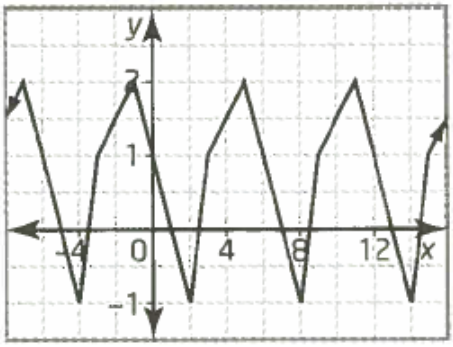
Chapter 5 Review
MCR3U
Jensen

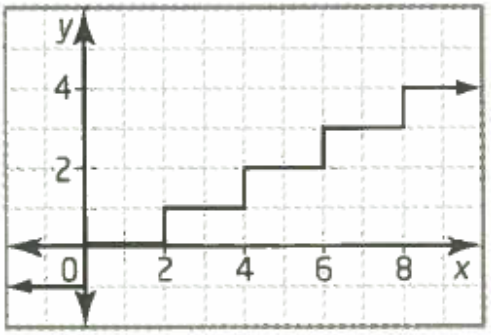
Section 1: Periodic Behaviour

1) Classify each graph as periodic or not periodic. If it is periodic, determine the amplitude and period.

a)



b)



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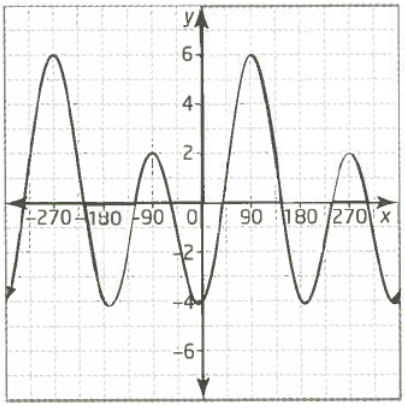

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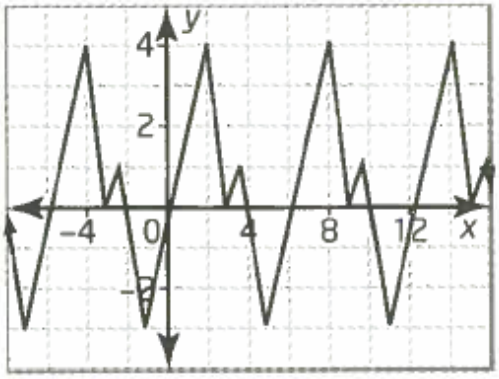
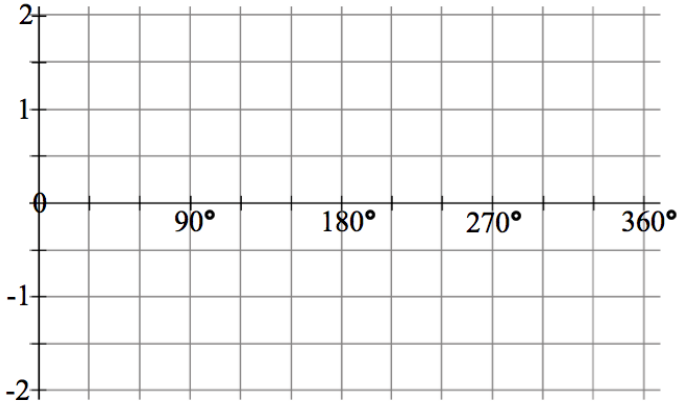


Section 2: Graphing Sine and Cosine Functions

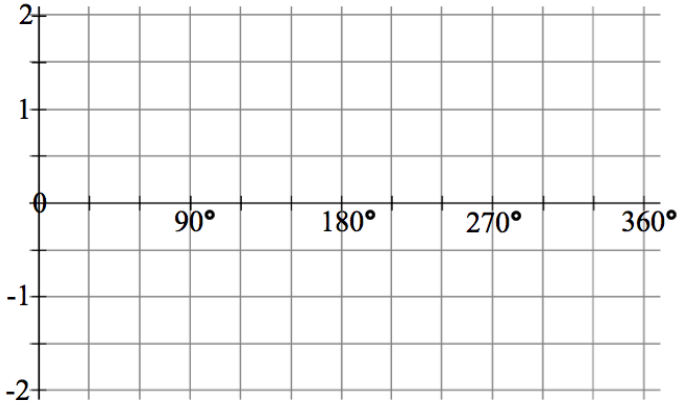
2) Graph the function $y = sinx$ using key points between 0° and 360°.

