$

d) $y = \frac{1}{2}f[-2(x+2)] - 3$



1) vertical compression bafo $\frac{1}{2}$ $\left(\frac{y}{2}\right)$

2) horizontal compression bafo $\frac{1}{2}$; horizontal reflection $\left(\frac{x}{-2}\right)$

3) left 2 units $(x-2)$

4) down 3 units $(y-3)$

$\frac{x}{-2} - 2$	$\frac{y}{2} - 3$
-0.5	1.5
-1	-1
-1.5	-2.5
-2	-3
-2.5	-2.5
-3	-1
-3.5	1.5

$g(x) = \frac{1}{2}\left[-2(x+2)\right]^2 - 3$

$g(x) = \frac{1}{2}(4)(x+2)^2 - 3$

$g(x) = 2(x+2)^2 - 3$

Transformations of $\sqrt{x}$ - Worksheet

SOLUTIONS

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Jensen

Key points of
$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

1) 1) State the transformations to the parent function $f(x) = \sqrt{x}$ in the order that you would do them.

) $f(x) = 2\sqrt{x+1} - 3$

- vertical stretch bafo 2
- shift left 1 unit
- shift down 3 units

b) $f(x) = 3\sqrt{\frac{1}{2}(x-5)} + 4$

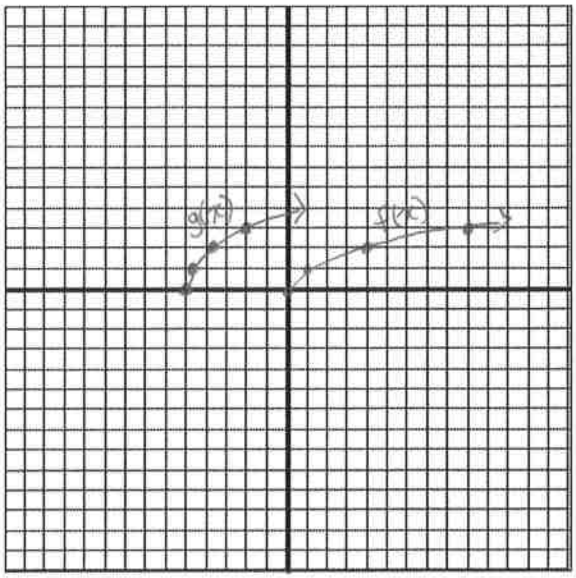
- vertical stretch bafo 3
- horizontal stretch bafo 2
- shift right 5 units
- shift up 4 units

c) $f(x) = -\frac{1}{2}\sqrt{-3(x)} - 6$

- vertical compression bafo $\frac{1}{2}$
- vertical reflection
- horizontal compression bafo $\frac{1}{3}$
- horizontal reflection
- shift down 6 units.

2) Graph the parent function, $f(x) = \sqrt{x}$. Describe the transformations in order, make a table of values of image points, write the equation of the transformed function and graph it.

a) $g(x) = f[3(x + 5)]$



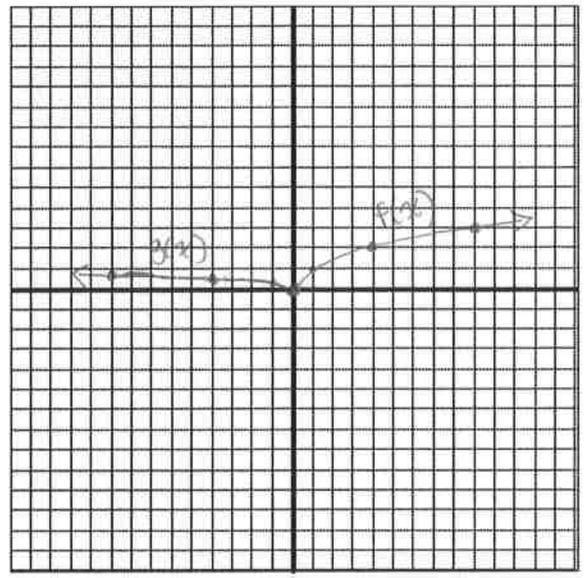
1) horizontal compression bafo $\frac{1}{3}$ $\left(\frac{x}{3}\right)$

2) shift left 5 units $(x-5)$

f(x)	$\frac{x}{3}-5$	y
(0,0)	-5	0
(1,1)	-4.7	1
(4,2)	-3.7	2
(9,3)	-2	3

$g(x) = \sqrt{3(x+5)}$

b) $g(x) = \frac{1}{4}f(-x)$



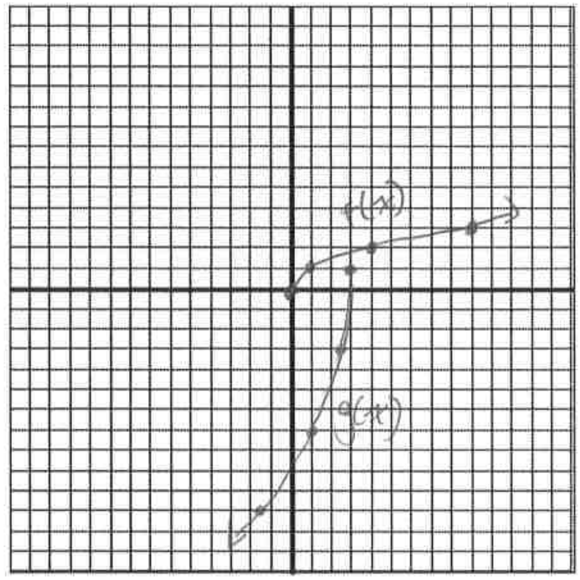
1) vertical compression bafo $\frac{1}{4}$ $\left(\frac{y}{4}\right)$

2) horizontal reflection $(-x)$

f(x)	$-x$	$\frac{y}{4}$
(0,0)	0	0
(1,1)	-1	0.25
(4,2)	-4	0.5
(9,3)	-9	0.75

$g(x) = \frac{1}{4}\sqrt{(-x)}$

c) $g(x) = -4f[-2(x-3)] + 1$



- vertical stretch bafo 4 ; vertical reflection $(-4y)$
- horizontal compression bafo $\frac{1}{2}$; horizontal reflection $\left(\frac{x}{-2}\right)$
- shift right 3 units $(x+3)$
- shift up 1 unit $(y+1)$

g(x)

f(x)	$\frac{x}{-2}+3$	$-4y+1$
(0,0)	3	1
(1,1)	2.5	-3
(4,2)	1	-7
(9,3)	-1.5	-11

$g(x) = -4\sqrt{-2(x-3)} + 1$

3) Use the description to write the transformed function, *g(x)*.

a) The parent function $f(x) = \sqrt{x}$ is compressed vertically by a factor of $\frac{1}{3}$ and then translated (shifted) 3 units left.

$a = \frac{1}{3}$

$d = -3$

$g(x) = \frac{1}{3}\sqrt{x+3}$

b) The parent function $f(x) = \sqrt{x}$ is reflected over the x-axis, stretch horizontally by a factor of 3 and then translated 1 unit left and 4 units down.

