

# Exam Review Part 2b - Transformations of Functions

MCR3U

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SOLUTIONS

## Section 1: Transformations of $f(x) = x^2$ , $f(x) = \sqrt{x}$ , and $f(x) = \frac{1}{x}$

1) Below is the graph of  $f(x) = x^2$ . Describe the transformations to a) and b) and use transformations to graph them.

a)  $f(x) = -(x+6)^2 + 4$

- vertical reflection ( $-y$ )
- shift left 6 ( $x-6$ )
- shift up 4 ( $y+4$ )

$f(x) = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

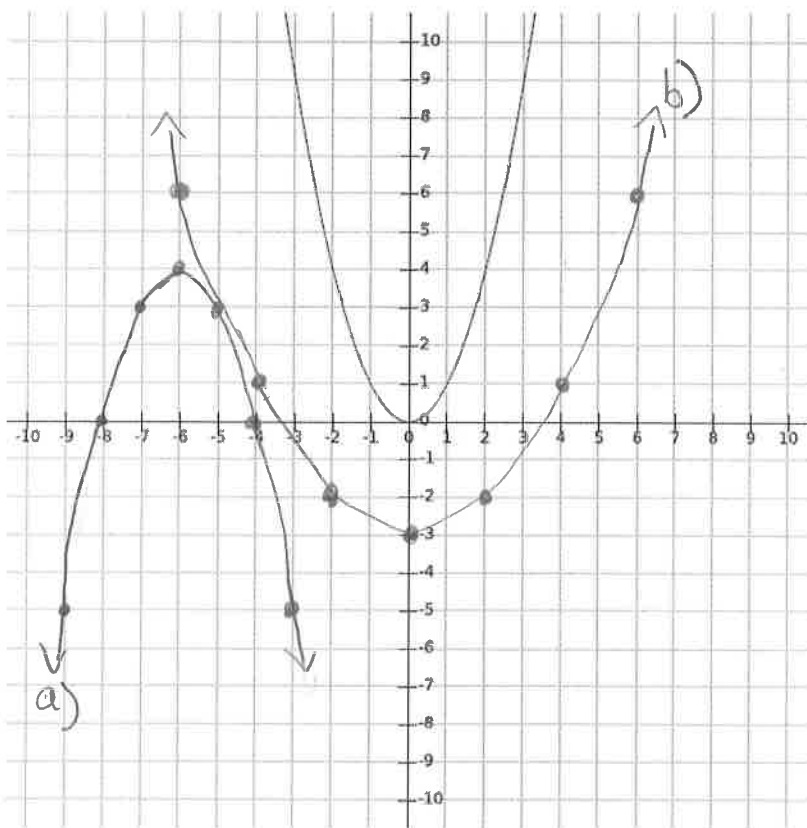
$f(x) = -(x+6)^2 + 4$

$x-6$	$-y+4$
-9	-5
-8	0
-7	3
-6	4
-5	3
-4	0
-3	-5

b)  $f(x) = \left(-\frac{1}{2}x\right)^2 - 3$

- horizontal reflection ( $-x$ )
- horizontal stretch b.a.f.o. 2 ( $2x$ )
- shift down 3 ( $y-3$ )

$-2x$	$y-3$
6	6
4	1
2	-2
0	-3
-2	-2
-4	1
-6	6



$$g(x) = a f[k(x-d)] + c$$

2) For the function  $f(x) = \sqrt{x}$ , write the new function equation for each transformation.

a) translation up 4 and right 9.

$$c=4$$

$$d=9$$

$$g(x) = \sqrt{x-9} + 4$$

b) vertical stretch by 6 and translation left 5.

$$a=6$$

$$d=-5$$

$$g(x) = 6\sqrt{x+5}$$

c) horizontal reflection in the y-axis and horizontal compression by  $\frac{1}{4}$ .

$$k=-4$$

$$g(x) = \sqrt{-4x}$$

3) Write the new function for the following description, given that the transformations are applied to the parent function  $f(x) = x^2$ .