x	y

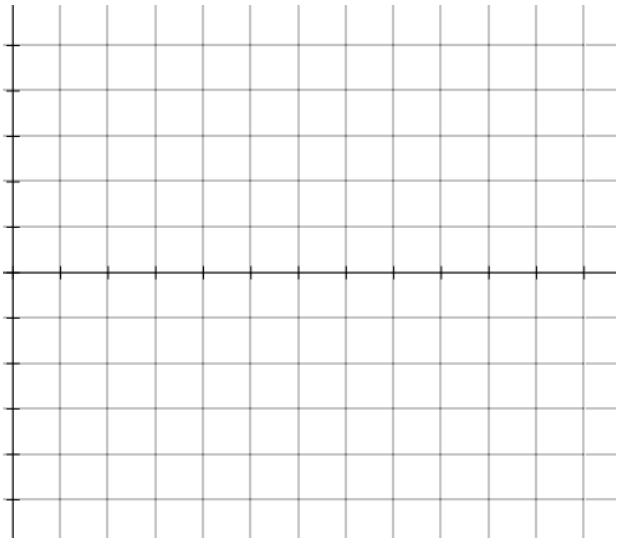


3) Graph the function $y = cosx$ using key points between 0° and 360°.

x	y



4) You are in a car of a Ferris wheel. The wheel has a radius of 10 m and turns counterclockwise. Let the origin be at the center of the wheel. Sketch a graph of your horizontal displacement versus the angle through which you turn for one rotation of the wheel. Begin the sketch when the radius from the center of the wheel to your car is along the positive x-axis. Which function models the horizontal displacement?



Section 3: Transformations of Sine and Cosine Functions

5) Determine the amplitude, the period, phase shift, vertical shift, maximum and minimum for each of the following.

