$y = a(b)^x$ $y = a(1+r)^x$ $y = a(1-r)^x$ $A = P(1+i)^n$

Exam Review Part 3 – Exponential Functions

MCR3U

Jensen

SOLUTIONS

Section 1: Exponential Growth

1) An insect colony as an initial population of 15. The number of insects quadruples every day.

a) Determine a function that models this exponential growth.

$a = 15$

$b = 4$

$P(n) = 15(4)^n$

b) How many insects will be present in 1 week?

$P(7) = 15(4)^7$

$= 245\,760$

$\boxed{243\,760}$

2) If the population of an ant colony is 213 and it doubles every week,

a) What will the population be in 4 weeks?

$y = 213(2)^x$

$y = 213(2)^4$

$y = 3408$

$\boxed{3\,408}$

b) How long will it take the population to reach 109 056 ants?

$109\,056 = 213(2)^x$

$512 = 2^x$

$2^9 = 2^x$

$9 = x$

$\boxed{\frac{\log 512}{\log 2} = 9}$

$\boxed{9 \text{ weeks}}$

3) The population of a town in the Northwest Territories starts off at 20,000 and grows by 13% each year. Find the populations after 10 years.

$y = 20\,000(1.13)^x$

$y = 20\,000(1.13)^{10}$

$y = 67891.35$

$\boxed{67\,891}$

4) There are 50 bacteria present initially in a culture. In 3min., the count is 204800.
What is the doubling period?

$y = 50(2)^x$

$204800 = 50(2)^{3/t}$

$4096 = 2^{3/t}$

$2^{12} = 2^{3/t}$

$12 = \frac{3}{t}$

$t = \frac{3}{12}$

$t = \frac{1}{4}$

$\boxed{\text{The doubling period is 0.25 minutes OR 15 seconds.}}$

$a \quad b = 2 \quad x = t/4$

5) A bacteria culture starts with a population of 12 000 and doubles every four hours.

a) How many bacteria are present after 12 hours?

$y = 12000(2)^{t/4}$

$y = 12000(2)^{12/4}$

$y = 96\,000$

$\boxed{96\,000}$

b) How many bacteria are present after 1 day?

$y = 12000(2)^{24/4}$

$y = 12000(2)^6$

$y = 768\,000$

$\boxed{768\,000}$

c) How long will it take for the population of the bacteria to reach 49 152 000?

$49\,152\,000 = 12000(2)^{t/4}$

$4096 = 2^{t/4}$

$\frac{\log 4096}{\log 2} = 12$

$2^{12} = 2^{t/4}$

$12 = \frac{t}{4}$

$48 = t$

$\boxed{\text{48 hours OR 2 days}}$

$b=2$ $x=t/15$ a

6) A bacteria culture doubles every 15 minutes. There were 20 individuals initially.

a) How many bacteria will be present after 3 hours

$=180$ minutes

$y=20(2)^{t/15}$

$y=20(2)^{180/15}$

$y=81920$

b) How long will it take to grow a population of 163 840?

$163\,840=20(2)^{t/15}$

$\frac{\log 8192}{\log 2}=13$

$8192=2^{t/15}$

$2^{13}=2^{t/15}$

$13=\frac{t}{15}$

$t=195$

$\boxed{195 \text{ minutes}}$

Section 2: Exponential Decay

) In 1976, a research hospital bought half of a gram of radium for cancer research. Assuming the hospital still exists, how much of this radium will the hospital have in the year 6836, if the half-life of the radium is 1620 years?

$a=0.5$

$b=\frac{1}{2}$

$x=t/1620$

$y=0.5\left(\frac{1}{2}\right)^{t/1620}$

$y=0.5\left(\frac{1}{2}\right)^{4860/1620}$

$y=0.0625$

$\boxed{0.0625 \text{ grams}}$

$b=\frac{1}{2}$ $x=t/20$ a

8) Polonium-210 is a radioactive isotope that has a half-life of 20 days. Suppose you start with a 40-mg sample.

a) Write an equation that relates the amount of polonium-210 remaining and time.