Vertical stretch by 2, horizontal stretch by 3, vertical reflection over the x-axis, a vertical translation 2 units up and a horizontal translation 6 units left.

$$a=-2$$

$$k=\frac{1}{3}$$

$$d=-6$$

$$c=2$$

$$g(x) = -2 \left[ \frac{1}{3}(x+6) \right]^2 + 2$$

4) List all the transformations, in words, of  $f'(x)$  for each of the following functions.

a)  $g(x) = -f(x-3) - 4$

- vertical reflection

- shift right 3

- shift down 4

b)  $h(x) = -\frac{1}{3}f(2x) + 10$

- vertical reflection

- vertical compression by  $\frac{1}{3}$

- horizontal compression by  $\frac{1}{2}$

- shift up 10 units.

c)  $j(x) = 5f(x+4) - 5$

- vertical stretch by 5

- shift left 4

- shift down 5

d)  $k(x) = -2f\left(-\frac{1}{6}x\right) + 6$

- vertical reflection

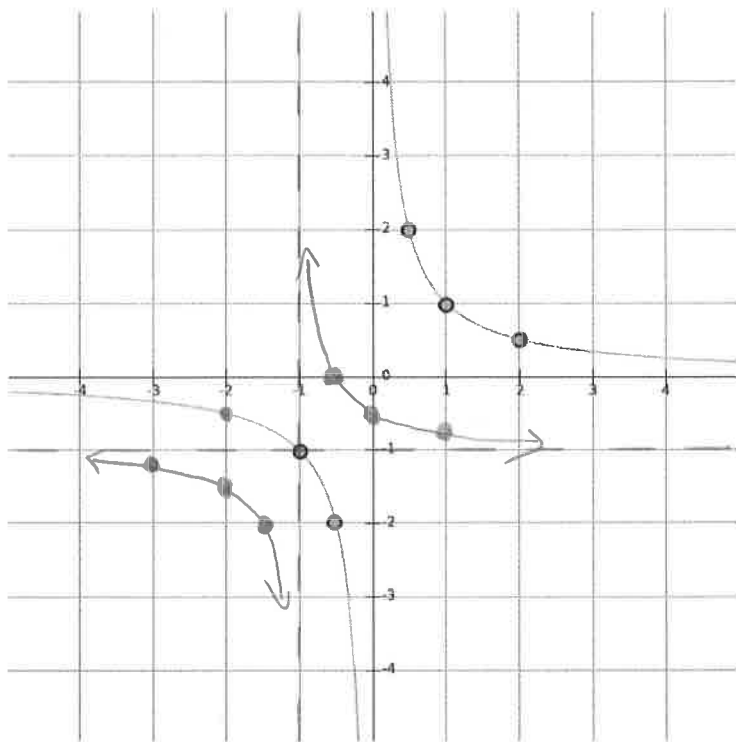
- vertical stretch by 2

- horizontal reflection

- horizontal stretch by 6

- shift up 6.

5) Graph  $g(x) = \frac{1}{2}f(x+1) - 1$  using transformations to the function  $f(x) = \frac{1}{x}$  that is shown.

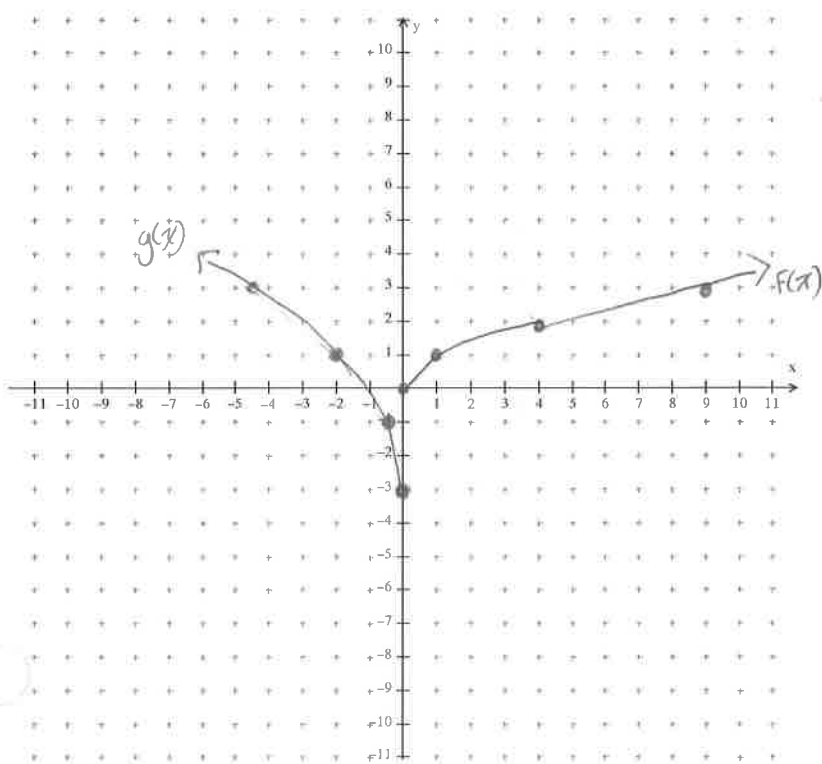


$f(x)$	
$x$	$y$
-2	-0.5
-1	-1
-0.5	-2
0	undefined
0.5	2
1	1
2	0.5

$g(x)$	
$x-1$	$\frac{y}{2} - 1$
-3	-1.25
-2	-1.5
-1.5	-2
-1	undefined
-0.5	0
0	-0.5
1	-0.75