a) $y = \sin(x - 40°) + 2$

b) $y = -3\sin(x + 38°) + 5$

c) $y = 4\sin[3(x + 30°)] - 6$

d) $y = 10\cos[3(x - 120°)] + 9$

e) $y = \frac{1}{2}\cos[3(x + 120°)] - 6$

f) $y = 4\sin\left[\frac{1}{4}(x + 45°)\right] - 3$

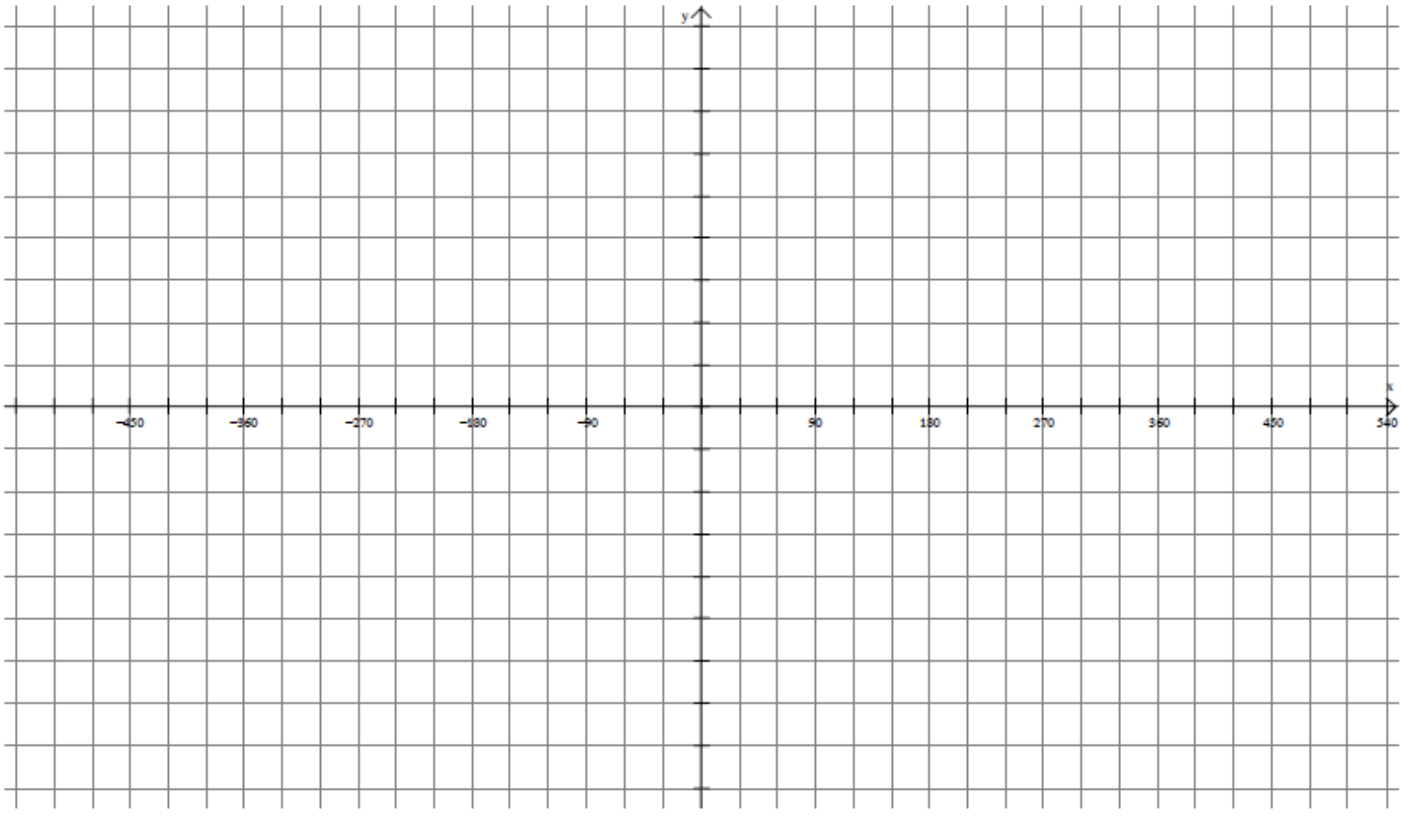
6) For the transformed function $y = 4\sin\left[\frac{3}{2}(x + 270°)\right] - 1$ .... (6 marks)

a) State the amplitude, the period, the phase shift and the vertical shift of the function with respect to the parent function. Then state the maximum and minimum values of the function

Amplitude = Period = Phase shift =

Vertical Shift = Maximum Value = Minimum Value =

b) Sketch two cycles of the parent function and two cycles of the transformed function on the graph provided … adjust the vertical scale appropriately