$f(t)=40\left(\frac{1}{2}\right)^{t/20}$

b) How much polonium-210 will remain after 10 weeks?

= 70 days

$f(70) = 40\left(\frac{1}{2}\right)^{70/20}$

$= 3.54$

$\boxed{3.54 \text{ mg}}$

c) How long will it take for the amount of polonium-210 to decay to 8% of its initial mass?

$0.08(40) = 40\left(\frac{1}{2}\right)^{t/20}$

$0.08 = \left(\frac{1}{2}\right)^{t/20}$

$\log 0.08 = \left(\frac{t}{20}\right)\log\left(\frac{1}{2}\right)$

$\frac{\log 0.08}{\log 0.5} = \frac{t}{20}$

$3.643856 = \frac{t}{20}$

$t = 72.88$

$\boxed{\text{About 73 days}}$

9) A cup of coffee contains approximately 96 mg of caffeine. When you drink the coffee, the caffeine is absorbed into the bloodstream and is eventually metabolized by the body. Every 5 hours the amount of caffeine present in the body is reduced by one-half. How many hours does it take for the amount of caffeine to be reduced to 12 mg?

$a = 96$

$b = \frac{1}{2}$

$x = t/5$

$y = 96\left(\frac{1}{2}\right)^{t/5}$

$12 = 96\left(\frac{1}{2}\right)^{t/5}$

$\frac{1}{8} = \left(\frac{1}{2}\right)^{t/5}$

$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/5}$

$3 = \frac{t}{5}$

$t = 15$

$\boxed{\text{15 hours}}$

10) Daniel is very excited about his new motorcycle. Although the motorcycle costs $13 500, its resale value will depreciate by 20% of its current value every year.

a) How much will the motorcycle be worth in 6 years?

$y = 13500(1-0.2)^x$

$y = 13500(0.8)^6$

$y = 3538.94$

$\boxed{\$3538.94}$

b) How long will it take for Daniel's motorcycle to depreciate to 50% of its original cost?

$0.5(13500) = 13500(0.8)^x$

$0.5 = 0.8^x$

$\frac{\log 0.5}{\log 0.8} = x$

$x = 3.1$

$\boxed{\text{3.1 years}}$

Section 3: Interest

.1) An investment opportunity is found that makes 7% per year compounded annually. How much should you invest now if you need $13, 450 at the end of 9 years?

$13450 = P(1.07)^9$

$P = \frac{13450}{(1.07)^9}$

$\boxed{\$7315.91}$

$P = 7315.91$

12) Jacqueline deposits an inheritance of $1500 into an account that earns interest of 3.5% per year, compounded annually.

a) How much is in the account after 8 years?

$A = 1500(1.035)^8$

$A = 1975.21$

$\boxed{\$1975.21}$

b) How long will it take for the money to double (round to the nearest year)?

$2(1500) = 1500(1.035)^n$

$2 = (1.035)^n$

$\frac{\log 2}{\log 1.035} = n$

$\boxed{\text{About 20 years}}$

$n = 20.1488$

13) Five years ago, Denise deposited an amount into an account that pays 7.5% per year, compounded annually. Today the account balance is $4200.

a) What was the amount of Denise's initial deposit?

$4200 = P(1.075)^5$

$P = \frac{4200}{(1.075)^5}$

$\boxed{\$2925.55}$

$P = 2925.55$

b) How much was in the account 2 years ago?

$4200 = P(1.075)^2$

$P = \frac{4200}{(1.075)^2}$

$P = 3634.40$

$\boxed{\$3634.40}$

c) How much will be in the account 2 years from now?