Horizontal asymptote at  $y=c \Rightarrow y = -1$   
 Vertical asymptote at  $x=d \Rightarrow x = -1$

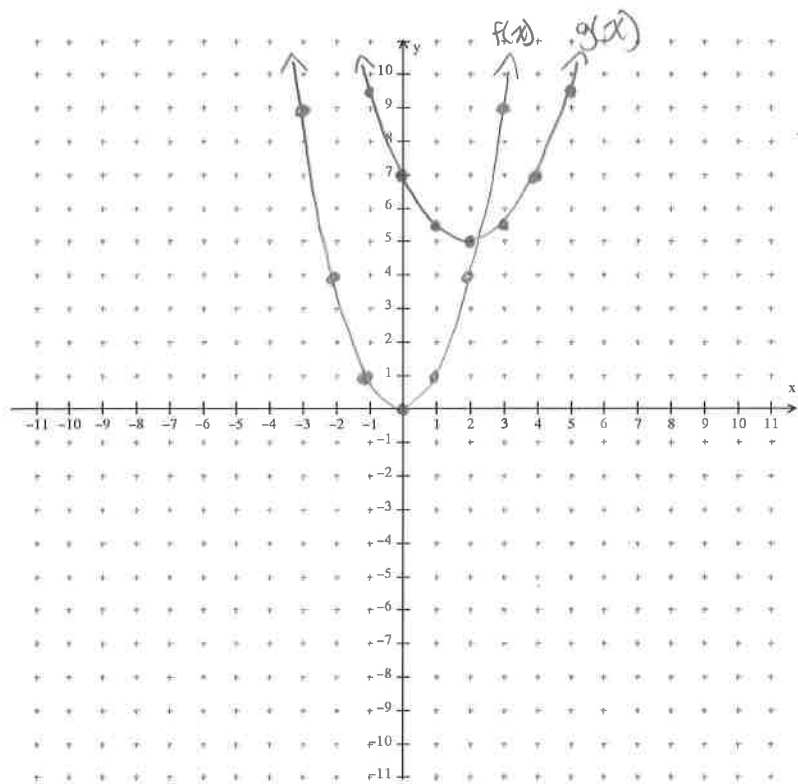
6)  $f(x) = \sqrt{x}$ . Graph  $g(x) = 2f(-2x) - 3$  using transformations.



$f(x)$	
$x$	$y$
0	0
1	1
4	2
9	3

$g(x)$	
$\frac{x}{-2}$	$2y-3$
0	-3
-0.5	-1
-2	1
-4.5	3

7) Graph the parent function of  $g(x) = \frac{1}{2}(x-2)^2 + 5$  and  $f(x)$  using transformations.



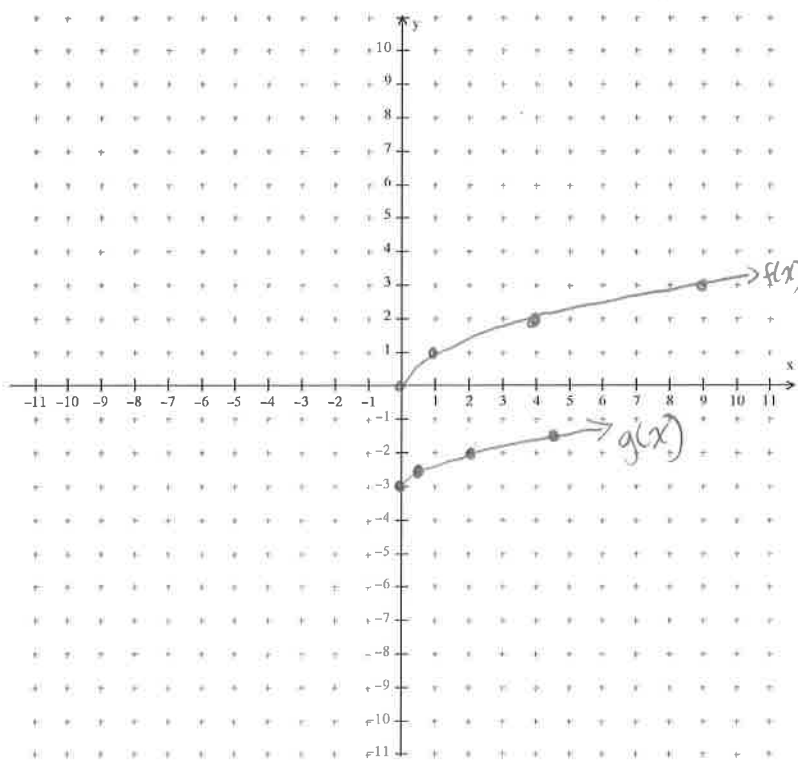
$$f(x) = x^2$$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$$g(x) = \frac{1}{2}(x-2)^2 + 5$$