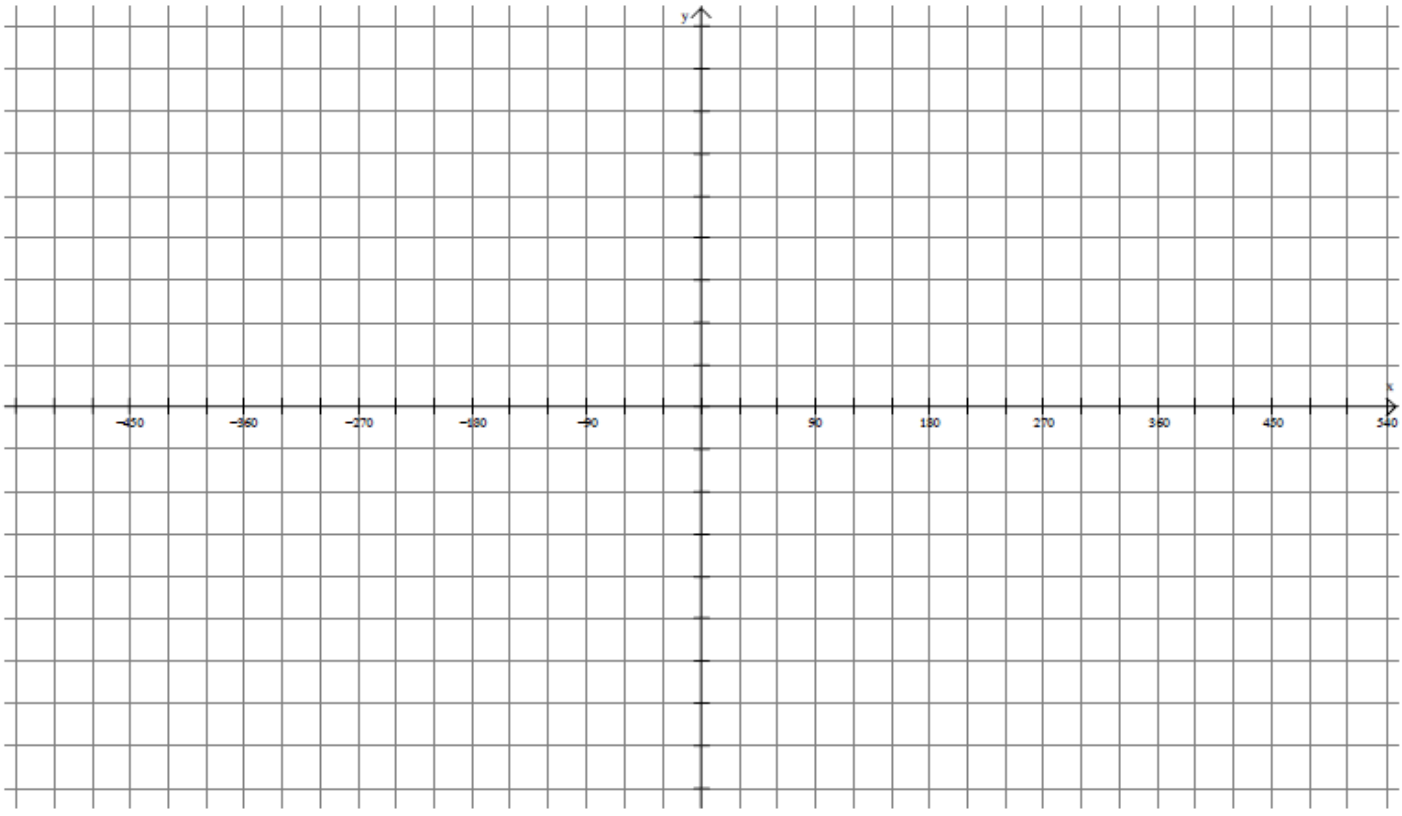
7) For the transformed function $y = -\frac{1}{2}\cos[2(x - 30°)] + \frac{3}{2}$ .... (6 marks)

a) State the amplitude, the period, the phase shift and the vertical shift of the function with respect to the parent function. Then state the maximum and minimum values of the function

Amplitude = Period = Phase shift =

Vertical Shift = Maximum Value = Minimum Value =

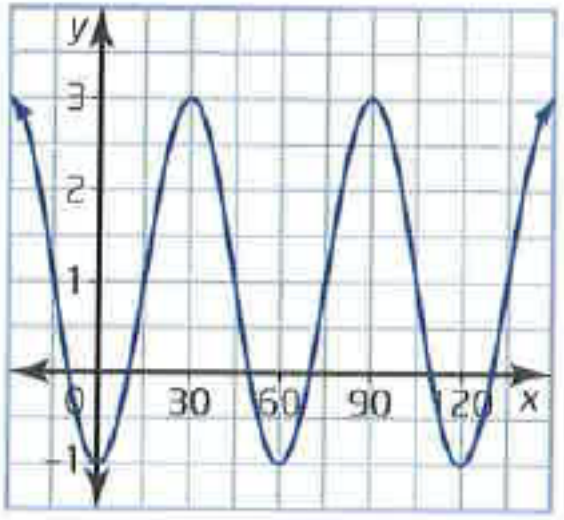
b) Sketch two cycles of the parent function and two cycles of the transformed function on the graph provided … adjust the vertical scale appropriately



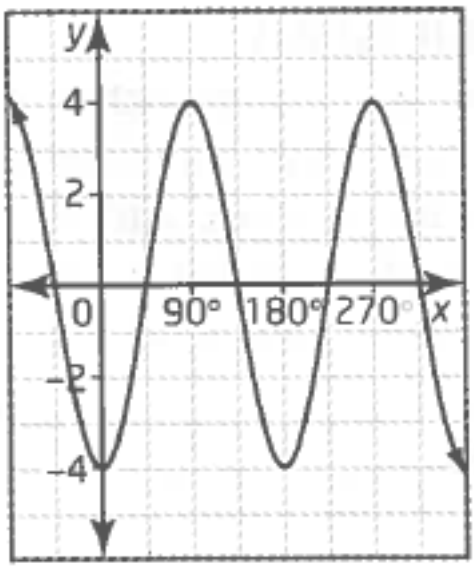
8) A sinusoidal functions has an amplitude of $\frac{1}{2}$ units, a period of 720° and a maximum at $(0, \frac{3}{2})$. Represent the function as a sine function and as a cosine function.

