Chapter 5 Review

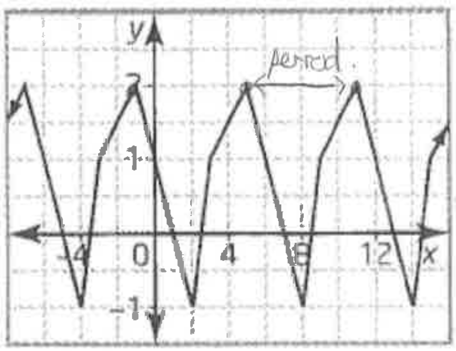
MCR3U
Jensen

SOLUTIONS

Section 1: Periodic Behaviour

1) Classify each graph as periodic or not periodic. If it is periodic, determine the amplitude and period.

a)

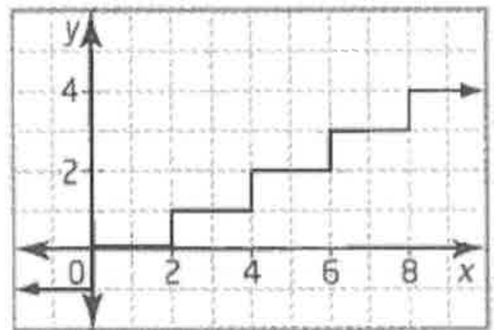


periodic.

amplitude $= \frac{2-(-1)}{2} = 1.5$

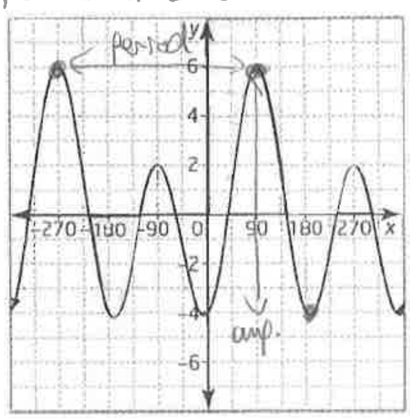
period $= 11 - 5 = 6$

b)



Not periodic

c)

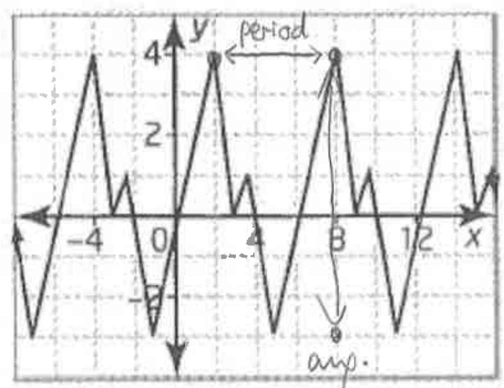


periodic.

amplitude $= \frac{6-(-4)}{2} = 5$

period $= 90 - (-270) = 360$

d)



