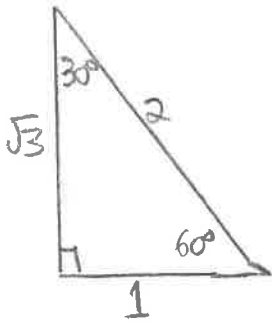
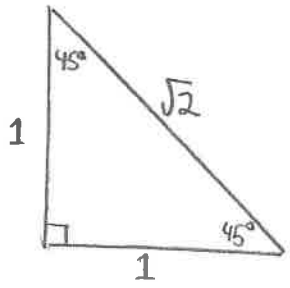
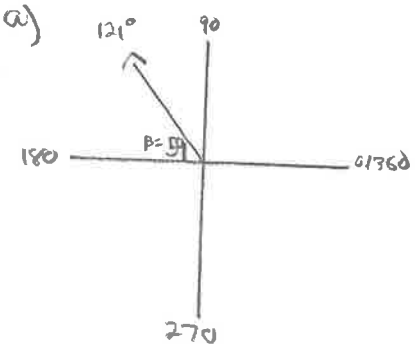
Chapter 4 Review

① 

② 

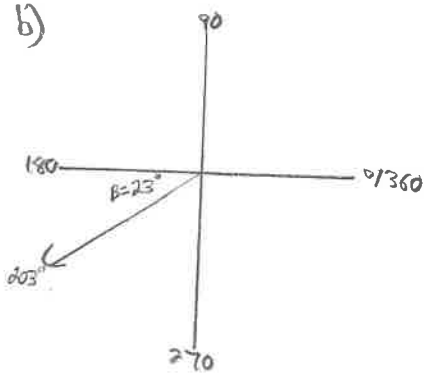
③ a) $\sin(45°) = \frac{1}{\sqrt{2}}$ b) $\cos(30°) = \frac{\sqrt{3}}{2}$ c) $\sin(60°) = \frac{\sqrt{3}}{2}$ d) $\tan(45°) = 1$

e) $\cos(60°) = \frac{1}{2}$ f) $\tan(30°) = \frac{1}{\sqrt{3}}$

④ a) 

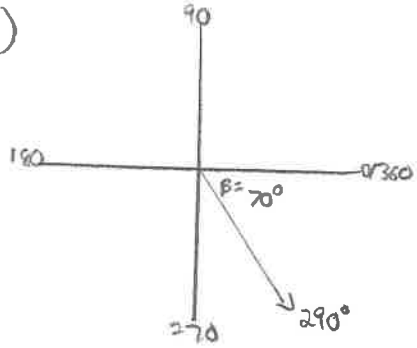
$\beta = 180 - 121$

$\beta = 59°$

b) 

$\beta = 203 - 180$

$\beta = 23°$

c) 

$\beta = 360° - 290°$

$\beta = 70°$

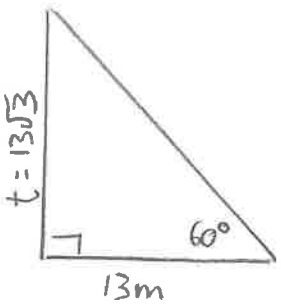
⑤ a) $-145° + 360° = 215°$ b) $-350° + 360° = 10°$ c) $-200° + 360° = 160°$

⑥ $\sin(30°) \times \tan(60°) - \cos(30°)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{1}\right) - \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$

$= 0$

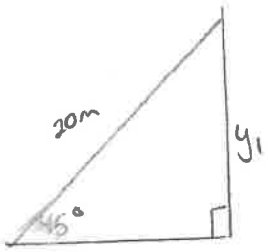
⑦ 

$\tan(60°) = \frac{t}{13}$

$\sqrt{3} = \frac{t}{13}$

$t = 13\sqrt{3}$ m

(8)