$a = -1$

$k = \frac{1}{3}$

$d = -1$

$c = -4$

$g(x) = -1\sqrt{\frac{1}{3}(x+1)} - 4$

Transformations of $\frac{1}{x}$ - Worksheet

SOLUTIONS

MCR3U
Jensen

Key points of
$$y = \frac{1}{x}$$

x	y
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$

1) State the transformations to the parent function $f(x) = \frac{1}{x}$ in the order that you would do them.

a) $g(x) = \frac{2}{3(x-1)}$

- vertical stretch bafo 2
- horizontal compression bafo $\frac{1}{3}$
- shift right 1 unit

b) $g(x) = \frac{-1}{x+2} - 1$

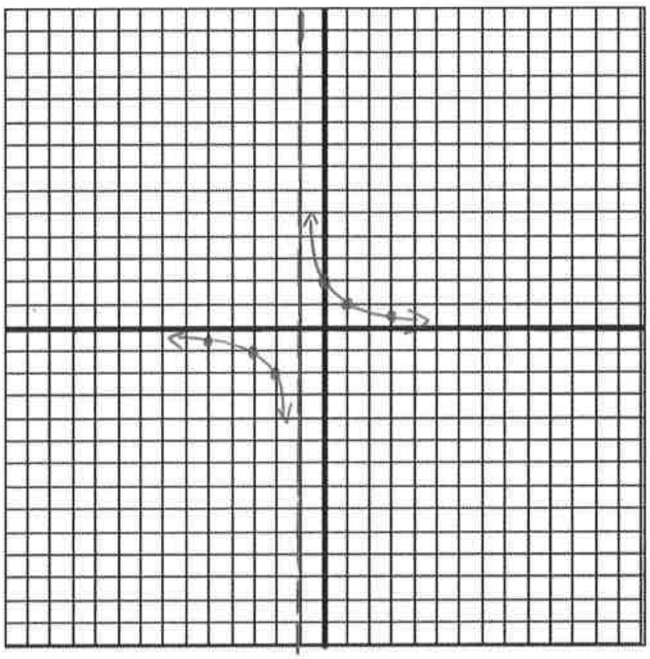
- vertical reflection
- shift left 2 units
- shift down 1 unit

c) $g(x) = \frac{1}{\frac{1}{2}(x+1)} - 0.5$

- horizontal stretch bafo 2
- shift left 1 unit
- shift down 0.5 units

2) Describe the transformations to the parent function $f(x) = \frac{1}{x}$ in order, make a table of values of image points, write the equation of the transformed function and graph it.

a) $g(x) = f[\frac{1}{2}(x+1)]$



- horizontal stretch bofo 2 $(2x)$
- shift left 1 unit $(x-1)$

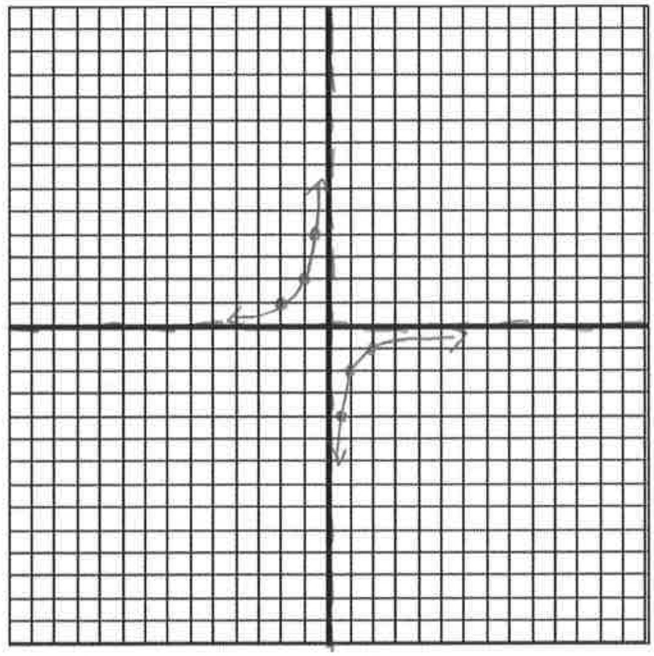
$f(x)$
$(-2,-0.5)$
$(-1,-1)$
$(-0.5,-2)$
$(0.5,2)$
$(1,1)$
$(2,0.5)$

$g(x)$

$2x-1$	y
-5	-0.5
-3	-1
-2	-2
0	2
1	1
3	0.5

$$g(x) = \frac{1}{\frac{1}{2}(x+1)}$$

b) $g(x) = 2f(-x)$



- vertical stretch bofo 2 $(2y)$
- horizontal reflection $(-x)$

$f(x)$
$(-2,-0.5)$
$(-1,-1)$
$(-0.5,-2)$
$(0.5,2)$
$(1,1)$
$(2,0.5)$

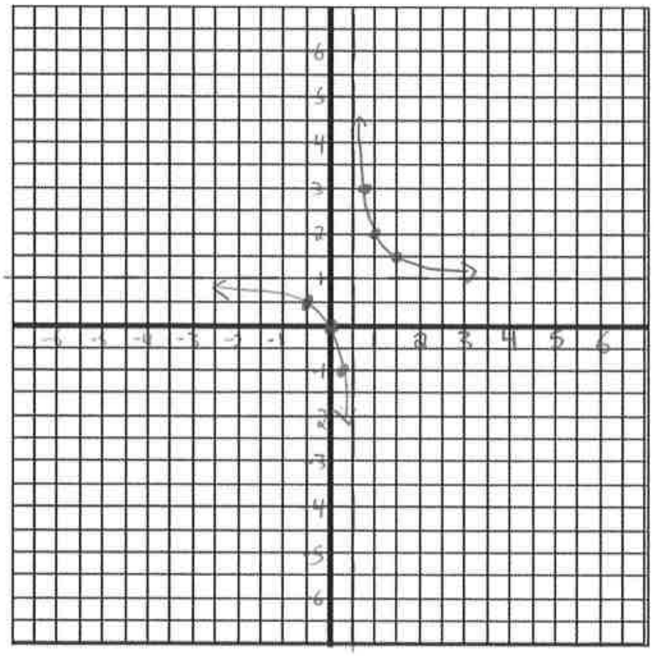
$g(x)$

$-x$	$2y$
2	-1
1	-2
0.5	-4
-0.5	4
-1	2
-2	1

$$g(x) = \frac{2}{-x}$$

c) $g(x) = -f[-2(x - 0.5)] + 1$

- vertical reflection $(-y)$
- horizontal reflection $(-x)$
- horizontal compression by a $\frac{1}{2}$ $\left(\frac{x}{2}\right)$
- shift right 0.5 units $(x+0.5)$
- shift up 1 unit $(y+1)$



f(x)
(-2, -0.5)
(-1, -1)
(-0.5, -2)
(0.5, 2)
(1, 1)
(2, 0.5)

$\frac{x}{-2}+0.5$	$-y+1$
1.5	1.5
1	2
0.75	3
0.25	-1
0	0
-0.5	0.5

3) Use the description to write the transformed function, *g(x)*.

a) The parent function, $f(x) = \frac{1}{x}$, is compressed vertically by a factor of $\frac{1}{3}$ and then translated (shifted) 3 units left.

$a = \frac{1}{3}$

$d = -3$

$g(x) = \frac{\frac{1}{3}}{x+3}$ OR $\frac{1}{3(x+3)}$

b) The parent function, $f(x) = \frac{1}{x}$, is reflected over the x-axis, stretch horizontally by a factor of 3 and then translated 1 unit left and 4 units down.

$a = -1$

$k = \frac{1}{3}$

$d = -1$

$c = -4$

$g(x) = \frac{-1}{\frac{1}{3}(x+1)} - 4$ OR $\frac{-3}{x+1} - 4$

2.6 - Inverse of a Function

Inverse of a function:

- The inverse of a function f is denoted as f^{-1}
- The function and its inverse have the property that if f(a) = b, then $f^{-1}(b) = a$
- So if f(5) = 13, then $f^{-1}(13) = 5$.

- More simply put: The inverse of a function has all the same points as the original function, except that the x's and y's have been reversed.

It is important to note that $f^{-1}(x)$ is read as "the inverse of f at x". The -1 does not behave like an exponent.

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

To draw an inverse, all you need to do is swap the x and y coordinates of each point.

Finding Inverses by Numerically

Example 1: The table shows ordered pairs belonging to a function $f(x)$. Determine $f^{-1}(x)$, then state the domain and range of $f(x)$ and its inverse.

