Section 1: Special Angles/ Co-terminal and Reference Angles

1) Draw a right triangle that has a 30° angle. Label the sides $\sqrt{3}$, 2, and 1.

2) Draw a right triangle that has a 45° angle. Label the side lengths 1, 1, and $\sqrt{2}$.

3) Use the triangles drawn in the previous questions to find the exact values of...

   a) $sin45°$
   b) $cos30°$
   c) $sin60°$
   d) $tan45°$
   e) $cos60°$
   f) $tan30°$

4) Find a reference angle for the following obtuse angles

   a) 121°
   b) 203°
   c) 290°

5) Find a co-terminal positive angle for each of the following negative angles.

   a) −145°
   b) −350°
   c) −200°

6) Determine the exact value of $sin30° \times tan60° - cos30°$

7) A tree is anchored by a guy wire that is attached 13 m from the base of the tree and makes an angle of 60° with the ground. Find the exact value of the height of the tree.

8) The arm of a crane is 20 m long. The angle of inclination of the boom of the crane has a minimum value of 45° and a maximum value of 60°.

   a) Find an exact value for the vertical displacement of the boom of the crane as it moves from its minimum to its maximum inclination angles.

   b) Find the value of this vertical displacement to the nearest tenth of a metre.

9) One of the primary trigonometric ratios is given, as well as the quadrant in which the angle is located. Determine an exact expression for the other two primary trigonometric ratios.

    a) $\sin\theta = \frac{3}{8}$, second quadrant

    b) $\cos\theta = -\frac{1}{4}$, third quadrant

    c) $\tan\theta = -\frac{4}{\sqrt{5}}$, fourth quadrant

10) If point C(-6, -8) lies on the terminal arm of an angle, determine the exact values for the primary trigonometric ratios of the angle.

11) Solve $\cos\theta = -\frac{\sqrt{3}}{2}$ for $0° \leq \theta \leq 360°$.

12) Solve $2\sin\theta - 1 = 0$ for $0° \leq \theta \leq 360°$.

13) Find two positive and two negative co-terminal angles of 80°.

Section 2: Reciprocal Trig Ratios

14) For $0° \leq x \leq 360°$, solve each of the following equations. Round to the nearest degree.

    a) $\sec x = -3$
    b) $4\cot x + 1 = 0$
    c) $\csc x = 5$

15) Determine the exact values of all six trig ratios for $\theta = 120°$.

Section 3: Problems in 2 Dimension

16) Jeremy has a new laser sight for golf that gives him the straight line distance to any object he points the sight at. From an elevated tee block, he uses the sight and finds that the straight line distance to the green is 378 m. He also determines that the angle of depression to the green is 52°. What is the height of the elevated tee block above the level of the green?

17) A carousel, with a diameter of 20 m, has 12 horses spaced around the circumference at equal distances. If the horses are numbered in order, how far is it directly from the second horse to the fourth?

18) The angle between the shaft and the blade of a hockey stick is 120°. The shaft is 1.5 m long and the blade is 25 cm long. Determine the straight line distance from the top of the shaft to the tip of the blade.

19) A support wire to the top of a newly planted tree is 15 m long. It forms an angle of 30° to the ground On the same side of the tree, a second wire is also attached to the top of the tree, but it makes an angle o 45° to the ground. Determine an exact expression for the distance between the points where the two support wires are attached to the ground.

Section 4: Problems in 3-Dimensions

20) Consider the diagram to the right.

   a) Determine the length of AC to the nearest cm.
   b) Determine the length of DC to the nearest cm.
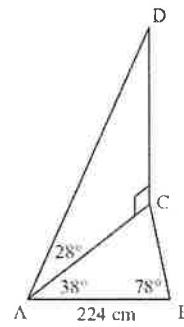
21) From the basket of a hot-air balloon that is 5 km in the air, Roxanna sees two small towns off in the distance. She measures the angle of depression to be 48° to one of the towns and the angle of depression to the other to be 45°. She estimates that if she were to land directly below her current position, the angle from one town to her position to the other town would be 120°. How far apart are the two towns in a straight line distance along the ground?

