Exam Review Chapter 1 – Functions
MCR3U
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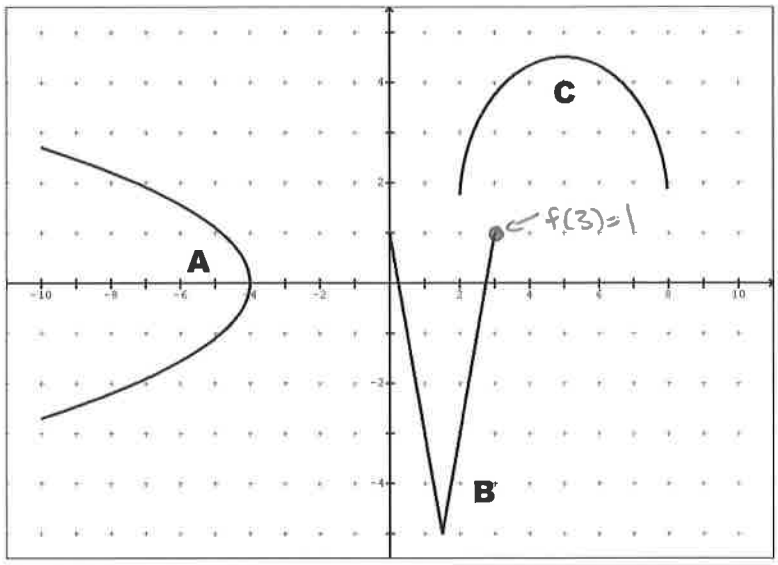
SOLUTIONS

Section 1: Functions, Domain, and Range

1)



a) List which graphs above are the graphs of functions, and which are not.

Function: B, C Not a function: A

b) Describe how you can tell whether a given graph is the graph of a function.

Vertical line test; each value of x has only 1 value of y.

c) For graph **B**, if $y = f(x)$, what is the value of $f(3)$?

$f(3) = 1$

2) State the domain and range of each relation. Is each relation a function? Justify your answer.

a) {(-6, 2), (-5, 2), (-4, 2), (-3, 2)}

D: $\{x = -6, -5, -4, -3\}$

R: $\{y = 2\}$

Is a function

b) {(5, -4), (5, -2), (5, 0), (5, 2), (5, 4)}

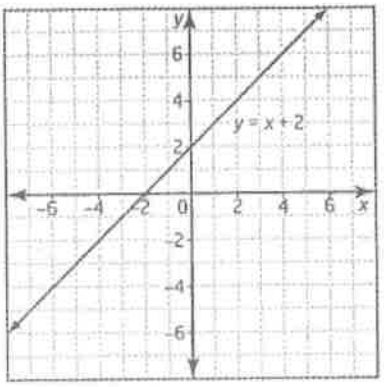
D: $\{x = 5\}$

R: $\{y = -4, -2, 0, 2, 4\}$

Not a function.

3) State the domain and range for each relation. Determine if each relation is a function.

a)

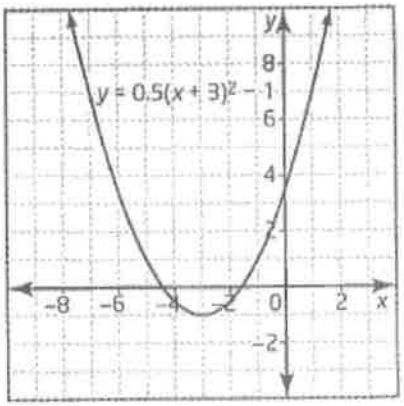


D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R}\}$

Is a function.

b)

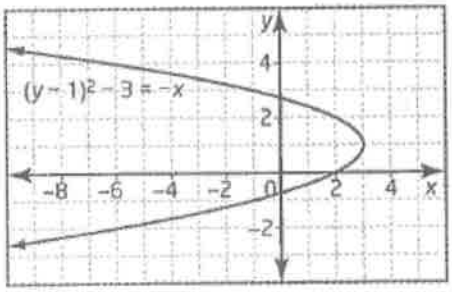


D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq -1\}$

Is a function

c)

