1.1 Functions, Domain, and Range - Lesson
MCR3U
Jensen

Section 1: Relation vs. Function

Definitions

Relation – an identified pattern between two variables that may be represented as a table of values, a graph, or an equation.

Functions – a relation in which each of value of the independent variable (*x*), corresponds to exactly one value of the dependent variable (*y*)

> ***Note:*** *All functions are relations but not all relations are functions. For a relation to be a function, there must be only one 'y' value that corresponds to a given 'x' value.*

Function or Relation Investigation

1) Complete the following tables of values for each relation:

$y = x^2$

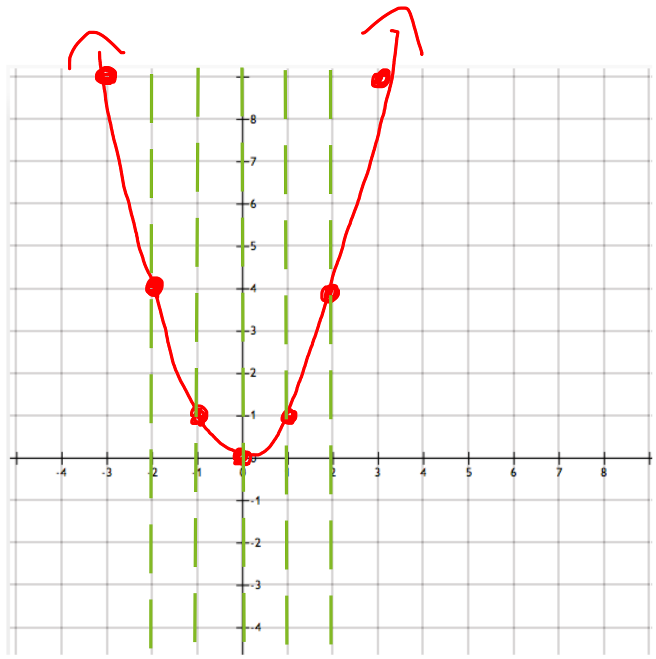
x	*y*
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$x = y^2$

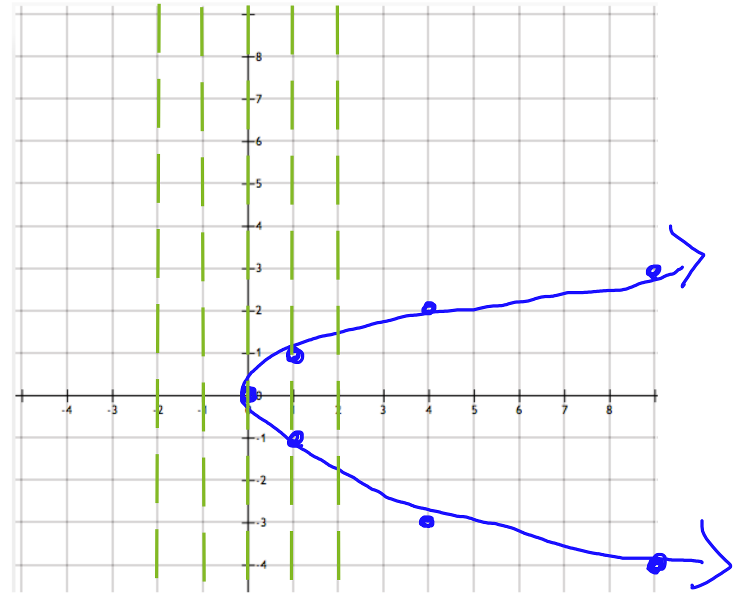
x	*y*
9	-3
4	-2
1	-1
0	0
1	1
4	2
9	3

2) Graph both relations

$y = x^2$



$x = y^2$



3) Draw the vertical lines $x = -2,\ x = -1,\ x = 0,\ x = 1$, and $x = 2$ on the graphs above.

4) Compare how the lines drawn in step 3 intersect each of the relations. Which relation is a function? Explain why.

For $y = x^2$, none of the vertical lines drawn intersect the graph at more than one point. That means that for each value of x, there is only 1 corresponding value of y. This means it is a function.