periodic

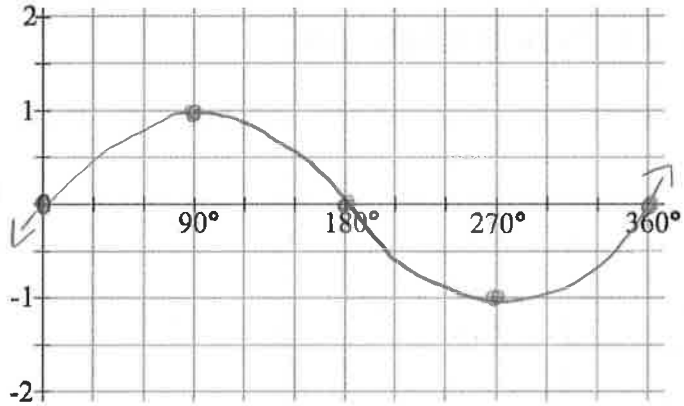
amplitude $= \frac{4-(-3)}{2} = 3.5$

period $= 8 - 2 = 6$

Section 2: Graphing Sine and Cosine Functions

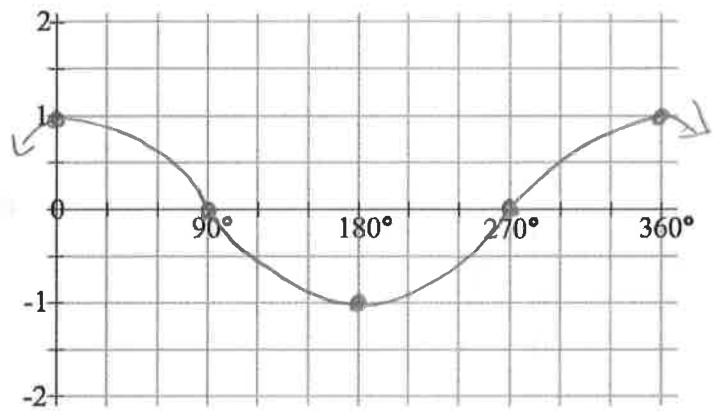
2) Graph the function $y = sinx$ using key points between 0° and 360°.

x	y
0	0
90	1
180	0
270	-1
360	0

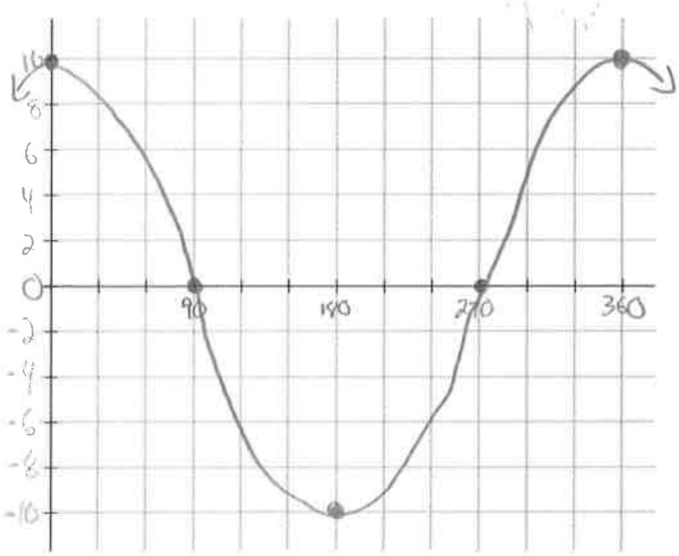


3) Graph the function $y = cosx$ using key points between 0° and 360°.

x	y
0	1
90	0
180	-1
270	0
360	1



4) You are in a car of a Ferris wheel. The wheel has a radius of 10 m and turns counterclockwise. Let the origin be at the center of the wheel. Sketch a graph of your horizontal displacement versus the angle through which you turn for one rotation of the wheel. Begin the sketch when the radius from the center of the wheel to your car is along the positive x-axis. Which function models the horizontal displacement?



A cosine function models the horizontal displacement.

$y = 10 \cos x$

Section 3: Transformations of Sine and Cosine Functions

5) Determine the amplitude, the period, phase shift, vertical shift, maximum and minimum for each of the llowing.

a) $y = \sin(x - 40°) + 2$

amp = 1
period = 30
phase shift: 40° right
vertical shift: up 2
max = 2+1 = 3
min = 2-1 = 1

b) $y = -3\sin(x + 38°) + 5$

amp = 3
period = 360
phase shift: 38° left
vertical shift: up 5
max = 5+3 = 8
min = 5-3 = 2

c) $y = 4\sin[3(x + 30°)] - 6$

amp = 4
period = $\frac{360}{3}$ = 120
p. shift: 30° left
v. shift: down 6
max = -6+4 = -2
min = -6-4 = -10

d) $y = 10\cos[3(x - 120°)] + 9$

amp = 10
period = $\frac{360}{3}$ = 120
p. shift: 120° right
v. shift: up 9
max = 9+10 = 19
min = 9-10 = -1

e) $y = \frac{1}{2}\cos[3(x + 120°)] - 6$

amp = $\frac{1}{2}$
period = $\frac{360}{3}$ = 120
p. shift: 120° left
v. shift: down 6
max = -6+0.5 = -5.5
min = -6-0.5 = -6.5

f) $y = 4\sin\left[\frac{1}{4}(x + 45°)\right] - 3$

amp = 4
period = $\frac{360}{(\frac{1}{4})}$ = 1440
p. shift: 45° left
v. shift: down 3
max = -3+4 = 1
min = -3-4 = -7

6) For the transformed function $y = 4\sin\left[\frac{3}{2}\text{[redacted]}(x + 270°)\right] - 1$ .... (6 marks)

a) State the amplitude, the period, the phase shift and the vertical shift of the function with respect to the parent function

b) State the maximum and minimum values of the function

c) Sketch ~~one~~ two cycles of the parent function and two cycles of the transformed function on the graph provided ... adjust the vertical scale appropriately