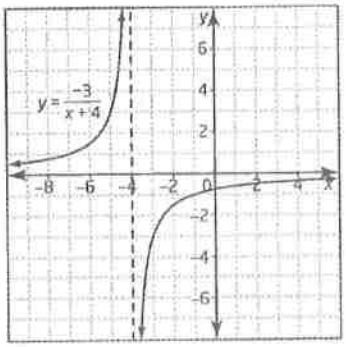


D: $\{x \in \mathbb{R} \mid x \leq 3\}$

R: $\{y \in \mathbb{R}\}$

Not a function

d)

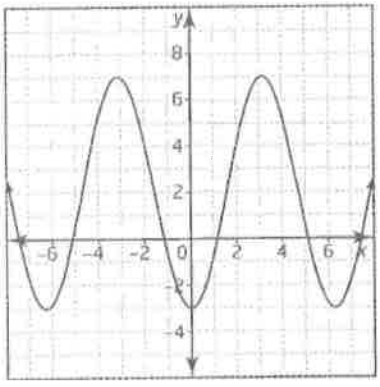


D: $\{x \in \mathbb{R} \mid x \neq -4\}$

R: $\{y \in \mathbb{R} \mid y \neq 0\}$

Is a function

e)



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid -3 \leq y \leq 7\}$

Is a function

Section 2: Function Notation

4) Suppose $f(x) = -2x^2 + 6$. Find each of the following and simplify.

a) $f(5)$

$f(5) = -2(5)^2 + 6$

$= -2(25) + 6$

$= -50 + 6$

$= -44$

b) $f(0)$

$f(0) = -2(0)^2 + 6$

$= 6$

c) $f(\frac{3}{4})$

$f(\frac{3}{4}) = -2(\frac{3}{4})^2 + 6$

$= -2(\frac{9}{16}) + 6$

$= -\frac{18}{16} + 6$

$= -\frac{9}{8} + \frac{48}{8}$

$= \frac{39}{8}$

5) For each function below, determine $f(-2), f(1), f\left(\frac{1}{2}\right)$

a) $f(x) = -\frac{3}{5}x + 2$

$f(-2) = -\frac{3}{5}(-2) + 2$

$= \frac{6}{5} + \frac{10}{5}$

$= \frac{16}{5}$

$f(1) = -\frac{3}{5}(1) + \frac{10}{5}$

$= \frac{7}{5}$

$f(\frac{1}{2}) = -\frac{3}{5}(\frac{1}{2}) + \frac{20}{10}$

$= -\frac{3}{10} + \frac{20}{10}$

$= \frac{17}{10}$

b) $f(x) = \sqrt{3 - 2x}$

$f(-2) = \sqrt{3-2(-2)}$

$= \sqrt{3+4}$

$= \sqrt{7}$

$f(1) = \sqrt{3-2(1)}$

$= \sqrt{1}$

$= 1$

$f(\frac{1}{2}) = \sqrt{3-2(\frac{1}{2})}$

$= \sqrt{2}$

Section 3: Max or Min of a Quadratic

6) Complete the square to determine the coordinates of the vertex. State if the vertex is a minimum or a maximum.

a) $f(x) = x^2 + 4x + 1$

$f(x) = (x^2+4x) + 1$

$f(x) = (x^2+4x+4-4) + 1$

$f(x) = (x^2+4x+4) - 4 + 1$

$f(x) = (x+2)^2 - 3$

Vertex $(-2,-3)$ is a min.

b) $f(x) = -2x^2 + 12x + 7$

$f(x) = (-2x^2+12x) + 7$

$f(x) = -2(x^2-6x) + 7$

$f(x) = -2(x^2-6x+9-9) + 7$

$f(x) = -2(x^2-6x+9) + 18 + 7$

$f(x) = -2(x-3)^2 + 25$

Vertex $(3,25)$ is a max.