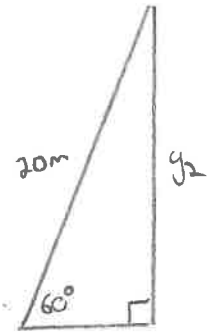
$\sin(45°) = \frac{y_1}{20}$

$\frac{1}{\sqrt{2}} = \frac{y_1}{20}$

$y_1 = \frac{20}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$y_1 = \frac{20\sqrt{2}}{2}$

$y = 10\sqrt{2}$



$\sin(60°) = \frac{y_2}{20}$

$\frac{\sqrt{3}}{2} = \frac{y_2}{20}$

$\frac{20\sqrt{3}}{2} = y_2$

$10\sqrt{3} = y_2$

a) vertical displacement $= y_2 - y_1$

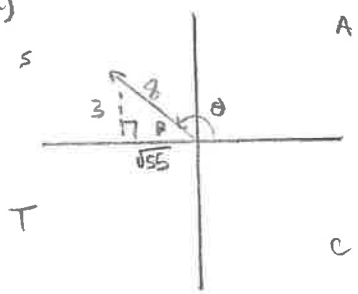
$= 10\sqrt{3} - 10\sqrt{2}$

$= 10(\sqrt{3} - \sqrt{2})$ m

b) vertical displacement $\simeq 3.2$ m

(9)

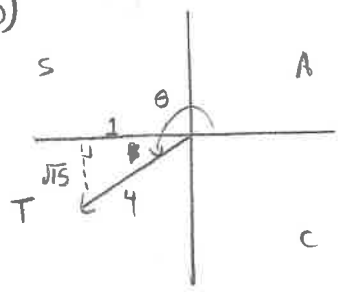
a)



$\cos\theta = -\cos\beta = -\frac{\sqrt{55}}{8}$

$\tan\theta = -\tan\beta = -\frac{3}{\sqrt{55}}$

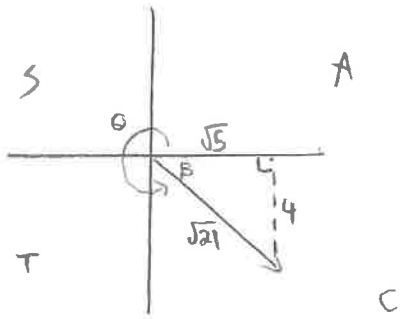
b)



$\sin\theta = -\sin\beta = -\frac{\sqrt{15}}{4}$

$\tan\theta = \tan\beta = \sqrt{15}$

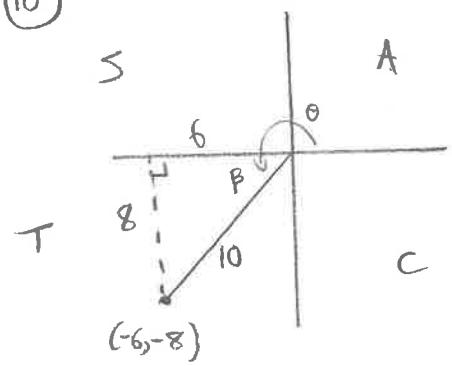
c)



$\sin\theta = -\sin\beta = -\frac{4}{\sqrt{21}}$

$\cos\theta = \cos\beta = \frac{\sqrt{5}}{\sqrt{21}}$

(10)

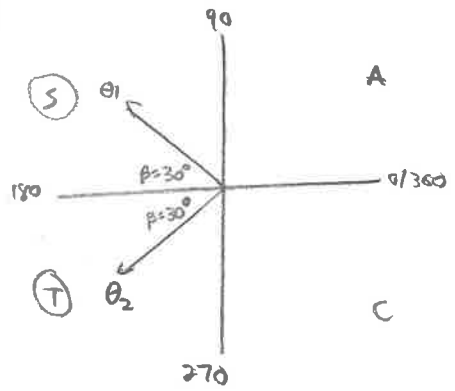


$\sin\theta = -\sin\beta = \frac{-8}{10} = -\frac{4}{5}$

$\cos\theta = -\cos\beta = \frac{-6}{10} = -\frac{3}{5}$

$\tan\theta = \tan\beta = \frac{8}{6} = \frac{4}{3}$

(11)