Amplitude = 4

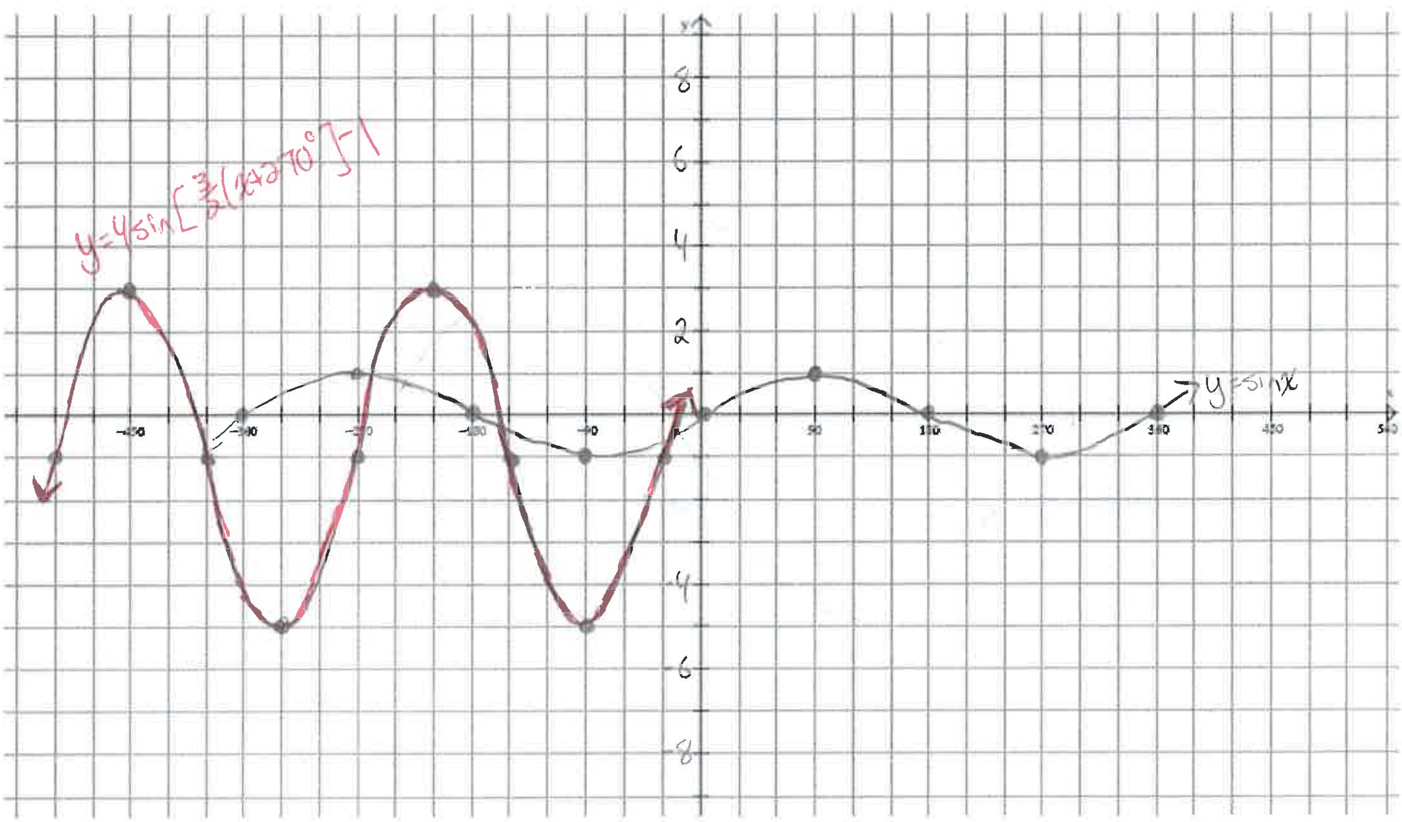
Period = $\frac{360}{\left(\frac{3}{2}\right)} = 240$