$f(x)$	$f^{-1}(x)$
(-5, 0)	(0, -5)
(-4, 2)	(2, -4)
(-3, 5)	(5, -3)
(-2, 6)	(6, -2)
(0, 7)	(7, 0)

$f(x)$

D: $\{x \in \mathbb{R} \mid x = -5, -4, -3, -2, 0\}$

R: $\{y \in \mathbb{R} \mid y = 0, 2, 5, 6, 7\}$

$f^{-1}(x)$

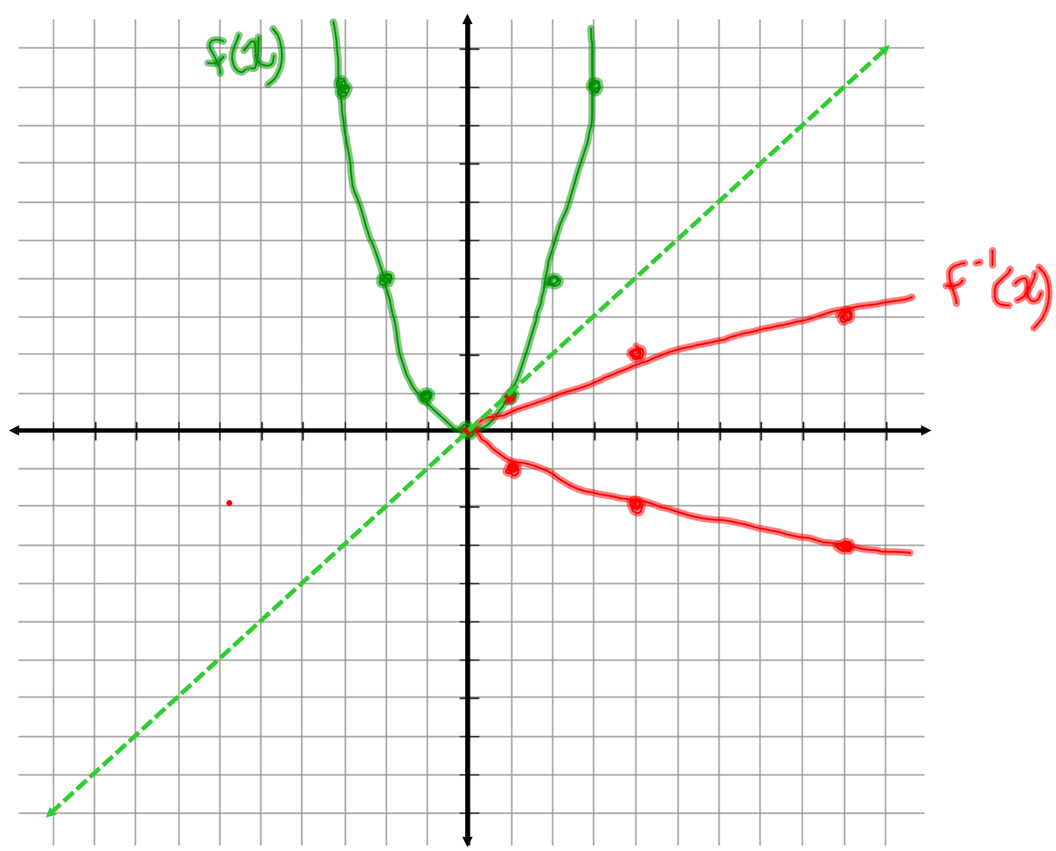
D: $\{x \in \mathbb{R} \mid x = 0, 2, 5, 6, 7\}$

R: $\{y \in \mathbb{R} \mid y = -5, -4, -3, -2, 0\}$

Example 2:

a) Graph the function $f(x) = x^2$ and its inverse $f^{-1}(x)$

$f(x)$	$f^{-1}(x)$
(-3,9)	(9,-3)
(-2,4)	(4,-2)
(-1,1)	(1,-1)
(0,0)	(0,0)
(1,1)	(1,1)
(2,4)	(4,2)
(3,9)	(9,3)



b) state the domain and range of both functions

$f(x)$

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$

$f^{-1}(x)$

D: $\{x \in \mathbb{R} \mid x \geq 0\}$

R: $\{y \in \mathbb{R}\}$

Note: *the domain and range of inverse functions are the reverse of each other.*

Example 3:

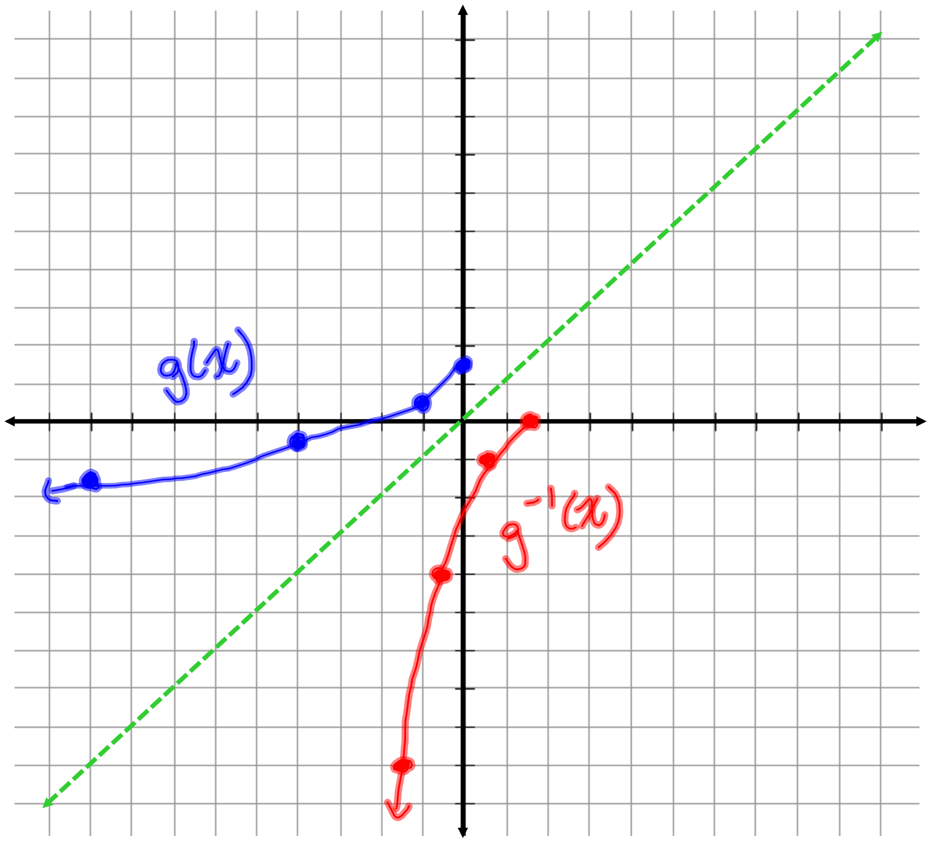
Sketch the graph of $g(x) = -2\sqrt{(-\frac{1}{2}x)} + 3$ then graph $g^{-1}(x)$.