x+2	$\frac{y}{2} + 5$
-1	9.5
0	7
1	5.5
2	5
3	5.5
4	7
5	9.5

8) Graph the parent function of  $h(x) = \frac{1}{2}\sqrt{2x} - 3$  and  $f(x)$  using transformations.



$$f(x) = \sqrt{x}$$

x	y
0	0
1	1
4	2
9	3

$$g(x) = \frac{1}{2}\sqrt{2x} - 3$$

$\frac{x}{2}$	$\frac{y}{2} - 3$
0	-3
0.5	-2.5
2	-2
4.5	-1.5

## Section 2: Inverse of a Function

9) For each function listed below, determine the equation of the inverse,  $f^{-1}(x)$ .

a)  $f(x) = 3x + 9$

$$y = 3x + 9$$

$$x = 3y + 9$$

$$x - 9 = 3y$$

$$\frac{x-9}{3} = y$$

$$f^{-1}(x) = \frac{x-9}{3}$$

b)  $f(x) = \frac{1}{3}x^2 - 4$

$$y = \frac{1}{3}x^2 - 4$$

$$x = \frac{1}{3}y^2 - 4$$

$$x + 4 = \frac{1}{3}y^2$$

$$3(x+4) = y^2$$

$$\pm \sqrt{3(x+4)} = y$$

$$f^{-1}(x) = \pm \sqrt{3(x+4)}$$

10) Determine the equation of the inverse of  $f(x) = 2x^2 + 16x + 30$  by first completing the square.

$$f(x) = 2(x^2 + 8x) + 30$$

$$f(x) = 2(x^2 + 8x + 16 - 16) + 30$$

$$f(x) = 2(x^2 + 8x + 16) - 32 + 30$$

$$f(x) = 2(x+4)^2 - 2$$

$$f(x) = 2(x+4)^2 - 2$$

$$y = 2(x+4)^2 - 2$$

$$x = 2(y+4)^2 - 2$$

$$\frac{x+2}{2} = (y+4)^2$$

$$\pm \sqrt{\frac{x+2}{2}} = y+4$$

$$-4 \pm \sqrt{\frac{x+2}{2}} = y$$

11) Calculate the inverse of  $f(x) = 2(x-1)^2 + 2$ .

$$y = 2(x-1)^2 + 2$$

$$x = 2(y-1)^2 + 2$$

$$\frac{x-2}{2} = (y-1)^2$$

$$\pm \sqrt{\frac{x-2}{2}} = y-1$$

$$1 \pm \sqrt{\frac{x-2}{2}} = y$$

$$f^{-1}(x) = 1 \pm \sqrt{\frac{x-2}{2}}$$

$$f^{-1}(x) = -4 \pm \sqrt{\frac{x+2}{2}}$$

12) Graph  $f(x)$  from the previous question and its inverse below.