Phase shift = 270° left

Vertical Shift = down 1

Maximum Value = $-1+4=3$

Minimum Value = $-1-4=-5$



$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

$\frac{2x}{3} - 270$	$4y-1$
-270	-1
-210	3
-150	-1
-90	-5
-30	-1

7) For the transformed function $y = -\frac{1}{2}\cos[2(x - 30°)] + \frac{3}{2}$ .... (6 marks)

a) State the amplitude, the period, the phase shift and the vertical shift of the function with respect to the parent function

b) State the maximum and minimum values of the function

c) Sketch ~~one~~ two cycles of the parent function and two cycles of the transformed function on the graph provided ... adjust the vertical scale appropriately

Amplitude = 0.5

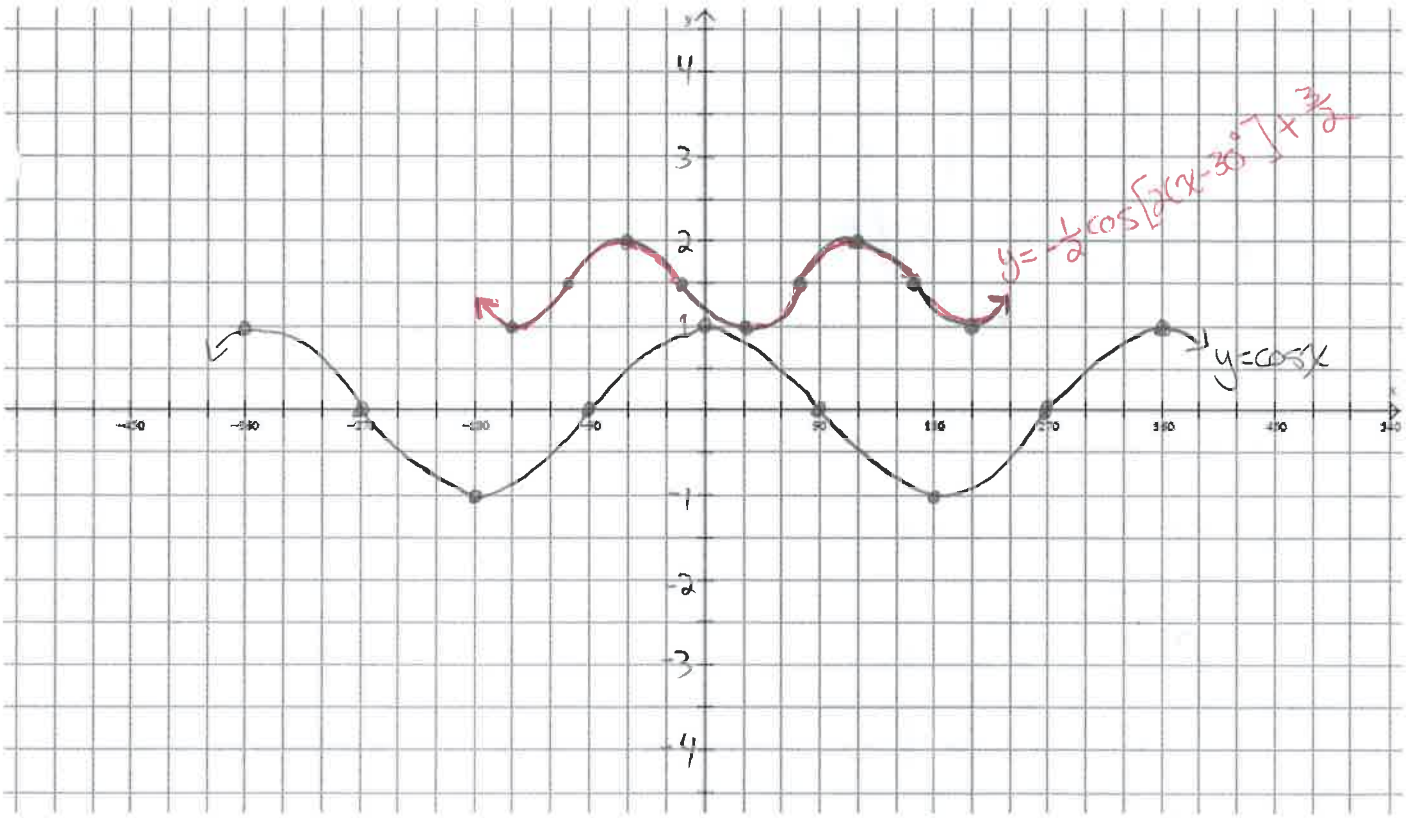
Period = $\frac{360}{2} = 180°$

Phase shift = Right 30°

Vertical Shift = up 1.5

Maximum Value = 1.5+0.5 = 2

Minimum Value = 1.5 - 0.5 = 1



y = cos x

x	y
0	1
90	0
180	-1
270	0
360	1

$\frac{x}{2} + 30$	$\frac{y}{-2} + 1.5$
30	1
75	1.5
120	2
165	1.5
210	1

8) A sinusoidal functions has an amplitude of $\frac{1}{2}$ units, a period of 720° and a maximum at $(0, \frac{3}{2})$. Represent the function as a sine function and as a cosine function.