$f(x) = \sqrt{x}$ $\longrightarrow$ $g(x) = -2\sqrt{(-\frac{1}{2}x)} + 3$ $\rightarrow$ $g^{-1}(x)$

x	*y*
0	0
1	1
4	2
9	3

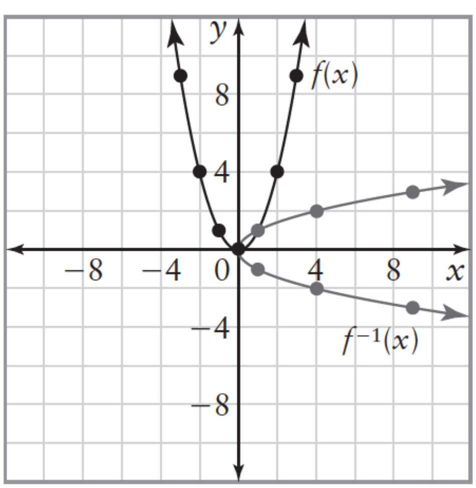
$-2x$	$-2y+3$
0	3
-2	1
-8	-1
-18	-3

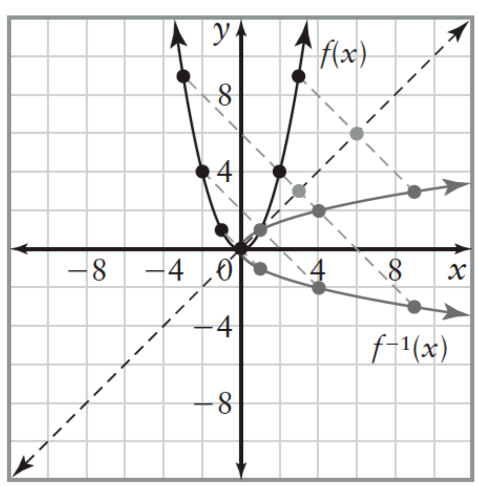
x	*y*
3	0
1	-2
-1	-8
-3	-18



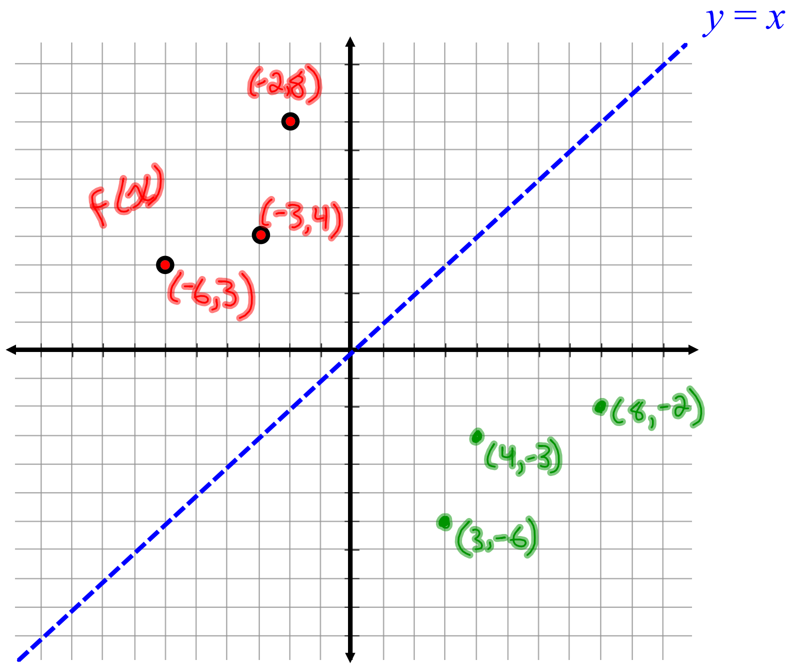
Finding Inverses by Graphing

The graph of $f^{-1}(x)$ is the graph of $f(x)$ reflected in the line $y = x$. This is true for all functions and their inverses. If you find the midpoint of each pair of points from example 2 and connect them you can prove this theorem.





Example 4: Sketch the inverse of the $f(x)$



<u>Finding Inverses Algebraically</u>

Algebraic Method for finding the inverse:

1. Replace $f(x)$ with "y"
2. Switch the x and y variables
3. Isolate for y
4. replace y with $f^{-1}(x)$

Example 5: Find the inverse of the following functions…

a) $g(x) = \dfrac{(3x)}{4}$

$y = \dfrac{3x}{4}$

$x = \dfrac{3y}{4}$

$4x = 3y$

$\dfrac{4x}{3} = y$

$g^{-1}(x) = \dfrac{4x}{3}$

b) $h(x) = 4x + 3$

$y = 4x+3$

$x = 4y+3$

$x-3 = 4y$

$\dfrac{x-3}{4} = y$

$h^{-1}(x) = \dfrac{x-3}{4}$

c) $f(x) = x^2 - 1$

$y = x^2 - 1$

$x = y^2 - 1$

$x+1 = y^2$

$\pm\sqrt{x+1} = y$

$f^{-1}(x) = \pm\sqrt{x+1}$

d) $h(x) = \dfrac{4x+3}{5}$

$y = \dfrac{4x+3}{5}$

$x = \dfrac{4y+3}{5}$

$5x = 4y+3$

$\dfrac{5x-3}{4} = y$

$h^{-1}(x) = \dfrac{5x-3}{4}$

e) $f(x) = 2x^2 + 16x + 29$

> **Note:** for algebraic inverses of quadratic functions, before interchanging x and y's you must re-write in vertex form.

$y = (2x^2 + 16x) + 29$

$y = 2(x^2 + 8x + 16 - 16) + 29$

$y = 2(x^2 + 8x + 16) - 32 + 29$

$y = 2(x+4)^2 - 3$

$x = 2(y+4)^2 - 3$

$\frac{x+3}{2} = (y+4)^2$

$\pm\sqrt{\frac{x+3}{2}} = y+4$

$-4 \pm \sqrt{\frac{x+3}{2}} = y$

$f^{-1}(x) = -4 \pm \sqrt{\frac{x+3}{2}}$

f) $r(x) = \sqrt{(x)} + 2$

$y = \sqrt{x} + 2$

$x = \sqrt{y} + 2$

$x - 2 = \sqrt{y}$

$(x-2)^2 = y$

$r^{-1}(x) = (x-2)^2$

Chapter 2b – Transformations of Functions – REVIEW
MCR3U
Jensen

SOLUTIONS

1) List the transformations of, in words, of $f(x)$ for each of the following functions.

a) $g(x) = 2f(x)$

- vertical stretch bafo 2

b) $h(x) = f(x-3)$

- phase shift right 3 units

c) $j(x) = f\left(\frac{1}{3}x\right)$

- horizontal stretch bafo 3

d) $k(x) = f(-x)$

- horizontal reflection

e) $m(x) = f(x) - 3$

- shift down 3 units.

2) List the transformations, in words, of $f(x)$ for each of the following functions in the order you would do them in.

a) $g(x) = -f(x+2)$

- vertical reflection
- phase shift left 2 units

b) $h(x) = f(3x) + 2$

- horizontal compression bafo $\frac{1}{3}$
- shift up 2 units

c) $j(x) = 3f(-x)$

- vertical stretch bafo 3
- horizontal reflection

3) List the transformations, in words, of $f(x) = x^2$ to $g(x) = 3(x-2)^2 - 11$ in the order you would do them.

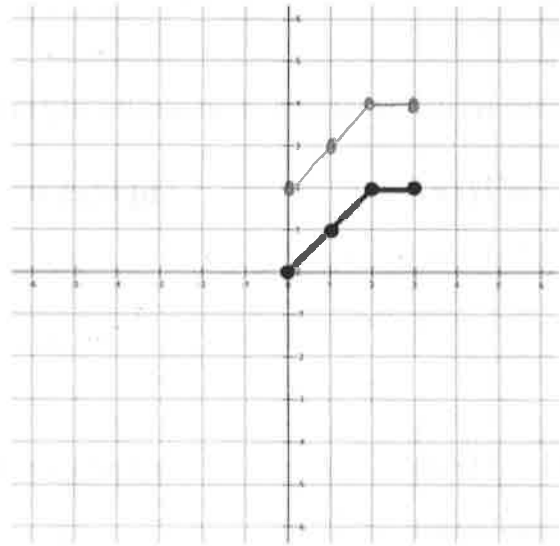
- vertical stretch bafo 3
- phase shift right 2 units
- shift down 11 units

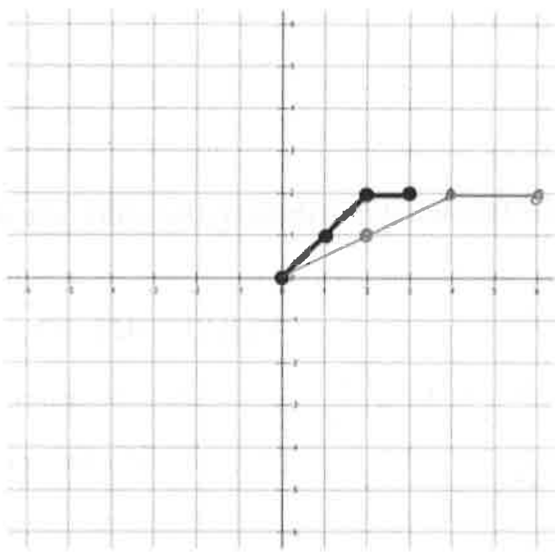
4) List the transformations, in words of $f(x) = \sqrt{x}$ to $g(x) = 2\sqrt{(x-3)} - 9$ in the order you would do them.