9) Determine the equation of a sine and cosine function that models the following graphs.

a)



b)



Section 4: Trig Applications

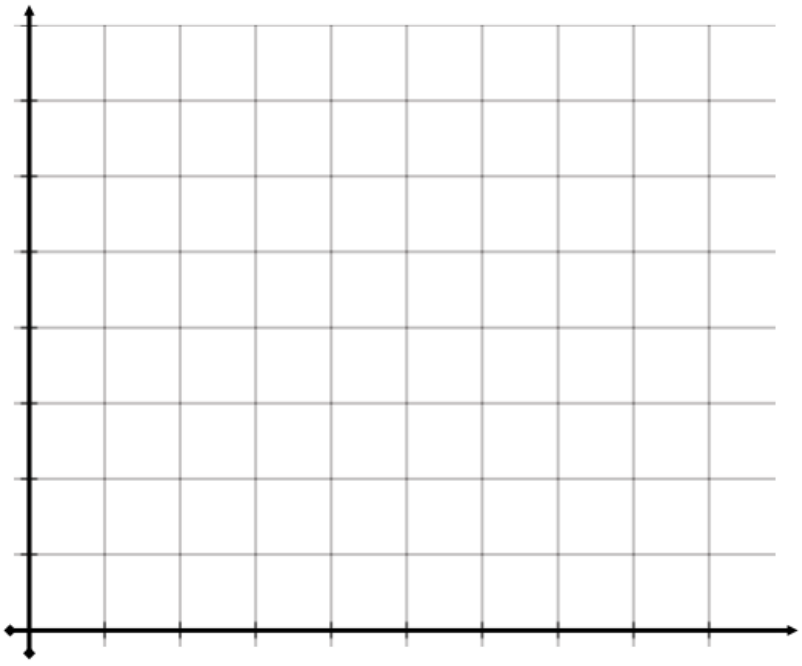
10) A robot arm is used to cap bottles on an assembly line. The vertical position, y, in centimetres, of the arm after *t* seconds can be modelled by the function:

$$y = 30\sin[360(t - 0.25)] + 45$$

a) Determine the amplitude, period, phase shift, and vertical shift:

b) What is the lowest vertical position that the arm reaches?

c) Graph the function below: (be sure to change the scale appropriately)



11) Smog is a generic term used to describe the pollutants in the air. A smog alert is usually issued when the air quality index is greater than 50. Air quality can vary throughout the day, increasing when more cars are on the road. Consider a model of the form $I = 30\sin[15(t-4)] + 25$, where I is the air quality index and t is the measure of time after midnight, in hours.

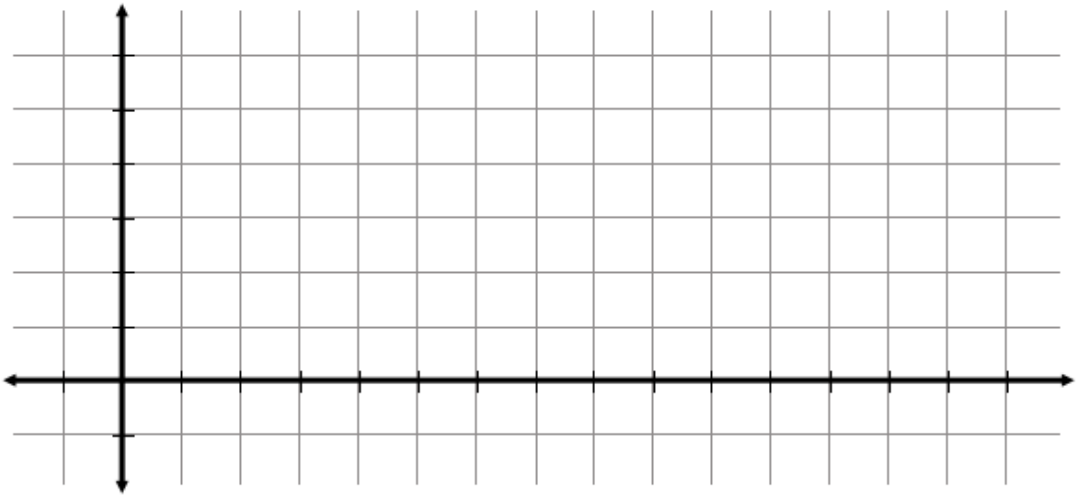
a) What is the period of the modelled function? Why does this make sense?