$\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta_1 = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

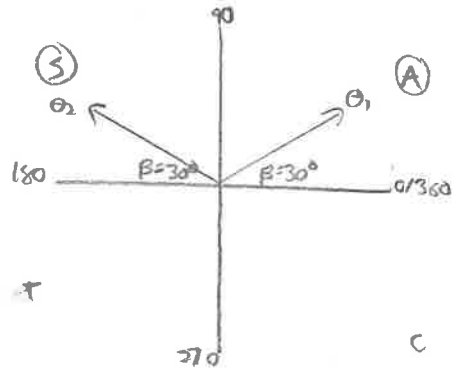
$\theta_1 = 150°$

$\theta_2 = 180° + \beta$

$\theta_2 = 180° + 30°$

$\theta_2 = 210°$

(12)



$2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$

$\theta_1 = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta_1 = 30°$

$\theta_2 = 180° - \beta$

$\theta_2 = 180° - 30°$

$\theta_2 = 150°$

(13)

$\theta_1 = 80° + 360°$
$= 440°$

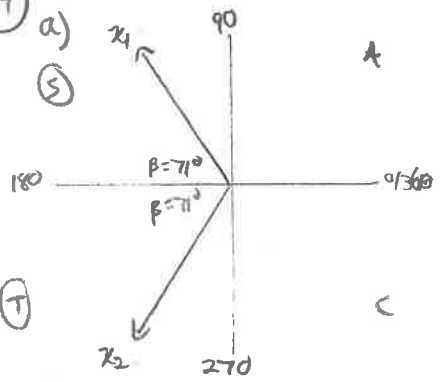
$\theta_2 = 440° + 360°$
$= 800°$

$\theta_3 = 80° - 360°$
$= -280°$

$\theta_4 = -280° - 360°$
$\theta_4 = -640°$

(14)

a)



$\sec x = -3$

$\cos x = -\frac{1}{3}$

$x_1 = \cos^{-1}\left(-\frac{1}{3}\right)$

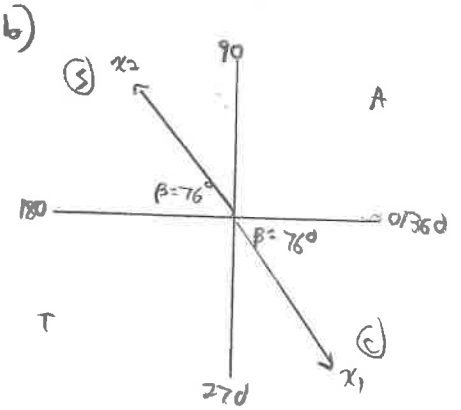
$x_1 = 109°$

$x_2 = 180 + \beta$

$x_2 = 180 + 71$

$x_2 = 251°$

b)



$4\cot x + 1 = 0$

$\cot x = -\frac{1}{4}$

$\tan x = -4$

$x_1 = \tan^{-1}(-4)$

$x_1 \simeq -76° + 360°$

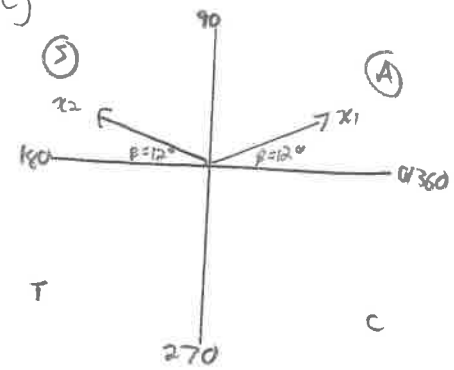
$x_1 \simeq 284°$

$x_2 = 180° - \beta$

$x_2 = 180° - 76°$

$x_2 \simeq 104°$

c)