- vertical stretch BAFO 2
- phase shift right 3 units
- shift down 9 units

5) Perform the following transformations on the graphs below.

a) translate up 2 units

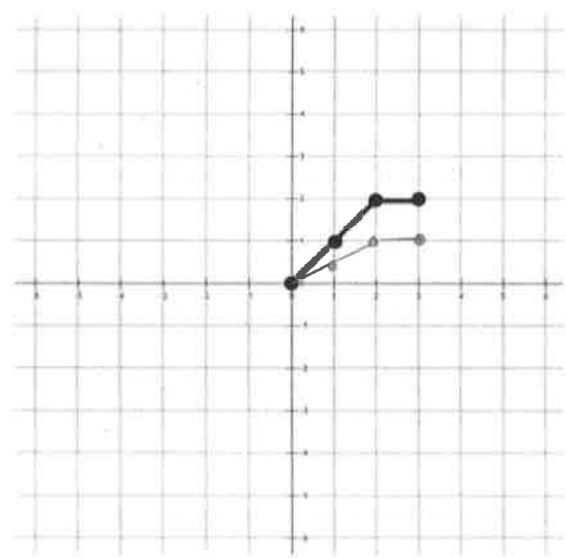




b) horizontal stretch by a factor of 2

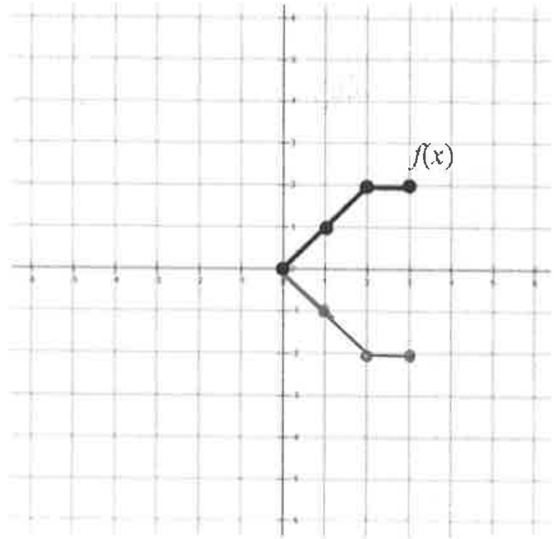
compression

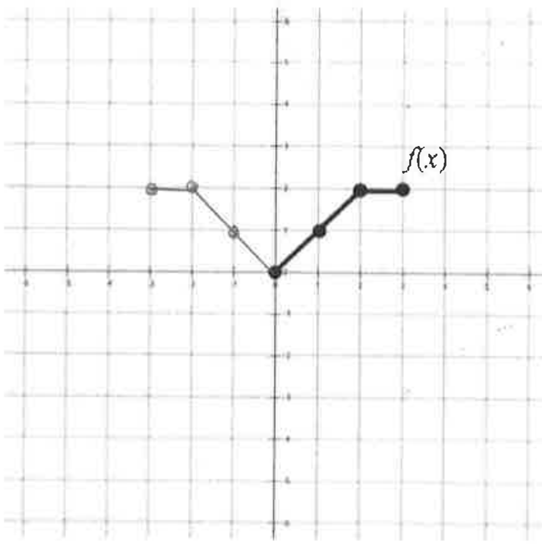
c) vertical ~~stretch~~ by a factor of $\frac{1}{2}$



6) For the graph of $f(x)$ given, sketch the graph of $g(x)$ after the given transformation. List the transformations in words as well.

a) $g(x) = -f(x)$





b) $g(x) = f(-x)$

7) For each function $g(x)$:

i) describe the transformations from the parent function $f(x)$
ii) create a table of values of image points for the transformed function
iii) graph the parent function and the transformed function and write its equation

a) $f(x) = x^2$. Graph $g(x) = 2f(x-2)$.

i) vertical stretch BAFO 2 (2y)
shift right 2 units (x+2)

ii)

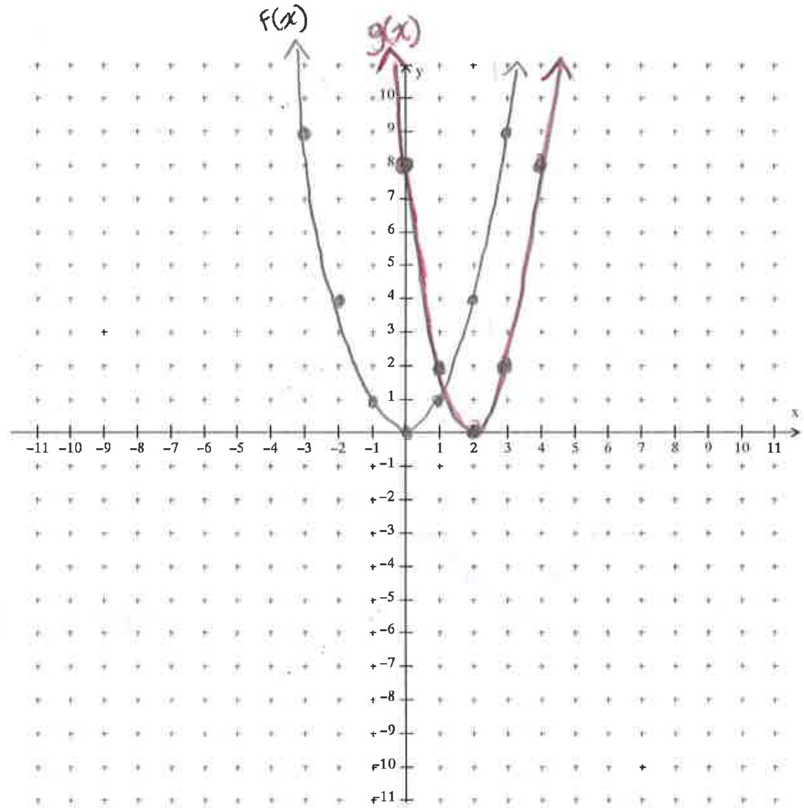
f(x): (-3,9), (-2,4), (-1,1), (0,0), (1,1), (2,4), (3,9)

g(x)

x+2	2y
-1	18
0	8
1	2
2	0
3	2
4	8
5	18

$f(x) = x^2$

$g(x) = 2(x-2)^2$



b) $f(x) = \sqrt{x}$. Graph $g(x) = -f(2x)$.

i) vertical reflection (-y)
horizontal compression bafo $\frac{1}{2}$ $(\frac{x}{2})$

ii)