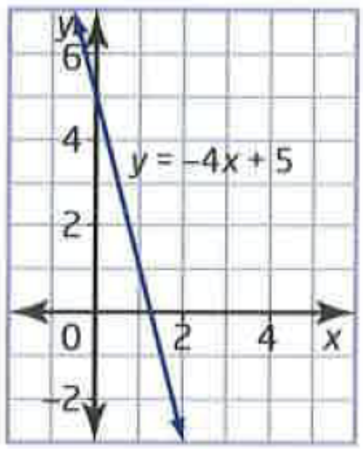
For $x = y^2$, some of the vertical lines drawn intersect the graph at more than one point. That means that some x-values correspond to more than one y-value. This means it is NOT a function.

Section 2: Vertical Line Test

Vertical line test: a method for determining if a relation is a function or not. If every possible vertical line intersects the graph of the relation at only one point, then the relation is a function.

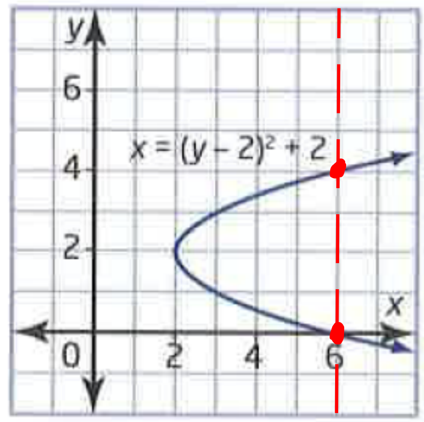
Example 1: Use the vertical line test to determine whether each relation is a function or not.

a)



Function

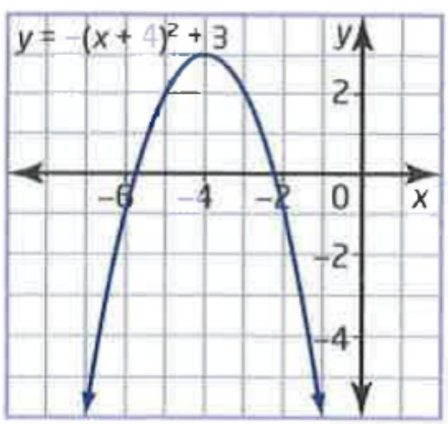
b)



Not a function

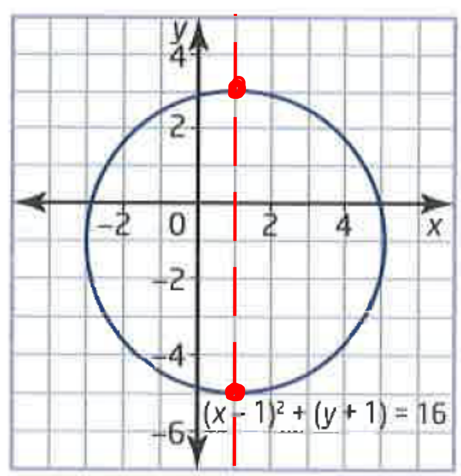
When x = 6, y = 0 and 4

c)



Function

d)



Not a function

When x = 1, y = -5 and 3

Section 3: Domain and Range

For any relation, the set of values of the independent variable (often the *x*-values) is called the domain of the relation. The set of the corresponding values of the dependent variable (often the *y*-values) is called the range of the relation.

> ***Note:*** *For a function, for each given element of the domain there must be exactly one element in the range.*

Domain: values *x* may take

Range: values *y* may take

General Notation

$D: \{X \in \mathbb{R} \mid \text{restrictions}\}$ or $D: \{x = \#, \#, \ldots\}$

$R: \{Y \in \mathbb{R} \mid \text{restrictions}\}$ or $R: \{y = \#, \#, \ldots\}$

Real number: *a number in the set of all integers, terminating decimals, repeating decimals, non-terminating decimals, and non repeating decimals. Represented by the symbol* $\mathbb{R}$

Example 2: Determine the domain and range of each relation from the data given.

a) { (-3, 4), (5, -6), (-2, 7), (5, 3), (6, -8) }

$D: \{x = -3, -2, 5, 6\}$

$R: \{y = -8, -6, 3, 4, 7\}$

b)

Age	*Number*
4	8
5	12
6	5
7	22
8	14
9	9
10	11

D: $\{x = 4, 5, 6, 7, 8, 9, 10\}$

R: $\{y = 5, 8, 9, 11, 12, 14, 22\}$

Are each of these relations functions?

part a) is NOT a function. There are multiple y-values that correspond to an x-value of 5

part b) is a function. Each value for x has exactly one value for y.