$A = 4200(1.075)^2$

$A = 4853.63$

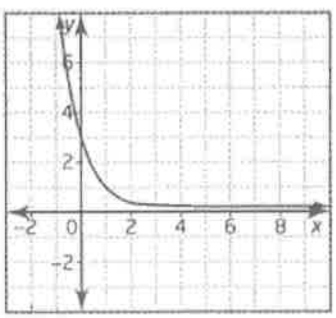
$\boxed{\$4853.63}$

Section 4: Properties of Exponential Functions and Transformations

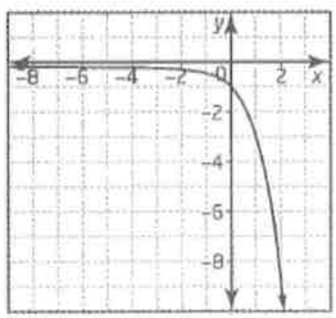
14) Match each graph with its corresponding equation

A $y = 3(3^x)$ **B** $y = 3\left(\frac{1}{3}\right)^x$ **C** $\frac{1}{3}(3^x)$ **D** $y = -3^x$

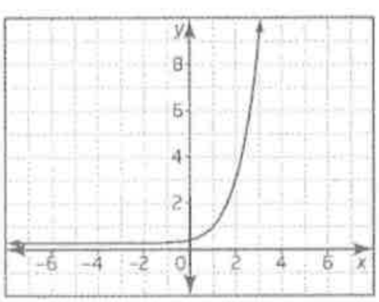
a) B



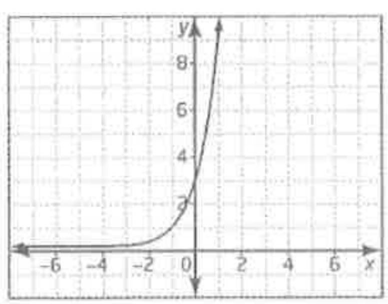
b) D



c) C

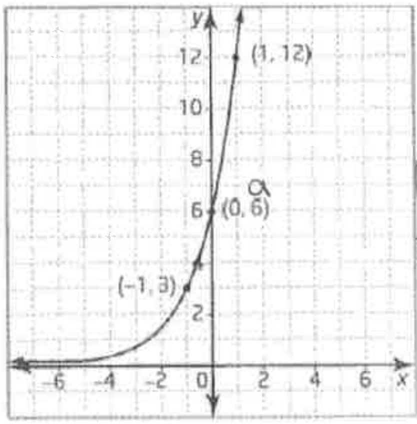


d) A



15) Write an exponential equation to match each graph.

a)

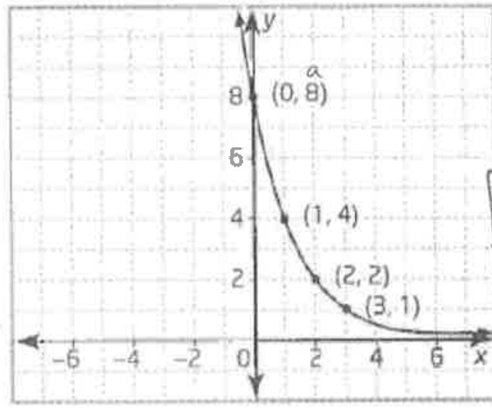


$a = 6$

$b = \frac{12}{6} = 2$

$\boxed{y = 6(2)^x}$

b)