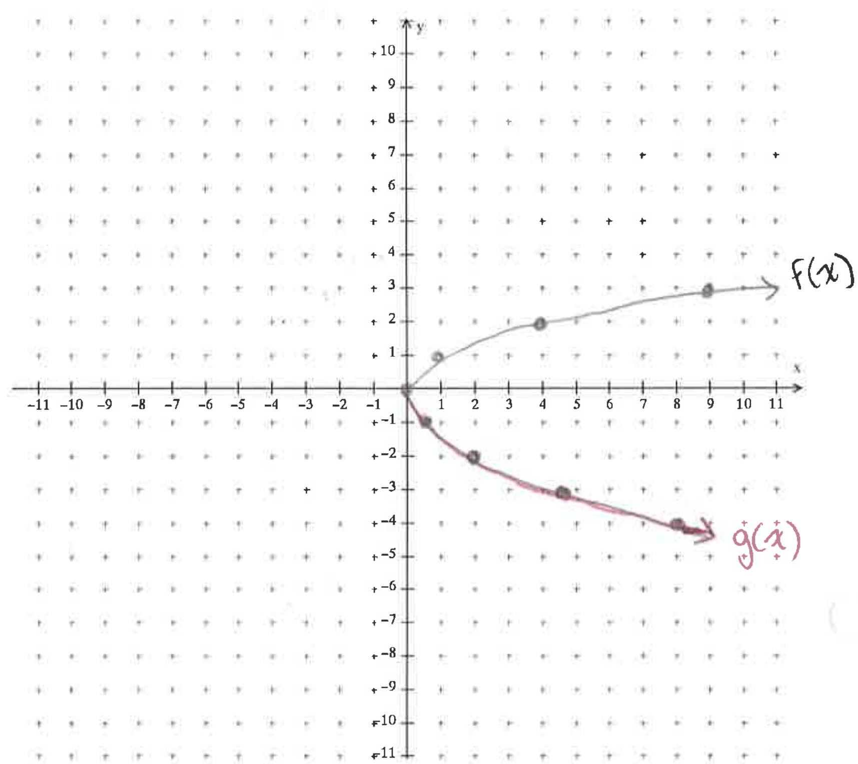
F(x): (0,0), (1,1), (4,2), (9,3)

g(x)

$\frac{x}{2}$	-y
0	0
0.5	-1
2	-2
4.5	-3

$f(x) = \sqrt{x}$

$g(x) = -\sqrt{2x}$



c) $f(x) = x^2$. Graph $g(x) = 4f(x-3)$

i) vertical stretch BAFO 4 $(4y)$
shift right 3 units $(x+3)$

ii)

f(x)

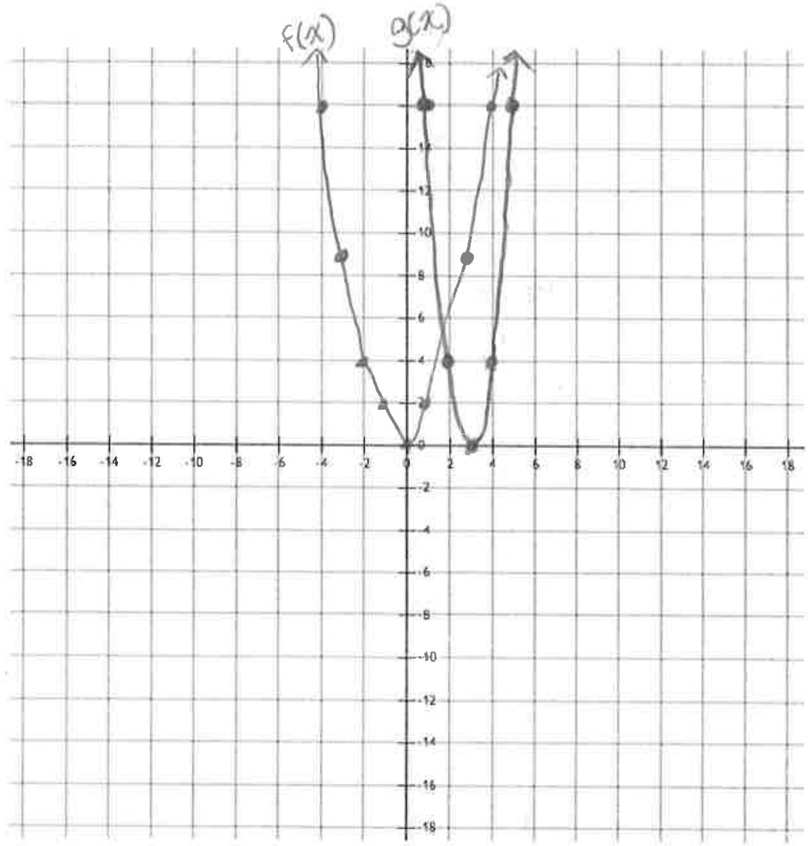
(-3,9)
(-2,4)
(-1,1)
(0,0)
(1,1)
(2,4)
(3,9)

g(x)

$x+3$	$4y$
0	36
1	16
2	4
3	0
4	4
5	16
6	36

$f(x) = x^2$

$g(x) = 4(x-3)^2$



d) $f(x) = \sqrt{x}$. Graph $g(x) = 3f(-x) - 2$.

i) vertical stretch BAFO 3 $(3y)$
horizontal reflection $(-x)$
shift down 2 units $(y-2)$

ii)

f(x)

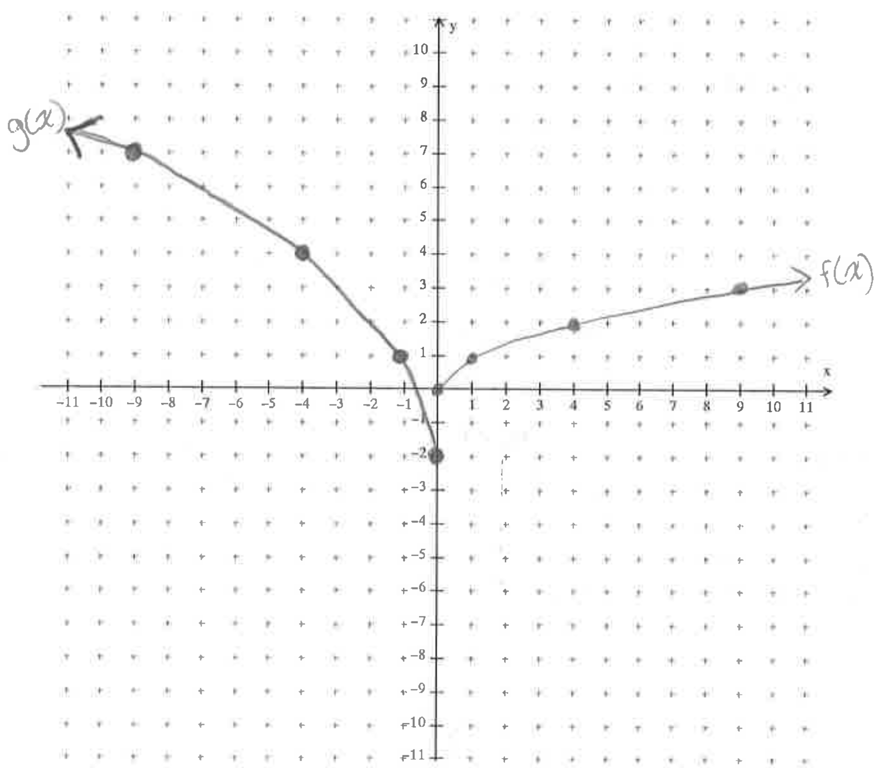
(0,0)
(1,1)
(4,2)
(9,3)

g(x)