$a = \frac{1}{2}$

$k = \frac{360}{720} = \frac{1}{2}$

$c = \text{max} - \text{amp}$
$= 1.5 - 0.5$
$= 1$

cosine

$d = 0$ since max is on y-axis.

$y = \frac{1}{2}\cos(\frac{1}{2}x) + 1$

sine

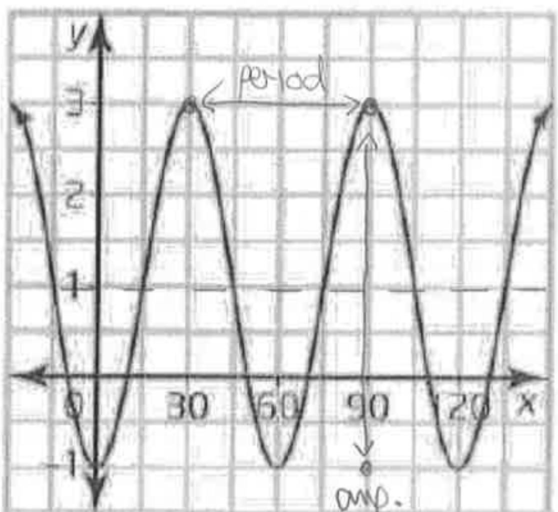
d-value: rising midline is $\frac{90}{k}$ to the left of the max.

$\frac{90}{k} = \frac{90}{(\frac{1}{2})} = 180$; ∴ $d = -180$

$y = \frac{1}{2}\sin[\frac{1}{2}(x+180)] + 1$

9) Determine the equation of a sine and cosine function that models the following graphs.

a)



$a = \frac{3-(-1)}{2} = 2$

period $= 90 - 30 = 60$

$k = \frac{360}{60} = 6$

$c = \text{max} - \text{amp}$
$= 3 - 2$
$= 1$

cosine

max at (30, 3); therefore $d = 30$

$y = 2\cos[6(x-30)] + 1$

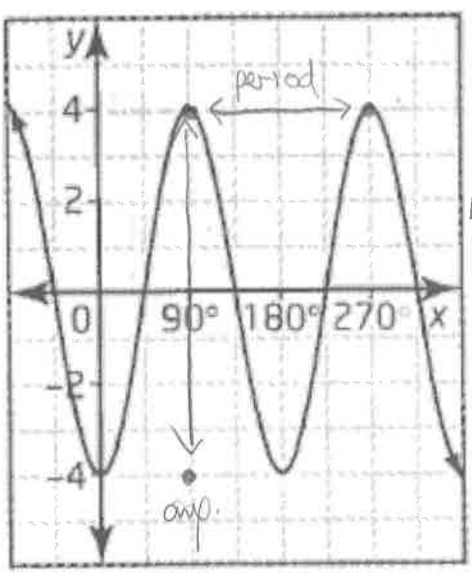
sine

rising midline is $\frac{90}{k} = \frac{90}{6} = 15$ to the left of the max.

∴ $d = 30 - 15 = 15$

$y = 2\sin[6(x-15)] + 1$

b)



$a = \frac{4-(-4)}{2} = 4$

period $= 270 - 90 = 180$

$k = \frac{360}{180} = 2$

$c = \text{max} - \text{amp}$
$= 4 - 4$
$= 0$

cosine

max at (90, 4); therefore $d = 90$

$y = 4\cos[2(x-90)]$

sine

rising midline is $\frac{90}{k} = \frac{90}{2} = 45$ to the left of the max;