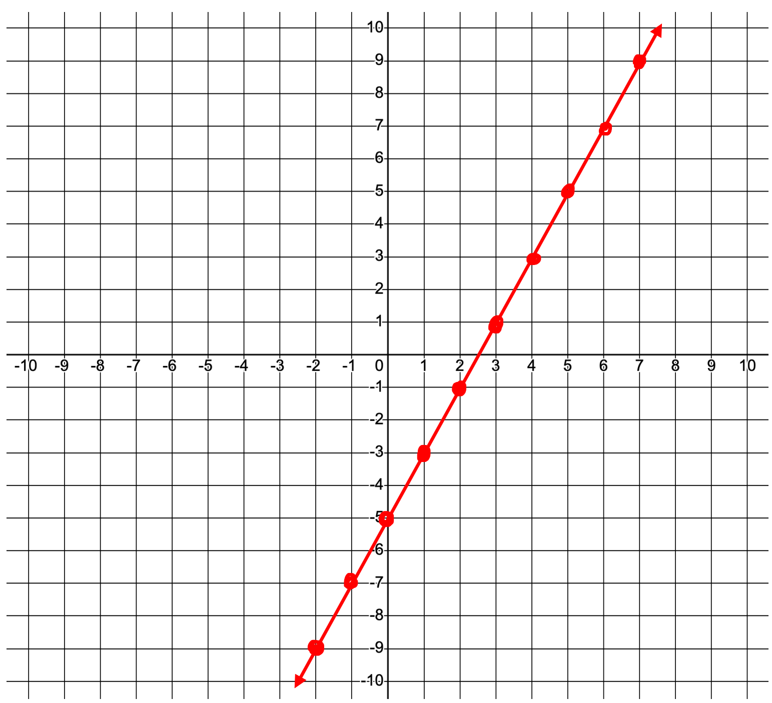
Example 3: Determine the domain and range of each relation. Graph the relation first.

a) $y = 2x - 5$ linear function

slope y-int

D: $\{X \in \mathbb{R}\}$

R: $\{Y \in \mathbb{R}\}$



b) $y = (x-1)^2 + 3$ quadratic function

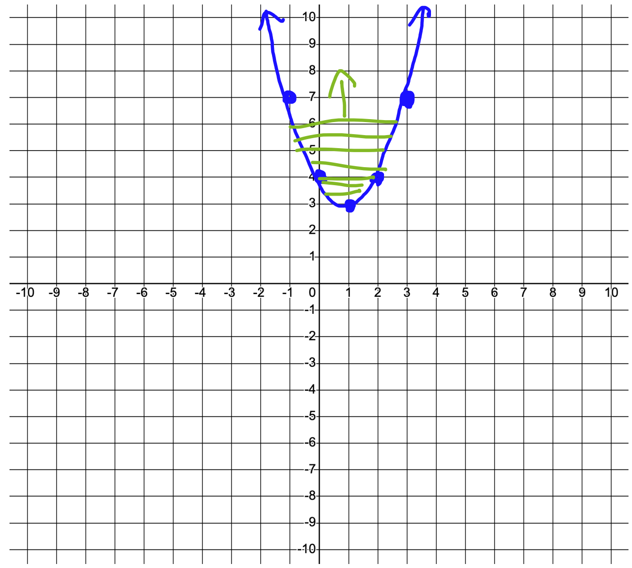
opens up

vertex at (1,3)