c) $f(x) = \frac{3}{4}x^2 - 3x + 6$

$f(x) = (\frac{3}{4}x^2 - 3x) + 6$

$f(x) = \frac{3}{4}(x^2 - 4x) + 6$

$f(x) = \frac{3}{4}(x^2 - 4x + 4 - 4) + 6$ Vertex (2, 3) is min

$f(x) = \frac{3}{4}(x^2 - 4x + 4) - 3 + 6$

$f(x) = \frac{3}{4}(x-2)^2 + 3$

7) Use partial factoring to determine the vertex of each function. State if the vertex is a min or max.

a) $f(x) = 4x^2 - 8x + 1$

$1 = 4x^2 - 8x + 1$

$0 = 4x^2 - 8x$

$0 = 4x(x-2)$

$4x = 0$ $\quad x - 2 = 0$

$x = 0$ $\quad x = 2$

x-vertex $= \frac{0+2}{2}$

$= 1$

y-vertex $= 4(1)^2 - 8(1) + 1$

$= 4 - 8 + 1$

$= -3$

Vertex (1, -3) is a min

b) $f(x) = -\frac{1}{2}x^2 - 4x - 3$

$-3 = -\frac{1}{2}x^2 - 4x - 3$

$0 = -\frac{1}{2}x^2 - 4x$

$0 = -\frac{1}{2}x(x+8)$

$-\frac{1}{2}x = 0$ $\quad x + 8 = 0$

$x = 0$ $\quad x = -8$

x-vertex $= \frac{0 + (-8)}{2}$

$= -4$

y-vert $= -\frac{1}{2}(-4)^2 - 4(-4) - 3$

$= -\frac{1}{2}(16) + 16 - 3$

$= 5$

Vertex (-4, 5) is a max.

8) Convert the parabola, $y = 3x^2 + 15x - 5$ into vertex form using any method. State if the vertex is a min or max point.

x-vertex $= \frac{-b}{2a}$

$= \frac{-15}{2(3)}$

$= \frac{-15}{6}$

$= \frac{-5}{2}$

y-vertex $= 3(\frac{-5}{2})^2 + 15(\frac{-5}{2}) - 5$

$= 3(\frac{25}{4}) - \frac{75}{2} - 5$

$= \frac{75}{4} - \frac{150}{4} - \frac{20}{4}$

$= \frac{-95}{4}$

Vertex $(-\frac{5}{2}, -\frac{95}{4})$ is a min.

9) A farmer has 5000 meters of fencing to enclose a rectangular field and subdivide it into three equal plots. The enclosed area is to be a maximum. Determine the dimensions of one plot of land, to the nearest meter.

$A = x(2500 - 2x)$

$0 = x(2500 - 2x)$

$x = 0$ $\quad 2500 - 2x = 0$

$2500 = 2x$

$1250 = x$

$x\text{-vertex} = \frac{0 + 1250}{2}$

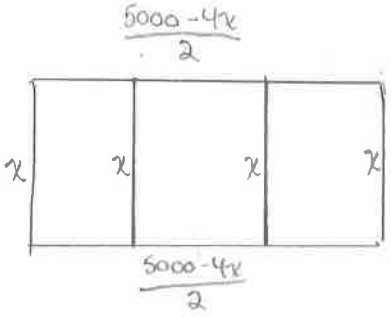
$= 625$

For 1 plot:

$\text{Length} = \frac{2500 - 2x}{3} = \frac{2500 - 2(625)}{3} \approx 417$

$\text{Width} = x = 625$

$\text{Area} = 417 \times 625 = 260\,625\ m^2$



10) The student council is organizing a trip to a rock concert. All proceeds from ticket sales will be donated to charity. Tickets to the concert cost $31.25 per person if a minimum of 104 people attend. For every 8 extra people that attend, the price will decrease by $1.25 per person.

a) How many tickets need to be sold to maximize the donation to charity?