∴ $d = 90 - 45 = 45$

$y = 4\sin[2(x-45)]$

Section 4: Trig Applications

10) A robot arm is used to cap bottles on an assembly line. The vertical position, y, in centimetres, of the m after t seconds can be modelled by the function:

$$y = 30\sin[360(t - 0.25)] + 45$$

a) Determine the amplitude, period, phase shift, and vertical shift:

$a = 30$

period $= \frac{360}{360} = 1$

p. shift: 0.25 right

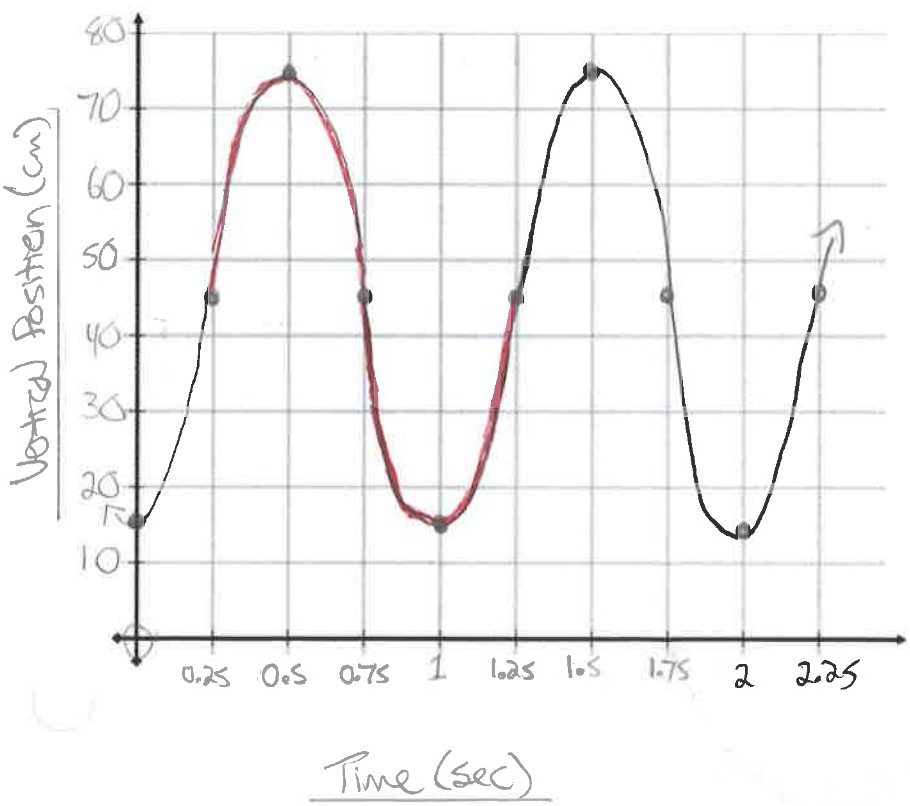
v. shift: up 45

b) What is the lowest vertical position that the arm reaches?

min $= 45 - 30$

$= 15$ cm

c) Graph the function below: (be sure to change the scale appropriately)



$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

$y = 30\sin[360(t - 0.25)] + 45$

$\frac{x}{360} + 0.25$	$30y + 45$
0.25	45
0.5	75
0.75	45
1	15
1.25	45

11) Smog is a generic term used to describe the pollutants in the air. A smog alert is usually issued when the air quality index is greater than 50. Air quality can vary throughout the day, increasing when more cars are on the road. Consider a model of the form $I = 30\sin[15(t-4)] + 25$, where I is the air quality index and t is the measure of time after midnight, in hours.

a) What is the period of the modelled function? Why does this make sense?

Period $= \frac{360}{15} = 24$ 24 hours in a day.

b) Determine the Maximum, Minimum, and Amplitude.

max $= 25+30 = 55$

min $= 25-30 = -5$

amp $= 30$

c) When do the max and min occur? (you can use your graph)

Rising midline at $t=4$

Max is $\frac{90}{k} = \frac{90}{15} = 6$ to the right; max at $t = 4+6 = 10$ → 10 a.m.

Min is $\frac{180}{15} = \frac{180}{6} = 12$ to the right of the max; ∴ min at $10+12=22$ → 10 p.m.

d) During what time interval would a smog alert be issued?