x	y
-1	7
0	4
1	3
2	4
3	7

$D: \{x \in \mathbb{R}\}$

$R: \{y \in \mathbb{R} \mid y \geq 3\}$

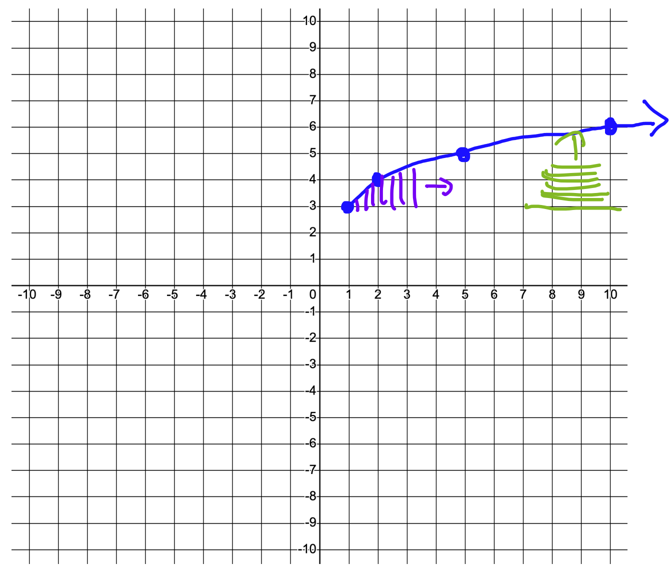


c) $y = \sqrt{x-1} + 3$ radical function

x	y
1	3
2	4
5	5
10	6

$D: \{x \in \mathbb{R} \mid x \geq 1\}$

$R: \{y \in \mathbb{R} \mid y \geq 3\}$

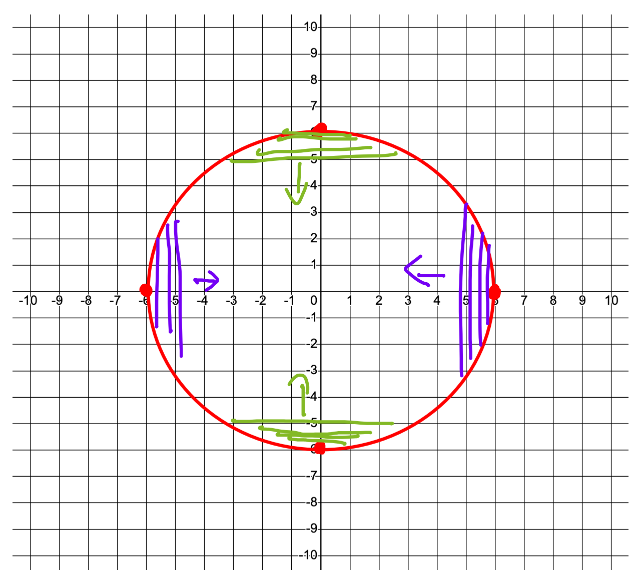


d) $x^2 + y^2 = 36$

circle centered at the origin with a radius of 6.

$D: \{x \in \mathbb{R} \mid -6 \leq x \leq 6\}$

$R: \{y \in \mathbb{R} \mid -6 \leq y \leq 6\}$



e) $y = \frac{1}{x+3}$

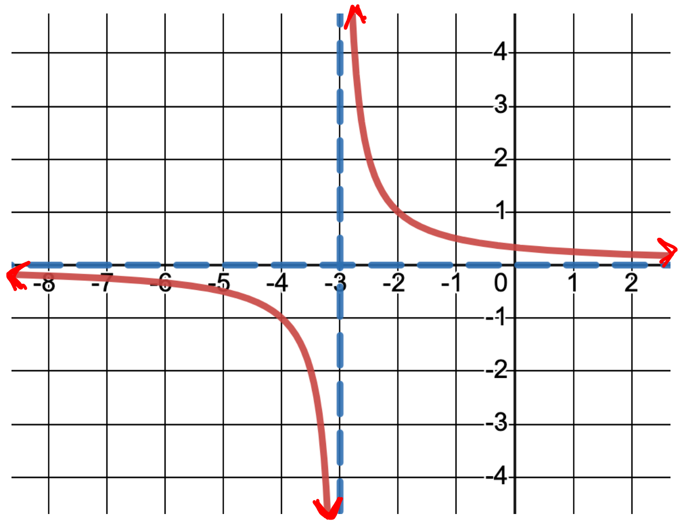
rational function

horizontal asymptote at $y = 0$

vertical asymptote at $x = -3$

D: $\{x \in \mathbb{R} \mid x \neq -3\}$

R: $\{y \in \mathbb{R} \mid y \neq 0\}$



<u>Asymptotes</u>

Asymptote:

The function $\boldsymbol{y = \frac{1}{x+3}}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq -3$. This is why the vertical line $x = -3$ is an asymptote for this function.

Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.