$\text{Donation} = (\text{cost})(\#\text{sold})$

$\text{Donation} = (31.25 - 1.25x)(104 + 8x)$

$0 = (31.25 - 1.25x)(104 + 8x)$

$31.25 - 1.25x = 0$ $\quad 104 + 8x = 0$

$x = 25$ $\quad x = -13$

$x\text{-vertex} = \frac{25 + (-13)}{2}$

$= 6$

$\#\text{sold} = 104 + 8(6)$

$= 104 + 48$

$= 152$

b) What is the price of each ticket that maximizes the donation?

$\text{Price} = 31.25 - 1.25(6)$

$= 23.75$

$\$23.75$

c) What is the maximum donation?

$\text{Donation} = (\text{cost})(\#\text{sold})$

$= (23.75)(152)$

$= \$3610$

$\$3610$

11) A ball is kicked into the air. It follows a path given by $h(t) = -4.9t^2 + 8t + 0.4$, where t is the time, in seconds, and $h(t)$ is the height, in meters.

a) Determine the maximum height of the ball to the nearest tenth of a meter.

$x\text{-vertex} = \frac{-b}{2a}$

$= \frac{-8}{2(-4.9)}$

$= \frac{40}{49}$

≈ 0.8

$y\text{-vertex} = -4.9(0.8)^2 + 8(0.8) + 0.4$

$\approx 3.7\text{ m}$

b) When does the ball reach its maximum height?

0.8 seconds.

Section 4: Radicals

12) Simplify

a) $2(7\sqrt{3})$

$= 14\sqrt{3}$

b) $\sqrt{5}(3\sqrt{6})$

$= 3\sqrt{30}$

c) $-3\sqrt{3}(5\sqrt{2})$

$= -15\sqrt{6}$

13) Express each as a mixed radical in simplest form

a) $\sqrt{54}$

$= \sqrt{9 \times 6}$

$= \sqrt{9} \times \sqrt{6}$

$= 3\sqrt{6}$

b) $\sqrt{84}$

$= \sqrt{4 \times 21}$

$= \sqrt{4} \times \sqrt{21}$

$= 2\sqrt{21}$

c) $\sqrt{18}$

$= \sqrt{9 \times 2}$

$= \sqrt{9} \times \sqrt{2}$

$= 3\sqrt{2}$

d) $\sqrt{48}$

$= \sqrt{16 \times 3}$

$= \sqrt{16} \times \sqrt{3}$

$= 4\sqrt{3}$

14) Simplify each radical first, and then add or subtract

a) $5\sqrt{12} - 2\sqrt{48} - 7\sqrt{75}$

$= 5\sqrt{4}(\sqrt{3}) - 2\sqrt{16}(\sqrt{3}) - 7\sqrt{25}(\sqrt{3})$

$= 5(2)(\sqrt{3}) - 2(4)(\sqrt{3}) - 7(5)(\sqrt{3})$

$= 10\sqrt{3} - 8\sqrt{3} - 35\sqrt{3}$

$= -33\sqrt{3}$

b) $\sqrt{20} - 3\sqrt{245} - 2\sqrt{20}$

$= \sqrt{4}(\sqrt{5}) - 3\sqrt{49}(\sqrt{5}) - 2\sqrt{4}(\sqrt{5})$

$= 2\sqrt{5} - 3(7)\sqrt{5} - 2(2)\sqrt{5}$

$= 2\sqrt{5} - 21\sqrt{5} - 4\sqrt{5}$

$= -23\sqrt{5}$

c) $9\sqrt{5} + 8\sqrt{6} - 13\sqrt{5} + 19\sqrt{6} + 4\sqrt{6}$

$= 9\sqrt{5} - 13\sqrt{5} + 8\sqrt{6} + 19\sqrt{6} + 4\sqrt{6}$

$= -4\sqrt{5} + 31\sqrt{6}$

d) $2\sqrt{12} + 4\sqrt{20} - 3\sqrt{27} - 5\sqrt{45}$

$= 2\sqrt{4}(\sqrt{3}) + 4\sqrt{4}(\sqrt{5}) - 3\sqrt{9}(\sqrt{3}) - 5\sqrt{9}(\sqrt{5})$

