

# Chapter 3 - Exponential Functions - Review

MCR3U

ensen

SOLUTIONS

## Section 1: Exponential Growth and Decay

1) The population of a pod of Southern right whales has doubled in size recently over a 9 year period. They have a current population of 38 individuals. What might the population be after 25 more years?

$$y = 38(2)^{25/9}$$

$$y = 260.6$$

The population will be approximately 261 whales

2) A bacteria culture doubles every 15 minutes. There were 20 individuals initially.

a) How many bacteria will be present after 2 hours?

$$y = 20(2)^{120/15}$$

$$y = 20(2)^8$$

$$y = 5120$$

There will be 5120 bacteria.

b) How long will it take to grow a population of 163 840?

$$163\,840 = 20(2)^x$$

$$\frac{163\,840}{20} = 2^x$$

$$8192 = 2^x$$

$$\log 8192 = \log 2^x$$

$$\log 8192 = x \cdot \log 2$$

$$x = \frac{\log 8192}{\log 2}$$

$$x = 13$$

$$x = \frac{t}{15}$$

$$13 = \frac{t}{15}$$

$$13(15) = t$$

$$t = 195$$

It will take 195 minutes.

$$r = 0.05$$

3) A weight loss program aims for its clients to lost 5% of their weight each week. If Helene currently weighs 280 pounds, what can she expect to weigh at the end of the 12-week program?

a

x

$$y = 280(1 - 0.05)^{12}$$

$$y = 280(0.95)^{12}$$

$$y = 151.3$$

Helene can expect to weigh 151.3 lbs.

$$r = 0.08$$

4) The population of a developing city increases by 8% per year. In 2005, 125 000 people lived in the city.

a

a) What is the population expected in 2020?

x = 15

$$y = 125000(1.08)^{15}$$

$$y = 396521.1$$

The population is expected to be approximately 396 521

b) In what year should the population reach half a million?

$$500000 = 125000(1.08)^x$$

$$4 = 1.08^x$$

$$\log 4 = \log 1.08^x$$

$$\log 4 = x \cdot \log 1.08$$

$$x = \frac{\log 4}{\log 1.08}$$

$$x = 18.0$$

$$\text{Year } 2005 + 18 = 2023$$

The population will reach a half million by 2023.

5) The half-life of Carbon-14 is approximately 6000 years. A fossil of Carbon-14 weighed 100 grams. How much would it weigh after 18 000 years?

b = 1/2

$$x = \frac{t}{6000}$$

a

$$y = 100\left(\frac{1}{2}\right)^{18000/6000}$$

$$y = 100\left(\frac{1}{2}\right)^3$$

$$y = 12.5$$

It will weigh 12.5 grams

6) A radioactive isotope decays rapidly with a half-life of 4 minutes. For a  $\overset{t}{12}$  minute test, technicians need at least 6 grams of the isotope to remain at the end of the test. How much must they start with?

$$y = a(b)^x$$

$$6 = a\left(\frac{1}{2}\right)^{12/4}$$

$$6 = a\left(\frac{1}{2}\right)^3$$

$$\frac{6}{\left(\frac{1}{2}\right)^3} = a$$

$$a = 48$$

They must start with 48 grams.

7) A nuclear power plant has a stockpile of  $\overset{a}{100}$  tonnes of radioactive waste. The radioactive waste has a half-life of 15 years. How long will it take for there to be only 1 tonne remaining?

$$b = \frac{1}{2} \quad x = \frac{t}{15}$$

$$y = a(b)^x$$

$$1 = 100\left(\frac{1}{2}\right)^x$$

$$0.01 = \left(\frac{1}{2}\right)^x$$

$$\log 0.01 = \log\left(\frac{1}{2}\right)^x$$

$$\log 0.01 = x \log\left(\frac{1}{2}\right)$$

$$x = \frac{\log 0.01}{\log 0.5}$$

$$x = 6.64$$

$$x = \frac{t}{15}$$

$$6.64 = \frac{t}{15}$$

$$t = 99.6 \text{ years}$$

It will take nearly 100 years for there to be only 1 tonne remaining.

