L1 - Intro to Transformations – Lesson

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In this chapter you will learn about transformations of functions. There are three main functions that we will use to learn about transformations:

1.

2.

3.

Note: *the equations given for each type of function are considered the base or parent functions of their respective families of functions. All transformations of these functions will be compared to these base functions.*

Before learning about transformations, you must understand what the base functions look like and be able to generate the key points for the graph of each function.

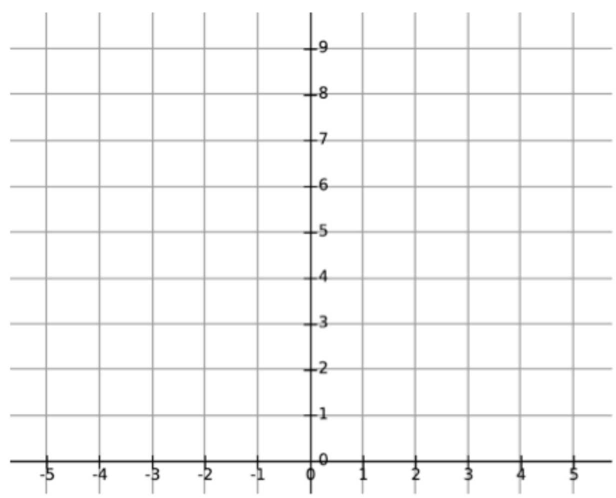
Quadratic Functions

Base Function:

Key Points:

x	y

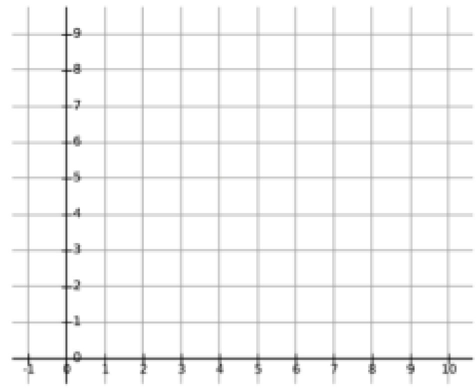
Graph of Base Function:



Radical (square root) Functions

Base Function:

Graph of Base Function:



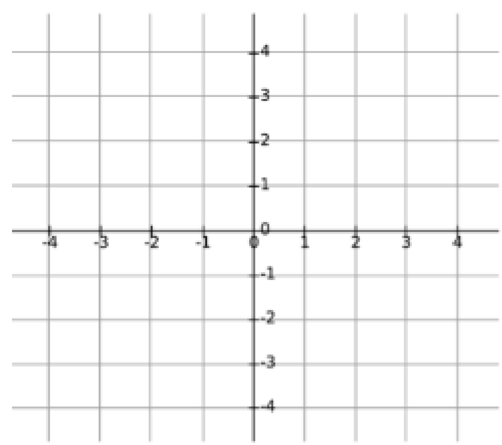
Key Points:

x	y

Rational Functions

Base Function:

Graph of Base Function:



Key Points:

x	y

Asymptotes

Asymptote:

The function $f(x) = \frac{1}{x}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq 0$. This is why the vertical line $x = 0$ is an asymptote for this function.

Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.

Transformations of Functions

Transformation:

The general function:

Changes to the *y*-coordinates (vertical changes)

c: vertical translation $g(x) = f(x) + c$

The graph of $g(x) = f(x) + c$ is a vertical translation of the graph of $f(x)$ by c units.

If $c > 0$, the graph shifts **UP**
If $c < 0$, the graph shifts **DOWN**

a: vertical stretch/compression $\boldsymbol{g(x) = a \cdot f(x)}$

The graph of $g(x) = af(x)$ is a vertical stretch or compression of the graph of $f(x)$ by a factor of a.

If $a > 1$ OR $a < -1$, **vertical stretch** by a factor of a
If -1< $a < 1$, **vertical compression** by a factor of a
If $a < 0$, **vertical reflection** (reflection over the x-axis)

Note: a vertical stretch or compression means that distance from the x-axis of each point of the parent function changes by a factor of a.

Note: for a vertical reflection, the point (x, y) becomes point $(x, -y)$

Changes to the *x*-coordinates (horizontal changes)

d: horizontal translation $g(x) = f(x - d)$

The graph of $g(x) = f(x - d)$ is a horizontal translation of the graph of $f(x)$ by d units.