$= 4\sqrt{3} + 8\sqrt{5} - 9\sqrt{3} - 15\sqrt{5}$

$= 4\sqrt{3} - 9\sqrt{3} + 8\sqrt{5} - 15\sqrt{5}$

$= -5\sqrt{3} - 7\sqrt{5}$

15) Expand. Simplify where possible.

a) $\sqrt{2}(\sqrt{6} - \sqrt{3})$

$= \sqrt{12} - \sqrt{6}$

$= \sqrt{4}(\sqrt{3}) - \sqrt{6}$

$= 2\sqrt{3} - \sqrt{6}$

b) $6\sqrt{6}(3\sqrt{2} - 4\sqrt{3})$

$= 18\sqrt{12} - 24\sqrt{18}$

$= 18\sqrt{4}(\sqrt{3}) - 24\sqrt{9}(\sqrt{2})$

$= 36\sqrt{3} - 72\sqrt{2}$

c) $(\sqrt{7} - 6)(\sqrt{7} + 1)$

$= 7 + \sqrt{7} - 6\sqrt{7} - 6$

$= 1 - 5\sqrt{7}$

d) $(3\sqrt{5} - 2\sqrt{3})(3\sqrt{5} + 2\sqrt{3})$ D.O.S.

$= (3\sqrt{5})^2 - (2\sqrt{3})^2$

$= 9(5) - 4(3)$

$= 45 - 12$

$= 33$

Section 5: Solving Quadratics

16) Solve each quadratic by factoring

a) $f(x) = x^2 + 7x + 12$ S: 7 P: 12 (3 and 4)

$0 = (x+3)(x+4)$

$x+3=0$ $x+4=0$

$x_1 = -3$ $x_2 = -4$

b) $f(x) = 3x^2 - 4x - 15$ S: -4 P: -45 (-9 and 5)

$0 = 3x^2 - 9x + 5x - 15$

$0 = 3x(x-3) + 5(x-3)$

$0 = (x-3)(3x+5)$

$x-3=0$ $3x+5=0$

$x_1 = 3$ $x_2 = -\frac{5}{3}$

17) Solve each quadratic using the quadratic formula

a) $f(x) = 3x^2 + 6x + 1$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(3)(1)}}{2(3)}$

$x = \frac{-6 \pm \sqrt{24}}{6}$

$x = \frac{-6 \pm 2\sqrt{6}}{6}$

$x = \frac{2(-3 \pm \sqrt{6})}{6}$

$x_1 = \frac{-3+\sqrt{6}}{3}$ $x_2 = \frac{-3-\sqrt{6}}{3}$

b) $f(x) = x^2 + 6x + 4$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{20}}{2}$

$x = \frac{-6 \pm 2\sqrt{5}}{2}$

$x = \frac{2(-3 \pm \sqrt{5})}{2}$

$x_1 = -3 + \sqrt{5}$ $x_2 = -3 - \sqrt{5}$

18) Use the discriminant to determine the number of roots for each quadratic equation

a) $f(x) = x^2 - 3x + 1$

$b^2 - 4ac = (-3)^2 - 4(1)(1)$

$= 5$

∴ 2 roots

b) $f(x) = 2x^2 - 5x + 7$

$b^2 - 4ac = (-5)^2 - 4(2)(7)$

$= -31$

∴ No roots

c) $f(x) = 4x^2 + 24x + 36$

$b^2 - 4ac = (24)^2 - 4(4)(36)$

$= 0$

∴ 1 root.