22) Albert wishes to determine the height of a small mountain that is at the other end of a provincial park, so he takes the measurements shown. Determine the height of the mountain.
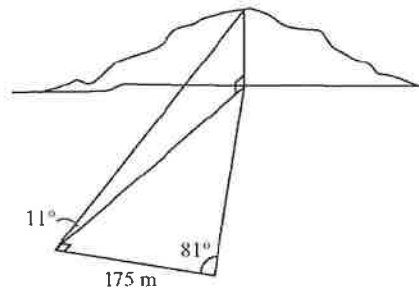
Section 5: Ambiguous Case of Sine

23) In triangle ABC, a=3cm, c=5cm and <A=30°.

   a) Does the ambiguous case need to be considered? Give proof. If so, draw both possible scenarios.
   b) Determine the measure of <C to 2 decimal places.
   c) Determine the length of AC to 2 decimal places.

Section 6: Trig Identities

24) Prove the following identities

a) $\sin^2 x(1+\cot^2 x)=1$

b) $1-\cos^2 x = \tan x \cos x \sin x$

c) $\cos x \tan^3 x = \sin x \tan^2 x$

d) $1-2\cos^2\theta = \sin^4\theta - \cos^4\theta$

e) $\cot\theta + \dfrac{\sin\theta}{1+\cos\theta} = \csc\theta$
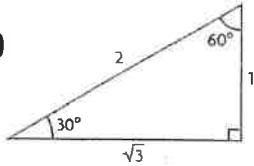
f) $\sec x(1-\cos x) = \sec x - 1$

g) $\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

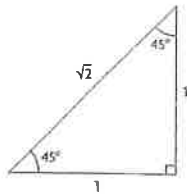**h)** $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x} = 2\csc^2 x$

i) $\sin^2\alpha + 2\cos^2\alpha - 1 = \cos^2\alpha$

Answers

1)

2)

3) a) $\frac{1}{\sqrt{2}}$ **b)** $\frac{\sqrt{3}}{2}$ **c)** $\frac{\sqrt{3}}{2}$ **d)** 1 **e)** $\frac{1}{2}$ **f)** $\frac{1}{\sqrt{3}}$

4) a) 59° **b)** 23° **c)** 70°

5) a) 215° **b)** 10° **c)** 160°

6) 0

7) $13\sqrt{3}$ m

8) a) $10(\sqrt{3}-\sqrt{2})$ m **b)** 3.2 m

9) a) $cos\theta = -\frac{\sqrt{55}}{8}$, $tan\theta = -\frac{3}{\sqrt{55}}$ **b)** $sin\theta = -\frac{\sqrt{15}}{4}$, $tan\theta = \sqrt{15}$ **c)** $sin\theta = -\frac{4}{\sqrt{21}} = -\frac{4\sqrt{21}}{21}$, $cos\theta = \frac{\sqrt{5}}{\sqrt{21}} = \frac{\sqrt{105}}{21}$

10) $sin\theta = -\frac{4}{5}$, $cos\theta = -\frac{3}{5}$, $tan\theta = \frac{4}{3}$

11) 150° and 210°

12) 30° and 150°

13) 440°, 800° and -280°, -640°

14) a) 109°, 251° **b)** 104°, 284° **c)** 12°, 168°

15) $sin\theta = \frac{\sqrt{3}}{2}$, $csc\theta = \frac{2}{\sqrt{3}}$, $cos\theta = -\frac{1}{2}$, $sec\theta = -2$, $tan\theta = -\sqrt{3}$, $cot\theta = -\frac{1}{\sqrt{3}}$

16) 298 m

17) 10 m

18) 1.64 m

19) $7.5(\sqrt{3}-1)$ m

20) a) 244 cm **b)** 130 cm

21) 8.23 km

22) 215 m

23) b) 56.44° or 123.56° **c)** 5.99 or 2.67 cm