b) Determine the Maximum, Minimum, and Amplitude.

c) When do the max and min occur? (you can use your graph)

d) During what time interval would a smog alert be issued?

e) Graph:

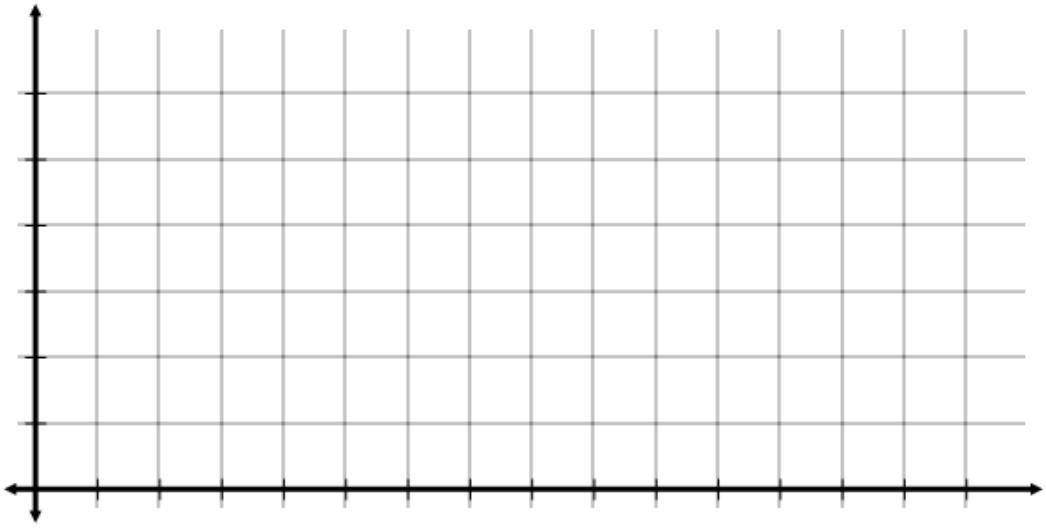


12) The Ferris wheel at a carnival rotates counterclockwise and has a diameter of 18 meters and descends to 3 meters above ground level at its lowest point. Assume that a rider enters a car from a platform that is located 40° around the rim before the car reaches its lowest point.

a) Model the rider's height above the ground versus angle using a transformed sine function.

b) Model the rider's height above the ground versus angle using a transformed cosine function.

13) The wind turbine at Exhibition Place in Toronto is 94 m tall (to the centre) and has three blades, each measuring 24 m in length. Draw a diagram, and then use a graph to model the height of one of the blades if it starts pointing straight down. Graph one full rotation of the blade.



14) A function has an amplitude of 12, a period of 90°, is translated 15° to the right and is moved up 8 units.

a) Use this information to write an equation using a sine function.

b) Determine an equivalent cosine function to the sine function in a).

15) A function that was developed to model the height of the tide at a small coastal village is $h = 6 \cos [30(t - 2)] + 8$. The height is measured on a pole that is placed out in the bay. Here h is in metres and t is in hours after midnight.

a) State the period, amplitude, phase shift and vertical shift of the function.

b) What is the water level at low tide?

c) What is the water level at high tide?

16) The sinusoidal function $h(t) = 7\sin[30(t - 2.5)]$ models the height, h, of tides in a particular location on a particular day at t hours after midnight.

a) Determine the max and min heights of the tides.

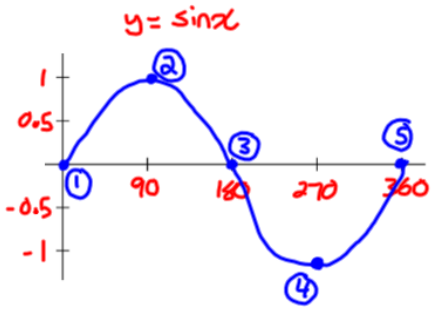
b) At what times do high tide and low tide occur?

c) Use a cosine function to write an equivalent equation.