$a = 8$

$b = \frac{4}{8} = \frac{1}{2}$

$\boxed{y = 8\left(\frac{1}{2}\right)^x}$

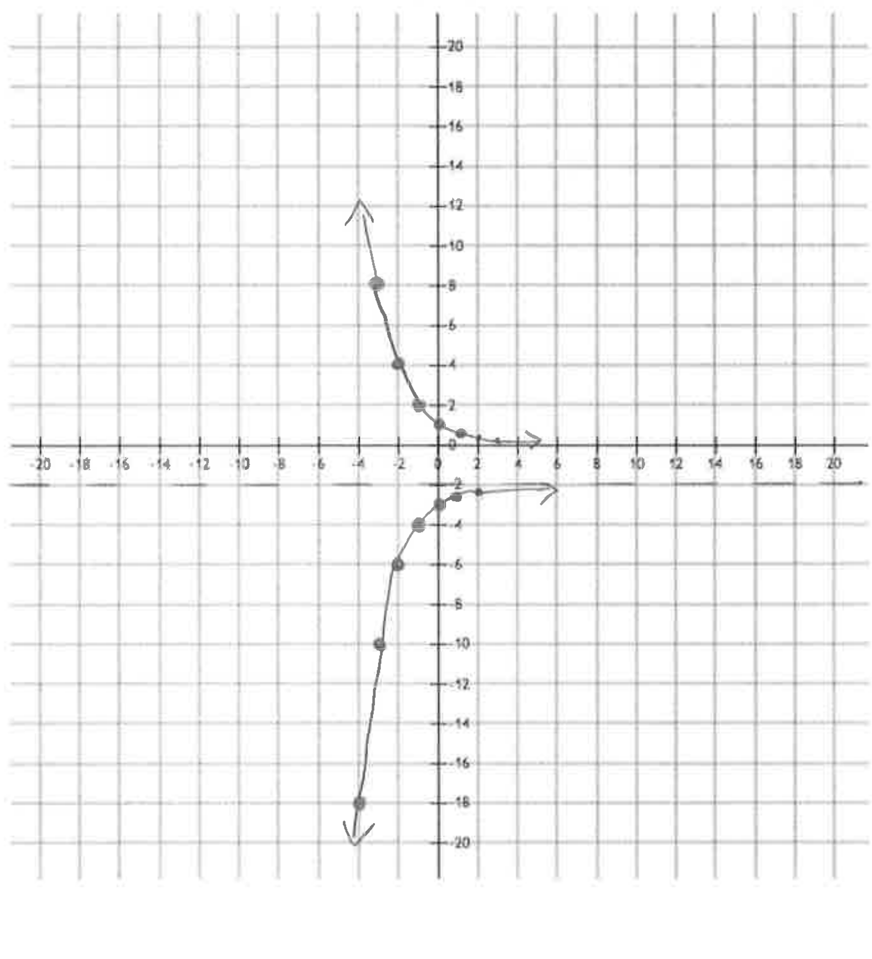
16) Sketch the graph base graph of $f(x) = -2\left(\frac{1}{2}\right)^{x+1}$ -2 and use transformations to graph f(x). Is this an increasing or decreasing function?

$y = \left(\frac{1}{2}\right)^x$

$f(x) = -2\left(\frac{1}{2}\right)^{x+1} - 2$

x	y
-3	8
-2	4
-1	2
0	1
1	0.5
2	0.25
3	0.125

$x-1$	$-2y-2$
-4	-18
-3	-10
-2	-6
-1	-4
0	-3
1	-2.5
2	-2.25



Answers

1) a) $P(n) = 15\times4^n$ **b)** 245 760

2) a) 3408 **b)** 9 weeks

3) 67 891

4) 15 seconds

5) a) 96 000 **b)** 768 000 **c)** 2 days

6) a) 81 920 **b)** 195 minutes

7) 0.0625 g

8) a) $f(t) = 40\left(\frac{1}{2}\right)^{\frac{t}{20}}$ **b)** 3.54 mg **c)** approximately 73 days

9) 15 hours

10) a) \$3538.94 **b)** 3.1 years

11) \$7315.91

12) a) \$1975.21 **b)** approximately 20 years

13) a) \$2925.55 **b)** \$3634.40 **c)** \$4853.63

14) a) B **b)** D **c)** C **d)** A

15) a) $y = 6(2^x)$ **b)** $y = 8\left(\frac{1}{2}\right)^x$

16) See posted solutions