$50 = 30\sin[15(t-4)]+25$

$\frac{25}{30} = \sin[15(t-4)]$

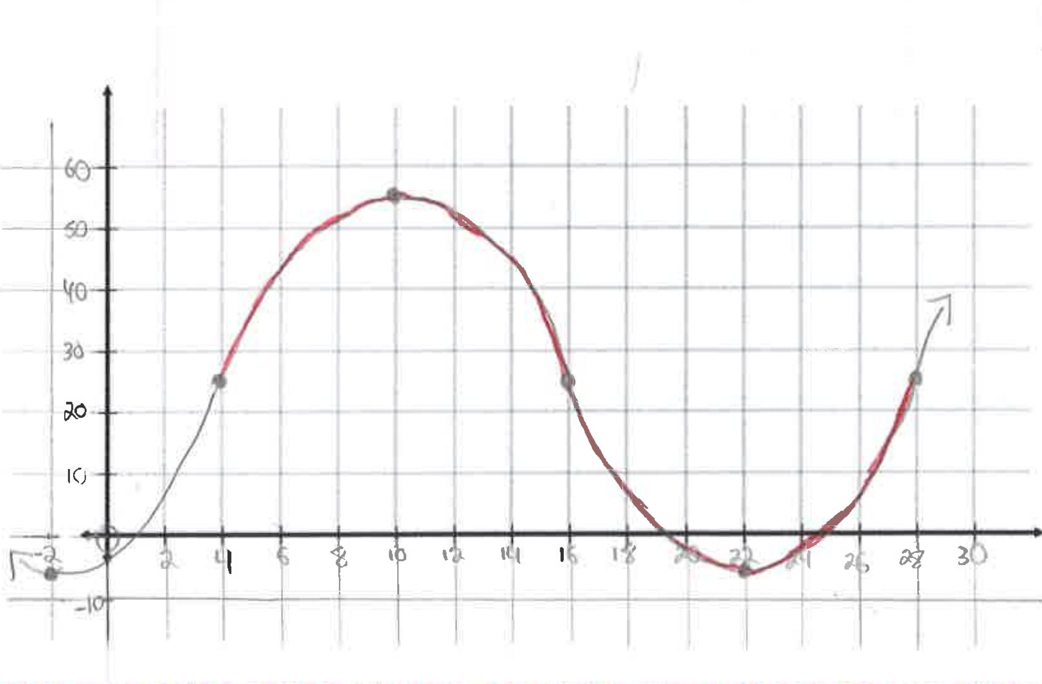
$t = 7.76$

approx 7 hours 45 mins

This is 2 hours and 15 minutes before the max. The air quality index will be above 50 until 2 hours and 15 after the max.

∴ The smog alert should be issued between 7:45 a.m. and 12:15 p.m.

e) Graph:



$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

$y = 30\sin[15(x-4)] + 25$

$\frac{x}{15} + 4$	$30y + 25$
4	25
10	55
16	25
22	-5
28	25

12) The Ferris wheel at a carnival rotates counterclockwise and has a diameter of 18 meters and descends to 3 meters above ground level at its lowest point. Assume that a rider enters a car from a platform that is located 40° around the rim before the car reaches its lowest point.

a) Model the rider's height above the ground versus angle using a transformed sine function.

$a = \frac{18}{2} = 9$

d-value: must rotate $40 + 90 = 130°$ before reaching rising midline; $d = 130$

period $= 360$

$k = \frac{360}{360} = 1$

$y = 9\sin(x - 130) + 12$

$c = 9 + 3 = 12$

b) Model the rider's height above the ground versus angle using a transformed cosine function.

Max is $\frac{90}{k} = \frac{90}{1} = 90°$ to right of rising midline; $d = 130 + 90 = 220°$