$\csc x = 5$

$\sin x = \frac{1}{5}$

$x_1 = \sin^{-1}\left(\frac{1}{5}\right)$

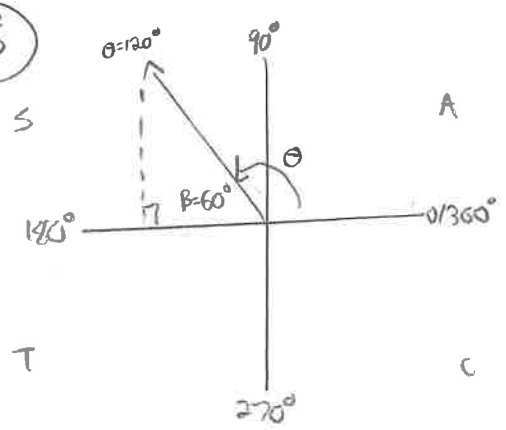
$x_1 \simeq 12°$

$x_2 = 180° - \beta$

$x_2 = 180 - 12$

$x_2 \simeq 168°$

(15)



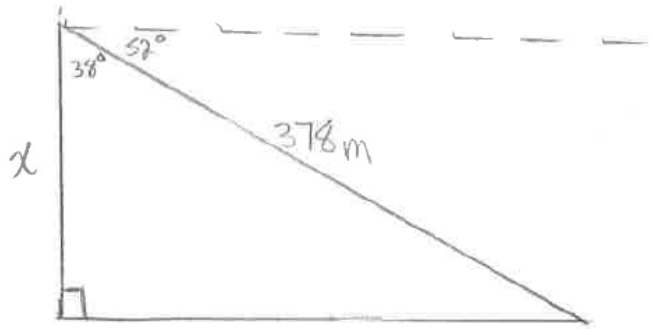
$\sin(120°) = \sin(60°) = \frac{\sqrt{3}}{2}$ $\quad \csc(120°) = \frac{2}{\sqrt{3}}$

$\cos(120°) = -\cos(60°) = \frac{-1}{2}$ $\quad \sec(120°) = -2$

$\tan(120°) = -\tan(60°) = -\sqrt{3}$ $\quad \cot(120°) = -\frac{1}{\sqrt{3}}$

(16)

(16)



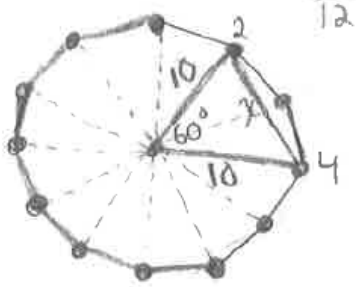
$\cos 38 = \frac{x}{378}$

$x = 378(\cos 38)$

$x \doteq 298$ m

(17)

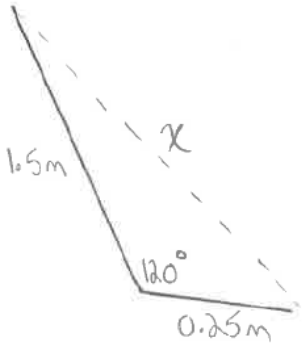
$\frac{360°}{12} = 30°$



$x^2 = 10^2 + 10^2 - 2(10)(10)\cos 60$

$x = 10$ m

(18)

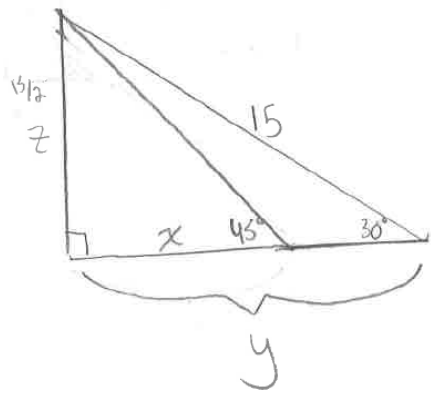


$x^2 = (1.5)^2 + (0.25)^2 - 2(1.5)(0.25)\cos 120$

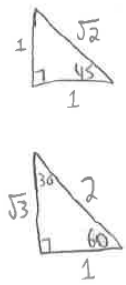
$x^2 = 2.6875$

$x = 1.64$ m or 164 cm

(19)