$-x$	$3y-2$
0	-2
-1	1
-4	4
-9	7

$f(x) = \sqrt{x}$

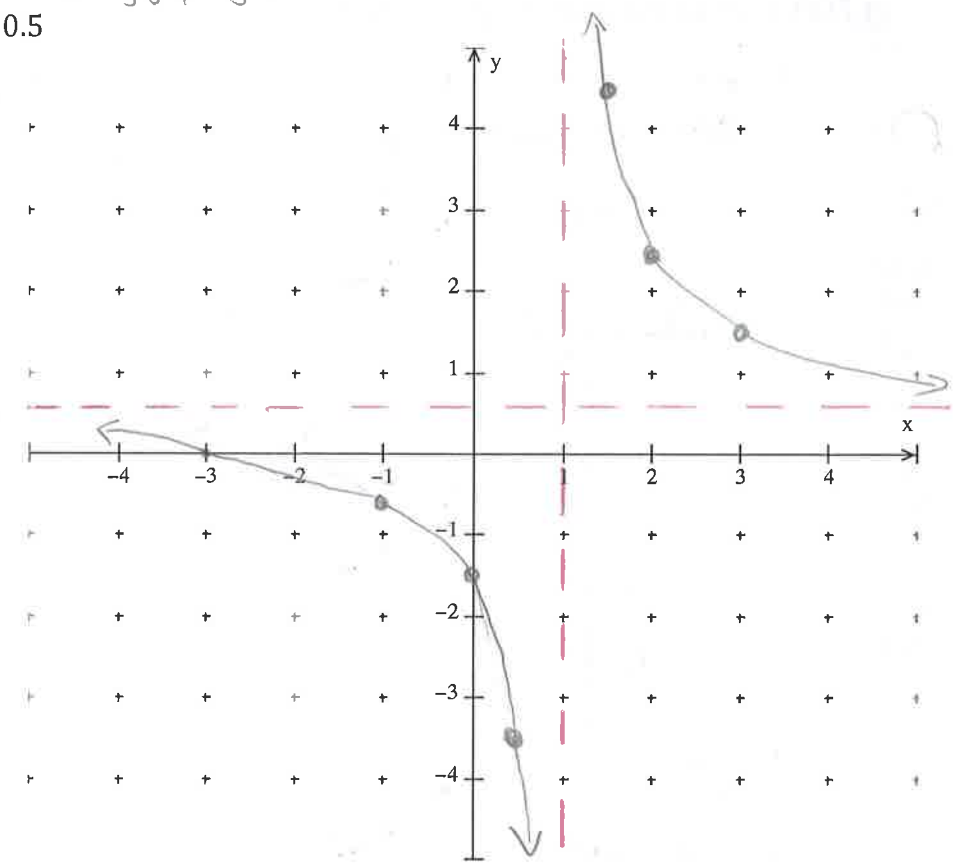
$g(x) = 3\sqrt{-x} - 2$



only graph g(x)

e) $f(x) = \frac{1}{x}$. Graph $g(x) = 2f(x - 1) + 0.5$

i) vertical stretch bafo 2 (2y)
shift 1 unit right (x+1)
shift up 0.5 units (y+0.5)



f(x)
(-2, -0.5)
(-1, -1)
(-0.5, -2)
(0.5, 2)
(1, 1)
(2, 0.5)

g(x)

x+1	2y+0.5
-1	-0.5
0	-1.5
0.5	-3.5
1.5	4.5
2	2.5
3	1.5

$g(x) = \frac{2}{x-1} + 0.5$

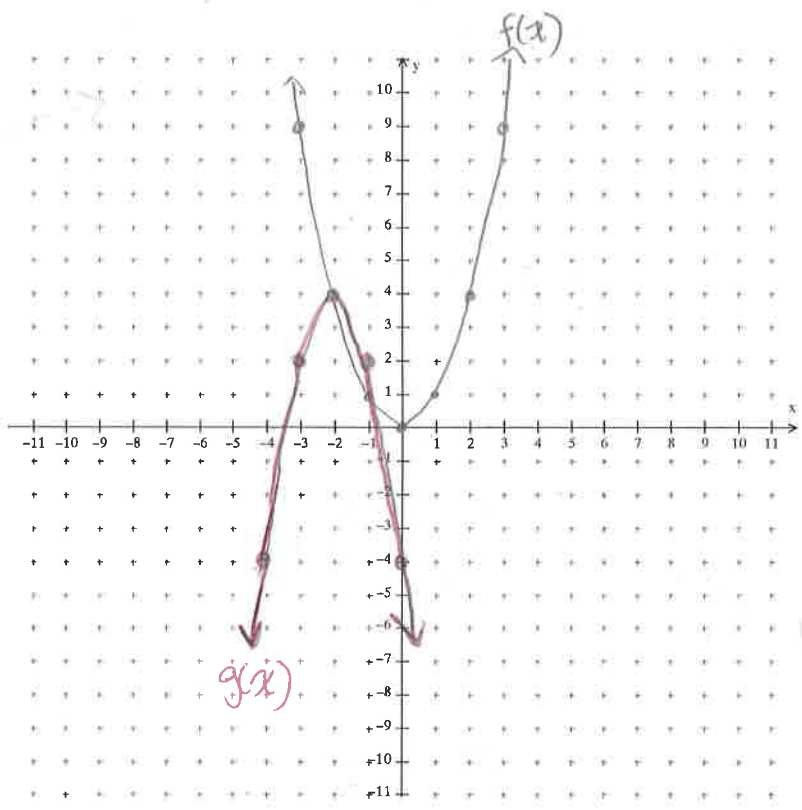
8) For each function $g(x)$:

i) determine the parent function and describe the transformations from the parent function $f(x)$
ii) create a table of values of image points for the transformed function
iii) graph the parent function and the transformed function

a) $g(x) = -2(x + 2)^2 + 4$

$f(x) = x^2$

Transformations: vertical stretch BAFO 2 (2y)
vertical reflection (-y)
left 2 units (x-2)
up 4 units (y+4)



F(x)
(-3, 9)
(-2, 4)
(-1, 1)
(0, 0)
(1, 1)
(2, 4)
(3, 9)

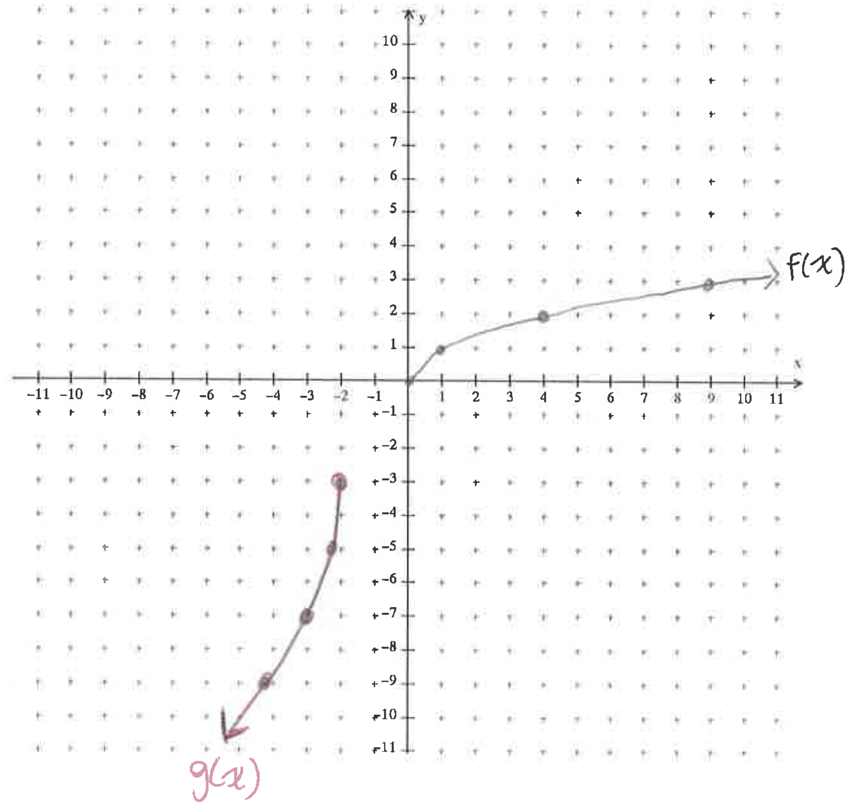
ii) g(x)

x-2	-2y+4
-5	-14
-4	-4
-3	2
-2	4
-1	2
0	-4
1	-14

$f(x) = \sqrt{x}$

b) $g(x) = -2\sqrt{-4(x+2)} - 3$

i) vertical stretch bafo 2 (2y)
vertical reflection (-y)
horizontal compression bafo $\frac{1}{4}$ ($\frac{x}{4}$)
horizontal reflection (-x)
shift left 2 (x-2)
down 3 (y-3)



f(x)