8) If Canada were to stop accepting immigrants the population would begin to decrease by 0.5% per year. Canada's population is currently 34,482,779. If it stopped accepting new immigrants today, what would the population be in 50 years?

$$y = 34\,482\,779 (1 - 0.005)^{50}$$

$$y = 34\,482\,779 (0.995)^{50}$$

$$y = 26\,838\,379.9$$

In 50 years the population will be approximately 26,838,380 people

$$b = \frac{1}{2}$$

$$x = \frac{m}{11}$$

9) The amount of sunlight a diver can see is halved for every 11 meters she dives. What percentage of light remains when the diver is 60 meters below the surface? (Hint: at the surface is 100% of the light)

$$y = 100 \left(\frac{1}{2}\right)^{60/11}$$

$$y = 2.28$$

2.28% of light remains 60m below the surface.

$$b = \frac{1}{2}$$

$$x = \frac{t}{180}$$

10) Lead-210 decays very slowly with a half-life of 180 years. After 1000 years, how much of a 40 kg sample remains?

$$y = 40 \left(\frac{1}{2}\right)^{1000/180}$$

$$y = 0.85$$

0.85 kg remains after 1000 years.

## Section 2: Interest

$$i = 0.06$$

$$P$$

11) A pension increases every year by 6%. If Opa has \$24 300 in his pension right now, how much will his pension be worth in 10 years?

$$n$$

$$A = P(1+i)^n$$

$$A = 24300(1.06)^{10}$$

$$A = \$43 517.60$$

His pension will be worth \$43 517.60

12) An investor invests  $\overset{P}{\$1000}$  into a mutual fund for  $\overset{n}{4}$  years at a growth rate of  $\overset{i=0.025}{2.5\%}$  per year. How much is the investment worth after 4 years?

$$A = 1000(1.025)^4$$

$$A = 1103.81$$

The investment will be worth \$1103.81

13) You have found an investment at the bank that pays  $\overset{i=0.024}{2.4\%}$  per year for  $\overset{n}{10}$  years. How much should you invest now so that you have  $\overset{A}{\$2000}$  after the 10 years?

$$2000 = P(1.024)^{10}$$

$$P = \frac{2000}{(1.024)^{10}}$$

$$P = \$1577.72$$

You should invest \$1577.72

14) Jeremiah's grandparents placed  $\overset{P}{\$3000}$  into an account for him when he was born. The investment is to be paid out when he turns 25. The account pays an annual interest rate of 8%, compounded semi-annually. How much will be in the account on Jeremiah's 25th birthday?  $i=0.08$