Remember:



$\cos 30 = \frac{y}{15}$ $\quad \tan 45 = \frac{(\frac{15}{2})}{x}$

$\frac{\sqrt{3}}{2} = \frac{y}{15}$ $\quad \frac{1}{1} = \frac{(\frac{15}{2})}{x}$

$\frac{15\sqrt{3}}{2} = y$ $\quad x = \frac{15}{2}$

Distance between wires $= y - x$

$= \frac{15\sqrt{3}}{2} - \frac{15}{2}$

$= \frac{15(\sqrt{3}-1)}{2}$ m

OR

$7.5(\sqrt{3}-1)$ m

$\sin 30 = \frac{z}{15}$

$\frac{1}{2} = \frac{z}{15}$

$\frac{15}{2} = z$

(20)

a)

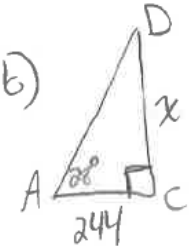
$\frac{224}{\sin 64} = \frac{b}{\sin 78}$

$b = \frac{224 \sin 78}{\sin 64}$

$b = 243.8$ cm

$b \approx 244$ cm

b)



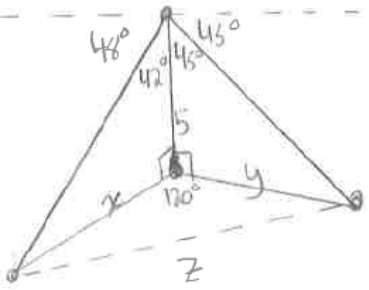
$\tan 28 = \frac{x}{244}$

$x = 244 \tan 28$

$x = 129.7$

$x \approx 130$ cm

(21)



$\tan 42 = \frac{x}{5}$ $\quad \tan 45 = \frac{y}{5}$

$x = 5 \tan 42$ $\quad y = 5 \tan 45$

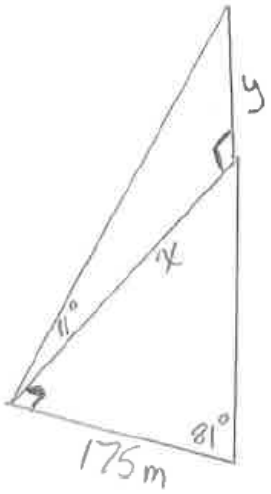
$x = 4.5$ $\quad y = 5$

$z^2 = 4.5^2 + 5^2 - 2(4.5)(5)\cos 120$

$z^2 = 67.75$

$z = 8.23$ km

(22)



$\tan 81 = \frac{x}{175}$ $\quad$ $\tan 11 = \frac{y}{1104.9}$

$x = 1104.9\text{ m}$ $\quad$ $y = 214.8$

$y \approx 215\text{ m}$

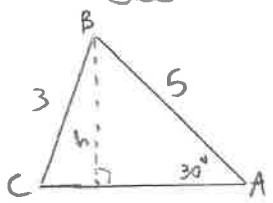
(23) a) $\sin(30^\circ) = \frac{h}{5}$

$h = 5\sin(30^\circ)$

$h = 2.5$

$h < a < c$, ∴ need to consider ambiguous case of sine.