Section 6: Linear-Quadratic Systems

19) Determine algebraically the coordinates of the points of intersection of each pair of functions.

a) $y = x^2 + 4x + 3$ and $y = 5x + 9$

$5x+9 = x^2+4x+3$

$0 = x^2 - x - 6$ S: -1, P: -6 (-3 and 2)

$0 = (x-3)(x+2)$

$x-3=0$ $x+2=0$

$x_1 = 3$ $x_2 = -2$

POI #1: $y = 5(3)+9$

$= 15+9$

$= 24$ $(3, 24)$

POI #2: $y = 5(-2)+9$

$= -10+9$

$= -1$ $(-2, -1)$

b) $y = -x^2 - 4x + 6$ and $y = x - 8$

$x-8 = -x^2-4x+6$

$0 = -x^2 - 5x + 14$

$0 = x^2 + 5x - 14$ S: 5, P: -14 (7 and -2)

$0 = (x+7)(x-2)$

$x+7=0$ $x-2=0$

$x_1 = -7$ $x_2 = 2$

POI #1: $y = -7-8$

$= -15$ $(-7, -15)$

POI #2: $y = 2-8$

$y = -6$ $(2, -6)$

20) Given the equation of a parabola and the slope of a line that is tangent to the parabola, determine the y-intercept of the tangent line.

$f(x) = -3x^2 + x - 4$, tangent line has slope 13 $y = 13x + k$

$13x + k = -3x^2 + x - 4$

$0 = -3x^2 - 12x - 4 - k$

Tangent has 1 POI; so $b^2 - 4ac = 0$

$0 = b^2 - 4ac$

$0 = (-12)^2 - 4(-3)(-4-k)$

$0 = 144 + 12(-4-k)$

$0 = 144 - 48 - 12k$

$0 = 96 - 12k$

$12k = 96$

$k = 8$

∴ The y-int is at 8

Answers

1) a) B&C are functions, A is not a function **b)** vertical line test: each value of x has only 1 value of y **c)** $f(3) = 1$

2) a) D: $\{x = -6, -5, -4, -3\}$; R: $\{y = 2\}$; is a function **b)** D: $\{x = 5\}$; R: $\{y = -4, -2, 0, 2, 4\}$; not a function

3) a) D: $\{x \in \mathbb{R}\}$; R: $\{y \in \mathbb{R}\}$; is a function **b)** D: $\{x \in \mathbb{R}\}$; R: $\{y \in \mathbb{R} | y \geq -1\}$, is a function

c) D: $\{x \in \mathbb{R} | x \leq 3\}$; R: $\{y \in \mathbb{R}\}$; not a function **d)** D: $\{x \in \mathbb{R} | x \neq -4\}$; R: $\{y \in \mathbb{R} | y \neq 0\}$; is a function

e) D: $\{x \in \mathbb{R}\}$; R: $\{y \in \mathbb{R} | -3 \leq y \leq 7\}$; is a function

4) a) -44 **b)** 6 **c)** $\frac{39}{8}$

5) a) $\frac{16}{5}, \frac{7}{5}, \frac{17}{10}$ **b)** $\sqrt{7}, 1, \sqrt{2}$

6) a) (-2, -3) min **b)** (3, 25) max **c)** (2, 3) min

7) a) (1, -3) min **b)** (-4, 5) max

8) $\left(-\frac{5}{2}, -\frac{95}{4}\right)$ min

9) 260 625 m²

10) a) 152 tickets **b)** \$23.75 **c)** \$3610

11) a) 3.7 m **b)** 0.8 s

12) a) $14\sqrt{3}$ **b)** $3\sqrt{30}$ **c)** $-15\sqrt{6}$

13) a) $3\sqrt{6}$ **b)** $2\sqrt{21}$ **c)** $3\sqrt{2}$ **d)** $4\sqrt{3}$

14) a) $-33\sqrt{3}$ **b)** $-23\sqrt{5}$ **c)** $-4\sqrt{5} + 31\sqrt{6}$ **d)** $-5\sqrt{3} - 7\sqrt{5}$

15) a) $2\sqrt{3} - \sqrt{6}$ **b)** $36\sqrt{3} - 72\sqrt{2}$ **c)** $1 - 5\sqrt{7}$ **d)** 33

16) a) -3 and -4 **b)** $-\frac{5}{3}$ and 3

17) a) $x = \frac{-3 \pm \sqrt{6}}{3}$ **b)** $x = -3 \pm \sqrt{5}$

18) a) 2 **b)** none **c)** 1

19) a) (3, 24), (-2, -1) **b)** (-7, -15), (2, -6)

20) 8