Answers

1) a) periodic; period is 6, amplitude is 1.5 **b)** not periodic **c)** periodic; period is 360, amplitude is 5
d) periodic; period is 6, amplitude is 3.5

2)



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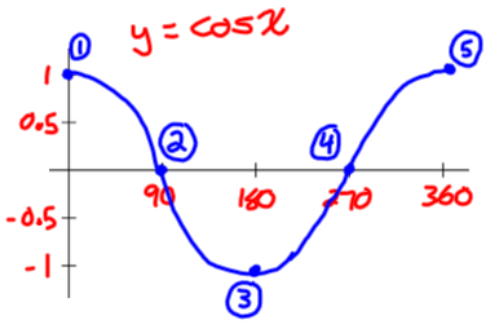
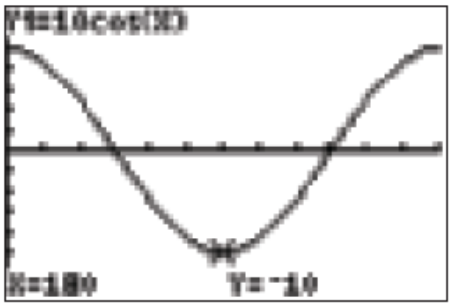


4)



A cosine function models the horizontal displacement.

5) a) amplitude = 1 period = 360 phase shift = 40 right vertical shift = 2 up max = 3 min = 1

b) amplitude = 3 period = 360 phase shift = 38 left vertical shift = 5 up max = 8 min = 2

c) amplitude = 4 period = 120 phase shift = 30 left vertical shift = 6 down max = -2 min = -10

d) amplitude = 10 period = 120 phase shift = 120 right vertical shift = 9 up max = 19 min = -1

e) amplitude = $\frac{1}{2}$ period = 120 phase shift = 120 left vertical shift = 6 down max = -5.5 min = -6.5

f) amplitude = 4 period = 1440 phase shift = 45 left vertical shift = 3 down max = 1 min = -7

6) a) amplitude = 4 period = 240 phase shift = 270 left vertical shift = 1 down max = 3 min = -5
b) see posted solutions

7) a) amplitude = $\frac{1}{2}$ period = 180 phase shift = 30 right vertical shift = 1.5 up max = 2 min = 1
b) see posted solutions

8) $y = \frac{1}{2}\sin\left[\frac{1}{2}(x + 180^\circ)\right] + 1$ $y = \frac{1}{2}\cos\left(\frac{1}{2}x\right) + 1$

9) a) $y = 2\cos[6(x - 30°)] + 1$; $y = 2\sin[6(x - 15°)] + 1$
b) $y = 4\cos[2(x - 90°)]$; $y = 4\sin[2(x - 45°)]$

10) a) amplitude = 30 period = 1 phase shift = 0.25 right vertical shift = 45 up

b) 15 **c)** see posted solutions

11) a) The period is 24 h. This model could be reasonable for a single day based on changes in human activity and the creation of pollutants.

b) a = 30, c = 25; The maximum value is 55 units and the minimum value is –5 units.

c) From the equation, d = 4, so the phase shift is 4 h right. The rising midline will occur at 4:00 a.m. The maximum occurs 6 h later, at 10:00 a.m., and the minimum occurs 12 h after the maximum, at 10:00 p.m.

d) The smog alert should be issued from 7:45 to 12:15.

e) See posted solutions

12) a) $y = 9\sin(x - 130°) + 12$ **b)** $y = 9\cos(x - 220°) + 12$

13) See posted solutions

14) a) $y = 12\sin[4(x - 15°)] + 8$ **b)** $y = 12\cos[4(x - 37.5)] + 8$

15) a) amplitude = 6 period = 12 phase shift = 2 right vertical shift = 8 up **b)** 2 m **c)** 14 m

16) a) max of 7 m, min of -7 m **b)** high tide at 5:30 am and 5:30 pm; low tide at 11:30 am and 11:30 pm
c) $y = 7\cos[30(t - 5.5)]$