$$A = ?$$

$$P = 3000$$

$$i = 0.08 \div 2 = 0.04$$

$$n = 25 \times 2 = 50$$

$$A = 3000(1.04)^{50}$$

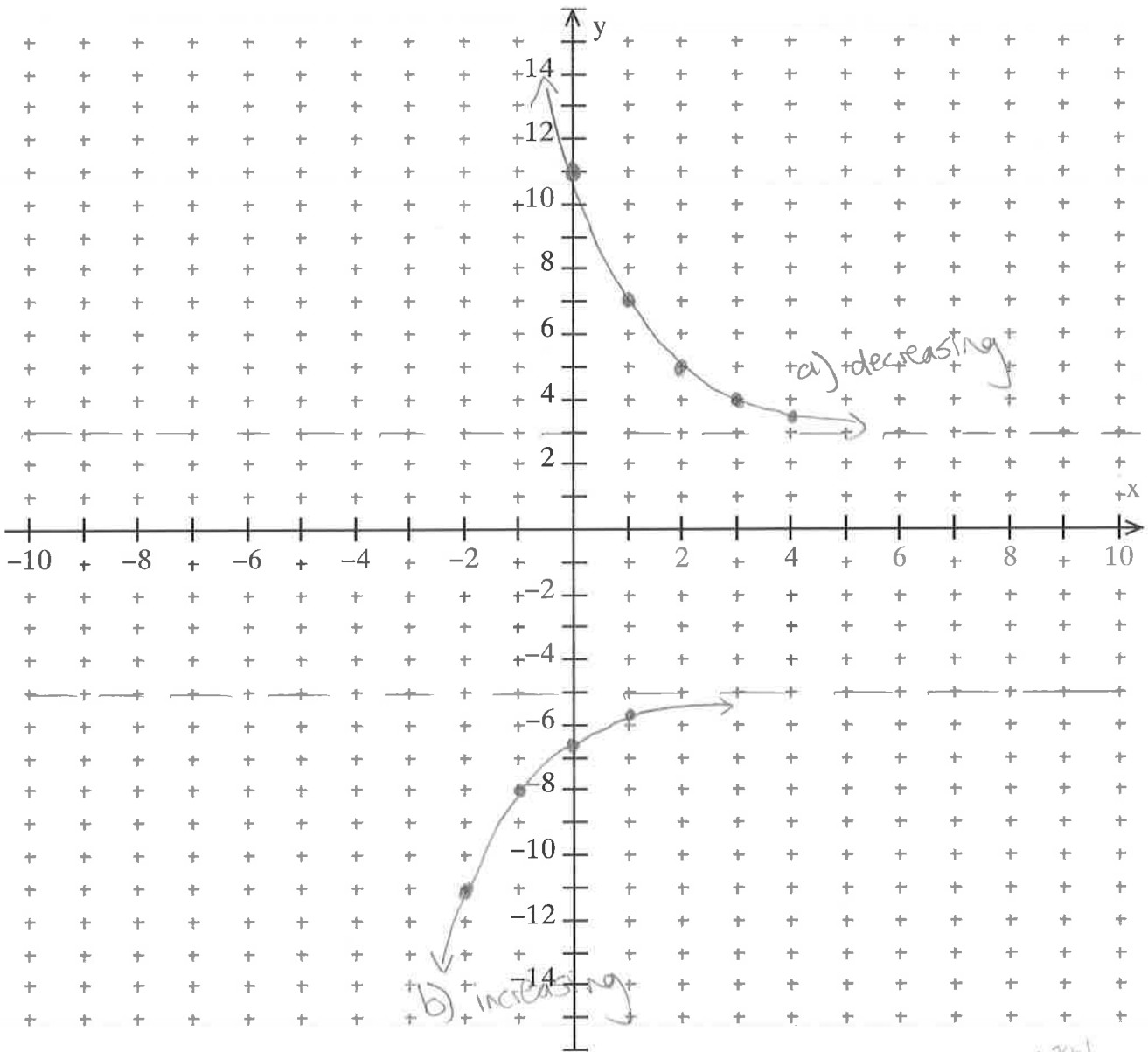
$$A = 21320.05$$

It will be worth \$21320.05

15) Graph of both of these functions on the graph below using transformations of the parent functions. State whether each function is increasing or decreasing:

a)  $y = 2(2)^{-(x-2)} + 3$

b)  $y = -3\left(\frac{1}{2}\right)^{x+1} - 5$



$y = 2^x$

$y = 2(2)^{-(x-2)} + 3$

$y = \left(\frac{1}{2}\right)^x$

$y = -3\left(\frac{1}{2}\right)^{x+1} - 5$

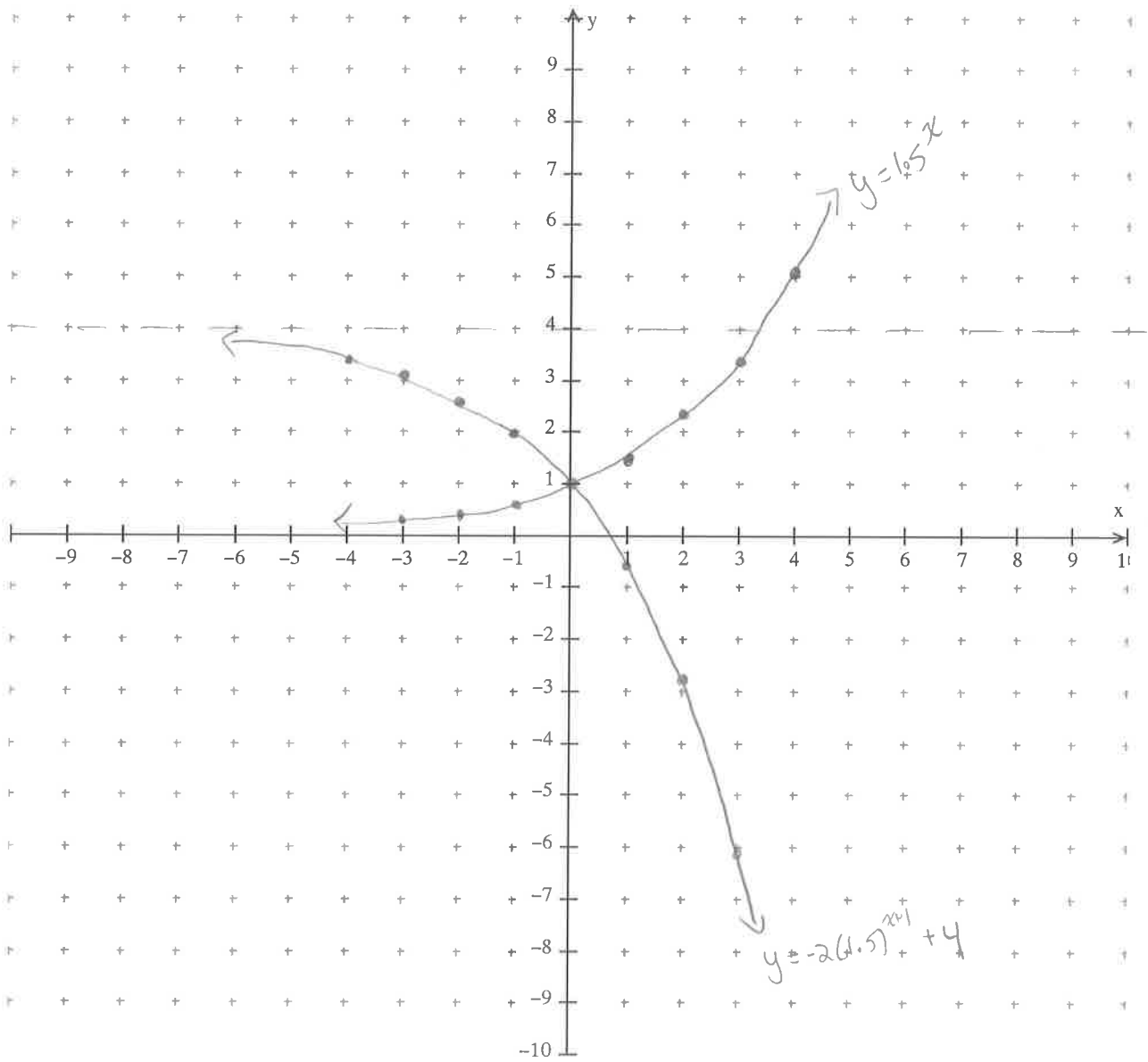
x	y
-2	0.25
-1	0.5
0	1
1	2
2	4

-x+2	2y+3
4	3.5
3	4
2	5
1	7
0	11

x	y
-2	4
-1	2
0	1
1	0.5
2	0.25

x-1	-3y-5
-3	-17
-2	-11
-1	-8
0	-6.5
1	-5.75

16) Graph the parent function  $y = 1.5^x$  and the transformed function  $y = -2(1.5)^{x+1} + 4$  onto the graph below:



$$y = 1.5^x$$

$$y = -2(1.5)^{x+1} + 4$$

x	y
-3	0.3
-2	0.44
-1	0.67
0	1
1	1.5
2	2.25
3	3.375

x-1	-2y+4
-4	3.4
-3	3.12
-2	2.66
-1	2
0	1
1	-0.5
2	-2.75

17) Match the graphs that are the same:

a)  $f(x) = 4^{2x}$

b)  $g(x) = 25^x$

c)  $p(x) = 2^{3x}$

d)  $h(x) = 32^x$

e)  $z(x) = 10^x$

i)  $r(x) = 5^{2x}$

ii)  $u(x) = 8^x$

iii)  $b(x) = 2^{5x}$

iv)  $a(x) = 16^x$

v) no match

B

C

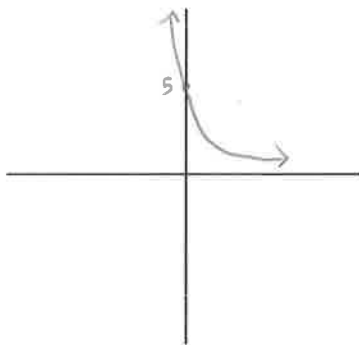
D

A

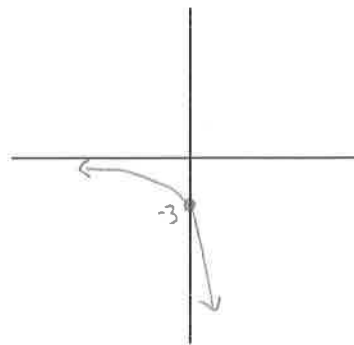
E

18) Sketch the graph using your knowledge of increasing and decreasing exponential functions.

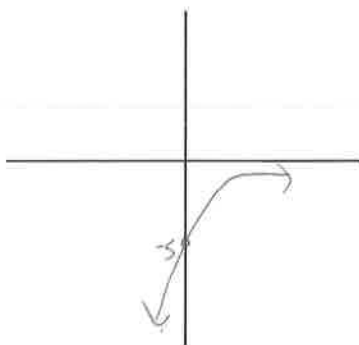
a)  $f(x) = 5\left(\frac{1}{2}\right)^x$