$$y = x^2$$

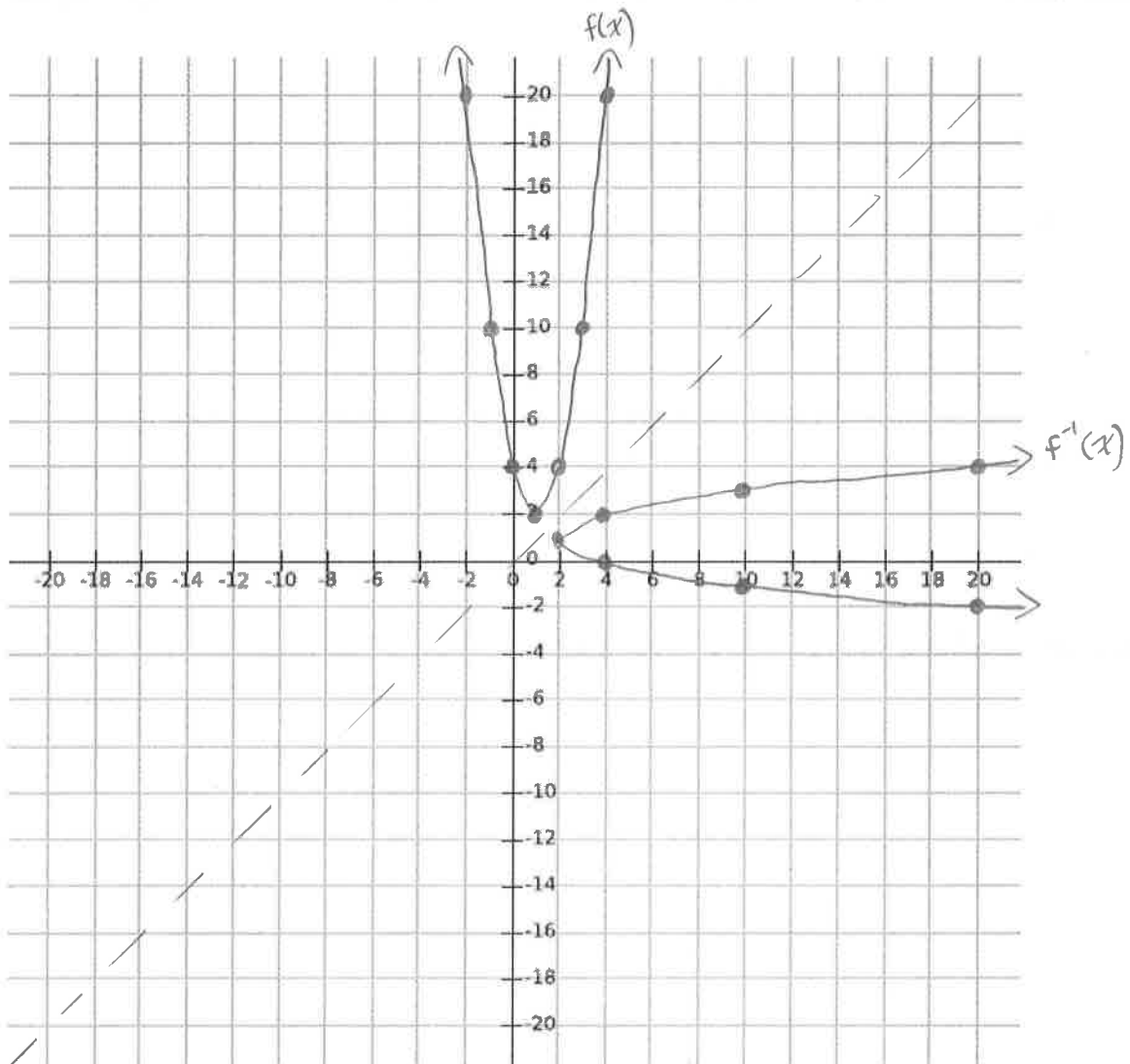
x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$$f(x) = 2(x-1)^2 + 2$$

x+1	2y+2
-2	20
-1	10
0	4
1	2
2	4
3	10
4	20

$$f^{-1}(x) = 1 \pm \sqrt{\frac{x-2}{2}}$$

x	y
20	-2
10	-1
4	0
2	1
4	2
10	3
20	4



## Answers

1) see posted solutions

2) a)  $g(x) = \sqrt{x-9} + 4$  b)  $g(x) = 6\sqrt{x+5}$  c)  $g(x) = \sqrt{-4x}$

3)  $g(x) = -2 \left| \frac{1}{3}(x+6) \right|^2 + 2$

4) a) vertical reflection, right 3 units, down 4 units

b) vertical reflection, vertical compression bafo  $\frac{1}{3}$ , horizontal compression bafo  $\frac{1}{2}$ , up 10 units

c) vertical stretch bafo 5, left 4 units, down 5 units

d) vertical reflection, vertical stretch bafo 2, horizontal reflection, horizontal stretch bafo 6, up 6 units

5) through 8) check posted solutions

9) a)  $f^{-1}(x) = \frac{x-9}{3}$  b)  $f^{-1}(x) = \pm \sqrt{3(x+4)}$

10)  $f^{-1}(x) = -4 \pm \sqrt{\frac{x+2}{2}}$

11)  $f^{-1}(x) = 1 \pm \sqrt{\frac{x-2}{2}}$

12) See posted solutions