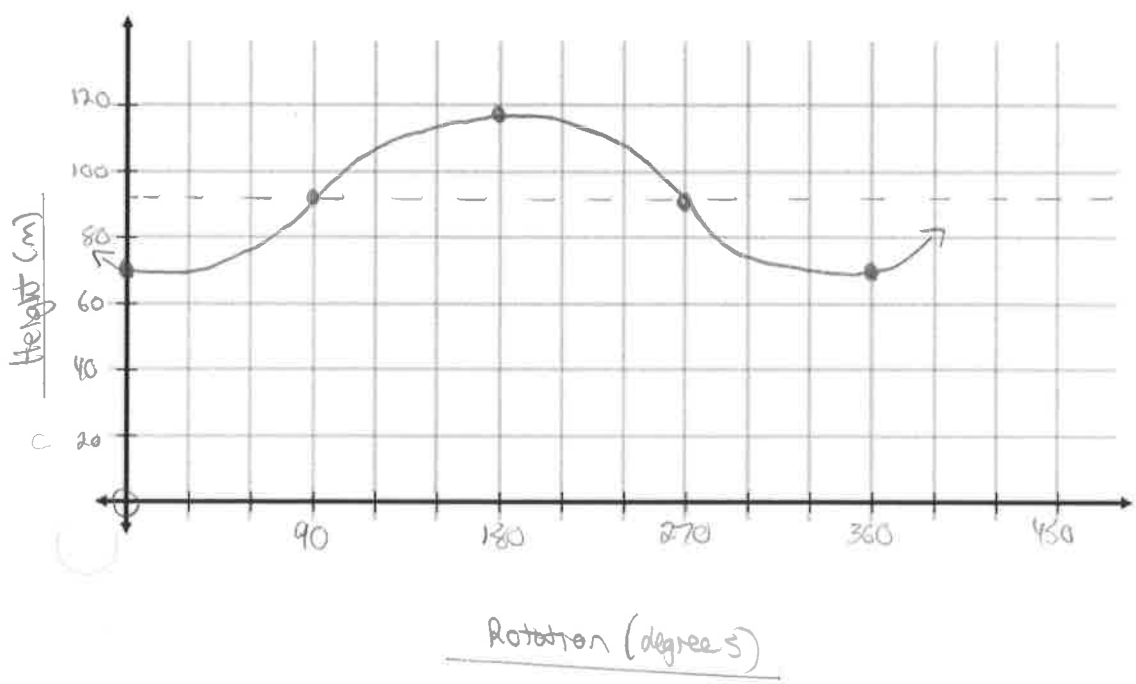
$y = 9\cos(x - 220) + 12$

13) The wind turbine at Exhibition Place in Toronto is 94 m tall (to the centre) and has three blades, each measuring 24 m in length. Draw a diagram, and then use a graph to model the height of one of the blades if it starts pointing straight down. Graph one full rotation of the blade.

Max $= 94 + 24 = 118$

Min $= 94 - 24 = 70$

Period $= 360$



x	y
0	70
90	94
180	118
270	94
360	70

14) A function has an amplitude of 12, a period of 90°, is translated 15° to the right and is moved up 8 units.

a) Use this information to write an equation using a sine function.

$a = 12$

$k = \frac{360}{90} = 4$

$d = 15$

$c = 8$

$y = 12\sin[4(x-15)] + 8$

b) Determine an equivalent cosine function to the sine function in a).

$y = \sin x = \cos(x - \frac{90}{k})$ → shift cosine function $\frac{90}{k} = \frac{90}{4} = 22.5$ to the right to make it equivalent to a sine function.

$\therefore d = 15 + 22.5 = 37.5$

$y = 12\cos[4(x - 37.5)] + 8$

15) A function that was developed to model the height of the tide at a small coastal village is $h = 6\cos[30(t-2)] + 8$. The height is measured on a pole that is placed out in the bay. Here h is in metres and t is in hours after midnight.

a) State the period, amplitude, phase shift and vertical shift of the function.

$a = 6$

period $= \frac{360}{30} = 12$

p. shift : 2 right

v. shift : 8 up

b) What is the water level at low tide?

min $= 8 - 6 = 2$ m

c) What is the water level at high tide?

max $= 8 + 6 = 14$ m

16) The sinusoidal function $h(t) = 7\sin[30(t-2.5)]$ models the height, h, of tides in a particular location on a particular day at t hours after midnight.

Determine the max and min heights of the tides.

max = 7

min = -7

b) At what times do high tide and low tide occur?

Rising midline at 2.5

Max at $\frac{90}{k} = \frac{90}{30} = 3$ to the right of rising midline ; ∴ high tide at 5:30 a.m. and 5:30 p.m.

Min $\frac{180}{k} = \frac{180}{30} = 6$ to the right of max ; ∴ low tide at 11:30 am and 11:30 p.m.

c) Use a cosine function to write an equivalent equation.

shift cosine function $\frac{90}{k} = \frac{90}{30} = 3$ to the right to make it equivalent to the sine function.

∴ $d = 2.5 + 3 = 5.5$

$y = 7\cos[30(t-5.5)]$