Exam Review Part 4 – Discrete Functions

MCR3U

Jensen

SOLUTIONS

1) Find the formula for the general term t_n and then use it to calculate t_{12} for each of the following sequences:

a) 9, 15, 21, … $\quad$ +6

$a=9$

$d=6$

$t_n = 9+(n-1)(6)$

$t_{12} = 9+(12-1)(6)$

$t_{12} = 75$

b) −1, 2, −4, 8, … $\quad \times(-2)$

$a=-1$

$r=-2$

$t_n = -1(-2)^{n-1}$

$t_{12} = -1(-2)^{12-1}$

$t_{12} = 2048$

2) Determine the general term for each of these sequences. Are they arithmetic, geometric or neither?

a) 1, 4, 7, 10, 13 $\quad$ +3 $\quad$ arithmetic

$a=1$

$d=3$

$t_n = 1+(n-1)(3)$

b) 2187, 729, 243, 81, 27 $\quad \times \frac{1}{3}$ $\quad$ geometric.

$a=2187$

$r=\frac{1}{3}$

$t_n = 2187\left(\frac{1}{3}\right)^{n-1}$

3) For those sequences which are arithmetic or geometric in question 2:

i) determine the value of the 10th term, t_{10}

a) $t_{10} = 1+(10-1)(3)$

$t_{10} = 28$

b) $t_{10} = 2187\left(\frac{1}{3}\right)^{10-1}$

$t_{10} = \frac{1}{9}$

ii) determine the sum of the series up to the 12th term, S_{12}.

a) $S_{12} = \frac{12}{2}\left[2(1)+(12-1)(3)\right]$

$= 6(35)$

$= 210$

b) $S_{12} = \dfrac{2187\left[\left(\frac{1}{3}\right)^{12}-1\right]}{\frac{1}{3}-1}$

$= \dfrac{\left(\frac{-531440}{243}\right)}{\left(\frac{-2}{3}\right)} = \dfrac{265720}{81}$

4) In an arithmetic series of 50 terms, the 17th term is 53 and the 28th term is 86. Determine, a, d and S_{50}.

$t_{17} = 53$

$53 = a + 16d$

$t_{28} = 86$

$86 = a + 27d$

$S_{50} = \frac{50}{2}\left[2(5) + (50-1)(3)\right]$

$= 25(157)$

$= 3925$

$86 = a + 27d$
$53 = a + 16d$ −

$33 = 11d$

$d = 3$

$86 = a + 27(3)$

$86 = a + 81$

$a = 5$

5) In an arithmetic series, the 12th term is 15 and the sum of the first 15 terms is 105. Determine the sum of the first three terms in the series.

$t_{12} = 15$

$15 = a + 11d$

$S_{15} = 105$

$105 = \frac{15}{2}\left[2a + 14d\right]$

$105 = 15(a + 7d)$

$7 = a + 7d$

$S_3 = \frac{3}{2}\left[2(-7) + (3-1)(2)\right]$

$S_3 = -15$

$15 = a + 11d$
$7 = a + 7d$ −

$8 = 4d$

$d = 2$

$7 = a + 7(2)$

$7 = a + 14$

$a = -7$

6) The fifth term of a geometric series is 405 and the sixth term is 1215. Find the sum of the first nine terms.

$t_5 = 405$

① $405 = a(r)^4$

$a = \frac{405}{r^4}$

$t_6 = 1215$

② $1215 = a(r)^5$

sub 'a' from ① into ②

$1215 = \frac{405}{r^4}(r)^5$

$1215 = 405r$

$r = 3$

$a = \frac{405}{r^4}$

$a = \frac{405}{3^4}$

$a = 5$

$S_9 = \frac{5\left[(3)^9 - 1\right]}{3-1}$

$S_9 = 49205$

7) Find the sum of each of the following series:

a) 251 + 243 + 235 +...−205

$a=251$

$d=-8$

$t_n=251+(n-1)(-8)$

$-205=251+(n-1)(-8)$

$-456=(n-1)(-8)$

$57=n-1$

$n=58$

$S_{58}=\frac{58}{2}(251-205)$

$=1334$

b) −4 − 12 − 36− ... −8748

$a=-4$

$r=3$

$t_n=-4(3)^{n-1}$

$-8748=-4(3)^{n-1}$

$2187=3^{n-1}$

$\boxed{\frac{\log 2187}{\log 3}=7}$

$3^7=3^{n-1}$

$7=n-1$

$n=8$

$S_8=\frac{-4(3^8-1)}{3-1}$

$S_8=-13\,120$

c) 21 + 23 + 25 + ... + 43

$a=21$

$d=2$

$t_n=21+(n-1)(2)$

$43=21+(n-1)(2)$

$22=(n-1)(2)$

$11=n-1$

$n=12$

$S_{12}=\frac{12}{2}(21+43)$

$S_{12}=384$

d) 1280 − 640 +320 − ... + 5

$a=1280$

$r=-\frac{1}{2}$

$t_n=1280\left(-\frac{1}{2}\right)^{n-1}$

$5=1280\left(-\frac{1}{2}\right)^{n-1}$

$\frac{1}{256}=\left(-\frac{1}{2}\right)^{n-1}$

$\left(-\frac{1}{2}\right)^8=\left(-\frac{1}{2}\right)^{n-1}$

$8=n-1$

$n=9$

$S_9=\frac{1280\left[\left(-\frac{1}{2}\right)^9-1\right]}{-\frac{1}{2}-1}$

$S_9=855$

8) Write the first 4 term of each of the following sequences:

a) $t_1 = -6$; $t_n = t_{n-1} + 5$

$t_1 = -6$

$t_2 = -6 + 5 = -1$

$t_3 = -1 + 5 = 4$

$t_4 = 4 + 5 = 9$

$\boxed{-6, -1, 4, 9}$

b) $t_1 = -2$; $t_2 = -1$; $t_n = t_{n-1} \times t_{n-2}$

$t_1 = -2$

$t_2 = -1$

$t_3 = (-1)(-2) = 2$

$t_4 = 2(-1) = -2$

$\boxed{-2, -1, 2, -2}$

9) Determine the recursive formula of each of these sequences. ~~Are they arithmetic, geometric or neither?~~

a) 1, 1, 2, 3, 5, 8,...

$t_n = t_{n-1} + t_{n-2}$

b) 3, 8, 13, 18, 23, 28, 33, 38

$t_n = t_{n-1} + 5$

10) In an arithmetic sequence, the 3rd term is 25 and the 9th term is 43. How many terms are less than 100?

$t_3 = 25$ $t_9 = 43$

$25 = a + 2d$ $43 = a + 8d$

$43 = a + 8d$
$25 = a + 2d$ $-$
$18 = 6d$
$d = 3$

$43 = a + 8(3)$
$a = 19$

$t_n = 19 + (n-1)(3)$

$100 = 19 + (n-1)(3)$

$81 = (n-1)(3)$

$27 = n-1$

$n = 28$

$\boxed{\therefore 27 \text{ terms are} < 100.}$

11) The sum of the first 6 terms is 297 and the sum of the first 8 terms is 500. Determine the 5^{th} term if the sequence is arithmetic.

$S_6 = 297$

$297 = \frac{6}{2}[2a + (6-1)d]$

$297 = 3(2a + 5d)$

$99 = 2a + 5d$

$S_8 = 500$

$500 = \frac{8}{2}[2a + (8-1)d]$

$500 = 4(2a + 7d)$

$125 = 2a + 7d$

$t_n = 17 + (n-1)(13)$

$t_5 = 17 + (5-1)(13)$

$t_5 = 69$

$99 = 2a + 5d$

$125 = 2a + 7d$ −

$-26 = -2d$

$d = 13$

$125 = 2a + 7(13)$

$34 = 2a$

$a = 17$

12) For $(1-x)^{11}$: use row 11 of Pascal's triangle.

```
            1
           1 1
          1 2 1
         1 3 3 1
        1 4 6 4 1
      1 5 10 10 5 1
            ⋮
1 11 55 165 330 462 462 330 165 55 11 1
```

a) find t_3

$= 55(1)^9(-x)^2$

$= 55x^2$

b) how many terms are in the expansion?

12

c) explain where the numerical coefficients of the expansion are coming from

Row 11 of Pascal's Triangle.

13) Expand $(x^2 - 2y)^4$ using binomial theorem; take the coefficients from Pascal triangle n = 4

$= 1(x^2)^4(-2y)^0 + 4(x^2)^3(-2y)^1 + 6(x^2)^2(-2y)^2 + 4(x^2)^1(-2y)^3 + 1(x^2)^0(-2y)^4$

$= x^8 - 8x^6y + 24x^4y^2 - 32x^2y^3 + 16y^4$

14) Expand $(4x + 2x^3)^3$ using binomial theorem.

1
1 1
1 2 1
1 3 3 1

$= 1(4x)^3(2x^3)^0 + 3(4x)^2(2x^3)^1 + 3(4x)^1(2x^3)^2 + 1(4x)^0(2x^3)^3$

$= 64x^3 + 96x^5 + 48x^7 + 8x^9$

Answers

1) a) $t_n = 9 + (n-1)6\,;\, t_{12} = 75$ **b)** $t_n = -1(-2)^{n-1}\,;\, t_{12} = 2048$

2) a) arithmetic; $t_n = 1 + (n-1)3$ **b)** geometric; $t_n = 2187\left(\frac{1}{3}\right)^{n-1}$

3) i) a) $t_{10} = 28$ **b)** $t_{10} = \frac{1}{9}$ **ii) a)** $S_{12} = 210$ **b)** $S_{12} = \frac{265720}{81}$

4) $S_{50} = 3925$

5) $S_3 = -15$

6) $S_9 = 49205$

7) a) $S_{58} = 1334$ **b)** $S_8 = -13120$ **c)** $S_{12} = 384$ **d)** $S_9 =$ ~~[illegible]~~ 855

8) a) -6, -1, 4, 9 **b)** -2, -1, 2, -2

9) a) $t_n = t_{n-1} + t_{n-2}$ **b)** $t_n = t_{n-1} + 5$

10) 27

11) $t_5 = 69$

12) a) $55x^2$ **b)** 12 **c)** 11th row of Pascal's triangle

13) $x^8 - 8x^6y + 24x^4y^2 - 32x^2y^3 + 16y^4$

14) $64x^3 + 96x^5 + 48x^7 + 8x^9$