If $d > 0$, the graph shifts **RIGHT**
If $d < 0$, the graph shifts **LEFT**

k: horizontal stretch/compression
$\boldsymbol{g(x) = f(kx)}$

The graph of $g(x) = f(kx)$ is a horizontal stretch or compression of the graph of $f(x)$ by a factor of $\frac{1}{k}$

If $k > 1$ OR $k < -1$, **horizontal compression** by a factor of $\frac{1}{k}$
If -1< $k < 1$, **horizontal stretch** by a factor of $\frac{1}{k}$
If $k < 0$, **horizontal reflection** (reflection over the y-axis)

Note: a vertical stretch or compression means that distance from the y-axis of each point of the parent function changes by a factor of $\frac{1}{k}$.

Note: for a horizontal reflection, the point (x, y) becomes point $(-x, y)$

Order of Transformations:

1. stretches, compressions, reflections

2. translations

Example 1: List the transformations and the order in which they should be done to a function *f*(*x*).

a) $g(x) = -f(x)$

b) $g(x) = 2f\left(\frac{1}{3}x\right)$

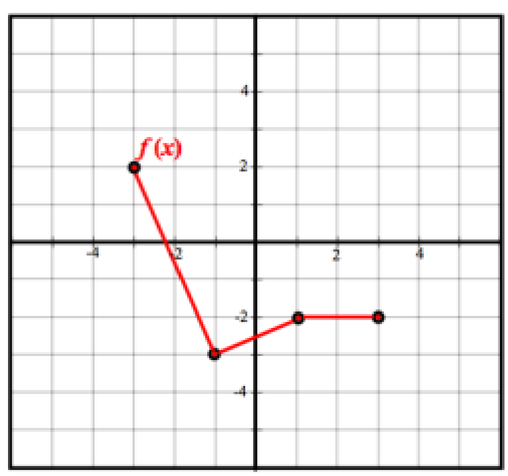
c) $g(x) = 3f(x+2) - 1$

d) $g(x) = \frac{1}{4}f[2(x-1)]$

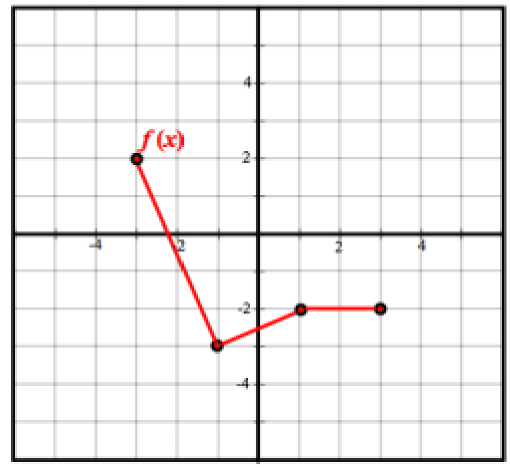
e) $g(x) = -5f\left[-\frac{1}{4}(x+2)\right] + 7$

Example 2: List the transformations and the order in which they should be done to the function *f*(*x*). Use the given graph of *f*(*x*) to sketch the graph of *g*(*x*)

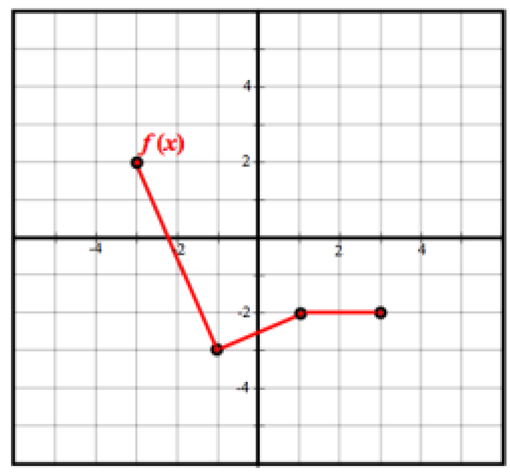
a) $g(x) = f(x+2)$



b) $g(x) = -f(x)$



c) $g(x) = f(x) + 3$



d) $g(x) = f(2x) - 1$