(0,0)
(1,1)
(4,2)
(9,3)

g(x)

$\frac{x}{-4} - 2$	$-2y-3$
-2	-3
-2.25	-5
-3	-7
-4.25	-9

$f(x) = \frac{1}{x}$

c) $g(x) = \frac{-1}{\frac{1}{2}(x+0.5)} + 1$ only graph g(x)

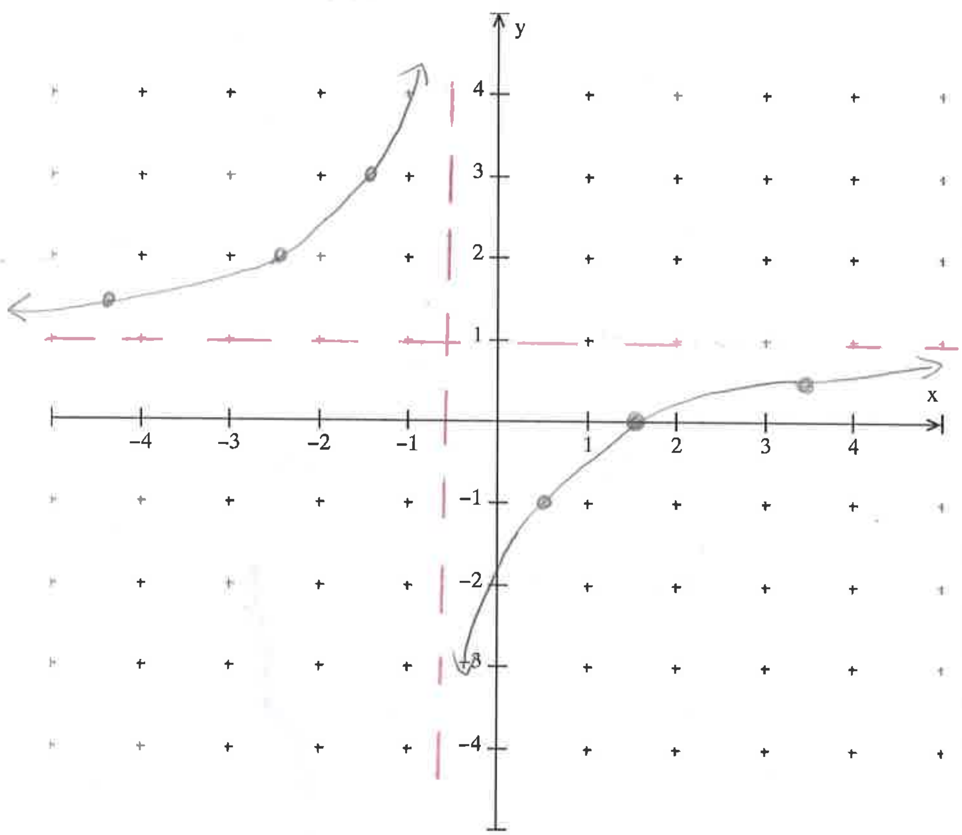
i) vertical reflection (-y)
horizontal stretch bafo 2 (2x)
left 0.5 units (x-0.5)
up 1 unit (y+1)

f(x)

(-2,-0.5)
(-1,-1)
(-0.5,-2)
(0.5,2)
(1, 1)
(2, 0.5)

g(x)

$2x-0.5$	$-y+1$
-4.5	1.5
-2.5	2
-1.5	3
0.5	-1
1.5	0
3.5	0.5



9) Find the inverse, $f^{-1}(x)$, algebraically if $f(x) = -2\sqrt{x+1} - 5$

$x = -2\sqrt{y+1} - 5$

$\frac{x+5}{-2} = \sqrt{y+1}$

$\left(\frac{x+5}{-2}\right)^2 = y+1$

$\left(\frac{x+5}{-2}\right)^2 - 1 = y$

$f^{-1}(x) = \left(\frac{x+5}{-2}\right)^2 - 1$

10) Find the inverse, $f^{-1}(x)$, algebraically if $f(x) = \frac{1}{3}(x-4)^2 + 2$

$x = \frac{1}{3}(y-4)^2 + 2$

$x - 2 = \frac{1}{3}(y-4)^2$

$3x - 6 = (y-4)^2$

$\pm\sqrt{3x-6} = y - 4$

$\pm\sqrt{3x-6} + 4 = y$

$f^{-1}(x) = \pm\sqrt{3x-6} + 4$

OR

$x = \frac{1}{3}(y-4)^2 + 2$

$3(x-2) = (y-4)^2$

$\pm\sqrt{3(x-2)} = y - 4$

$\pm\sqrt{3(x-2)} + 4 = y$

$f^{-1}(x) = \pm\sqrt{3(x-2)} + 4$

Answers

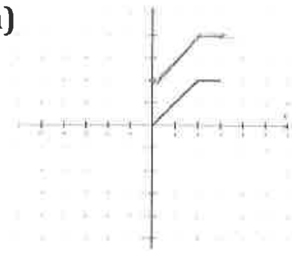
1) a) vertical stretch BAFO 2 **b)** phase shift right 3 units **c)** horizontal stretch BAFO 3
d) horizontal reflection in the y-axis **e)** shift down 3 units

2) a) vertical reflection in the x-axis and then shift left 3 units.
b) horizontal compression BAFO $\frac{1}{3}$ and then shift up 2 units.
c) vertical stretch BAFO 3 and then horizontal reflection in the y-axis.

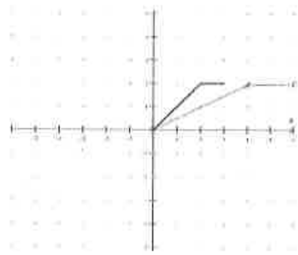
3) vertical stretch BAFO 3, then shift right 2 units and down 11 units.

4) vertical stretch by a factor of 2, then shift right 2 units and down 9 units.

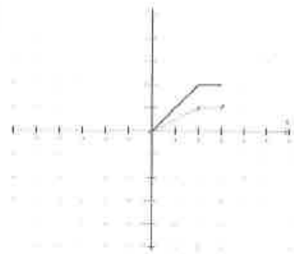
5) a)



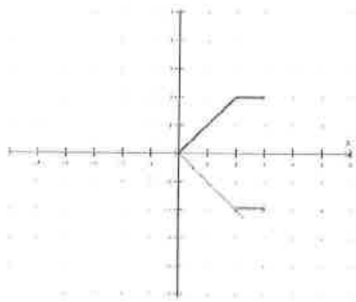
b)



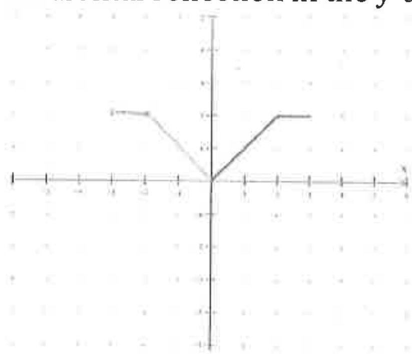
c)



6) a) vertical reflection in the x-axis



b) horizontal reflection in the y-axis



See posted solutions for 7&8

9) $f^{-1}(x) = \left(\frac{x+5}{-2}\right)^2 - 1$

10) $f^{-1}(x) = \pm\sqrt{3(x-2)} + 4$