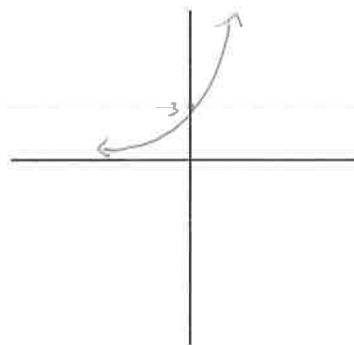
c)  $g(x) = -3(4)^x$



b)  $h(x) = -5\left(\frac{1}{2}\right)^x$

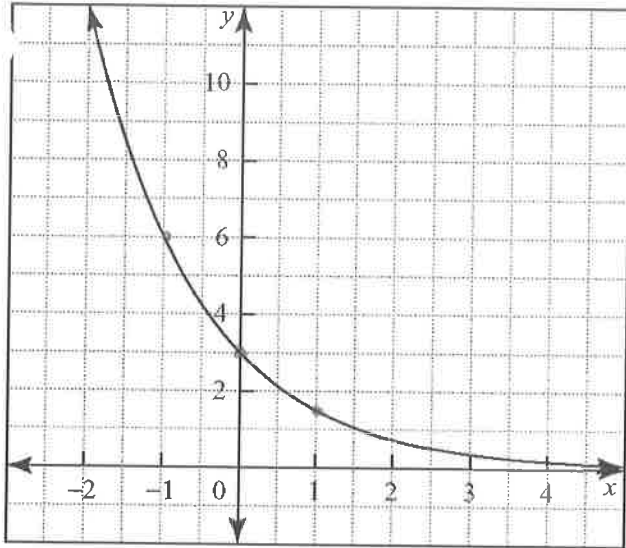


d)  $b(x) = 3(4)^x$





19) Write a function for the exponential graph shown



$$a = 3$$
$$b = \frac{1}{2}$$

$$y = 3\left(\frac{1}{2}\right)^x$$

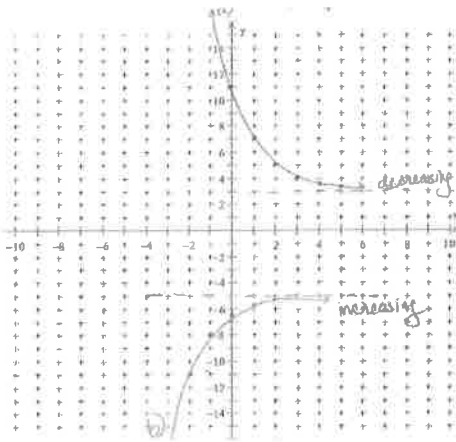
x	y
-1	6
0	3
1	1.5

$\left. \begin{array}{l} 6 \\ 3 \\ 1.5 \end{array} \right\} \times \frac{1}{2}$

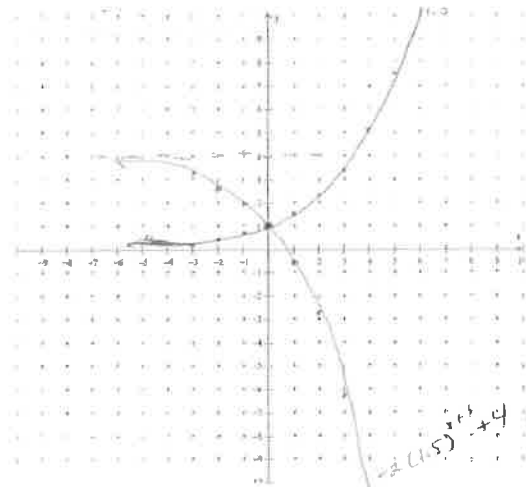
## Answers

- 1) 260 whales
- 2) a) 5120 bacteria b) 195 minutes
- 3) 151.3 lbs
- 4) a) 396 521 b) 2023
- 5) 12.5 grams
- 6) 48 grams
- 7) Almost 100 years
- 8) 26 838 379
- 9) 2.28%
- 10) 0.85 kg
- 11) \$43 517.60
- 12) \$1103.81
- 13) \$1577.72
- 14) \$21 320.05

15)

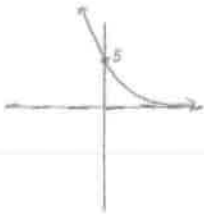


16)

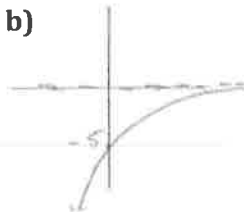


17) a) iv b) i c) ii d) iii e) v

18) a)



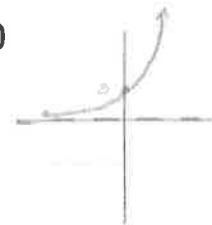
b)



c)



d)



19)  $y = 3 \left(\frac{1}{2}\right)^x$