b) Case 1



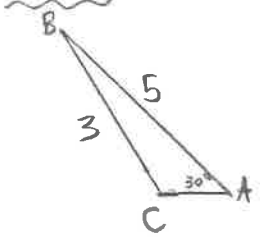
$\frac{5}{\sin C} = \frac{3}{\sin 30}$

$\sin C = \frac{5\sin 30}{3}$

$\angle C = \sin^{-1}\left(\frac{5\sin 30}{3}\right)$

$\angle C = 56.44^\circ$

Case 2



$\angle C = 180^\circ - \text{case 1}$

$\angle C = 180^\circ - 56.44$

$\angle C \simeq 123.56^\circ$

c) $\angle B = 180 - 30 - 56.44 = 93.56^\circ$

$\frac{b}{\sin 93.56} = \frac{3}{\sin 30}$

$b = \frac{3\sin 93.56}{\sin 30}$

$b \simeq 5.99\text{ cm}$

$\angle B = 180 - 30 - 123.56 = 26.44^\circ$

$\frac{b}{\sin 26.44} = \frac{3}{\sin 30}$

$b = \frac{3\sin 26.44}{\sin 30}$

$b \simeq 2.67\text{ cm}$

(24)

a)

LS

$= \sin^2 x(1+\cot^2 x)$

$= \sin^2 x + \sin^2 x \cdot \cot^2 x$

$= \sin^2 x + \sin^2 x \left(\frac{\cos^2 x}{\sin^2 x}\right)$

$= \sin^2 x + \cos^2 x$

$= 1$

RS

$= 1$

LS = RS

b)

LS

$= 1 - \cos^2 x$

$= \sin^2 x$

RS

$= \tan x \cos x \sin x$

$= \left(\frac{\sin x}{\cos x}\right) \cos x \sin x$

$= \sin^2 x$

LS = RS

c)

LS

$= \cos x \tan^3 x$

$= \cos x \left(\frac{\sin^3 x}{\cos^3 x}\right)$

$= \frac{\sin^3 x}{\cos^2 x}$

RS

$= \sin x \tan^2 x$

$= \sin x \left(\frac{\sin^2 x}{\cos^2 x}\right)$

$= \frac{\sin^3 x}{\cos^2 x}$

LS = RS

d)

LS

$= 1 - 2\cos^2 \theta$

RS

$= \sin^4 \theta - \cos^4 \theta$

$= (\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta)$

$= (\sin^2 \theta - \cos^2 \theta)(1)$

$= \sin^2 \theta - \cos^2 \theta$

$= (1 - \cos^2 \theta) - \cos^2 \theta$

$= 1 - 2\cos^2 \theta$

LS = RS.

e) LS

$$\cot\theta + \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)}{(1+\cos\theta)}\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta(\sin\theta)}{1+\cos\theta(\sin\theta)}$$

$$= \frac{\cos\theta(1+\cos\theta) + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta + 1}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1}{\sin\theta}$$

RS

$$= \csc\theta$$

$$= \frac{1}{\sin\theta}$$

LS = RS

f) LS

$$= \sec x(1-\cos x)$$

$$= \left(\frac{1}{\cos x}\right)(1-\cos x)$$

$$= \frac{1-\cos x}{\cos x}$$

RS

$$= \sec x - 1$$

$$= \frac{1}{\cos x} - \frac{\cos x}{\cos x}$$

$$= \frac{1-\cos x}{\cos x}$$

LS = RS

h) LS

$$= \frac{(1-\cos x)}{(1-\cos x)}\frac{1}{1+\cos x} + \frac{1}{1-\cos x}\frac{(1+\cos x)}{(1+\cos x)}$$

$$= \frac{1-\cos x + 1 + \cos x}{1-\cos^2 x}$$

$$= \frac{2}{\sin^2 x}$$

RS

$$= 2\csc^2 x$$

$$= 2\left(\frac{1}{\sin^2 x}\right)$$

$$= \frac{2}{\sin^2 x}$$

LS = RS

g)

LS

$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x}$

$= \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x(\cos x + \sin x)}$

$= \dfrac{\cos x - \sin x}{\cos x}$

RS

$= 1 - \tan x$

$= 1 - \dfrac{\sin x}{\cos x}$

$= \dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x}$

$= \dfrac{\cos x - \sin x}{\cos x}$

LS = RS

i)

LS

$= \sin^2\theta + 2\cos^2\theta - 1$

$= 1 - \cos^2\theta + 2\cos^2\theta - 1$

$= \cos^2\theta$

RS

$= \cos^2\theta$

LS = RS