L1 – Trig Review and Special Angles
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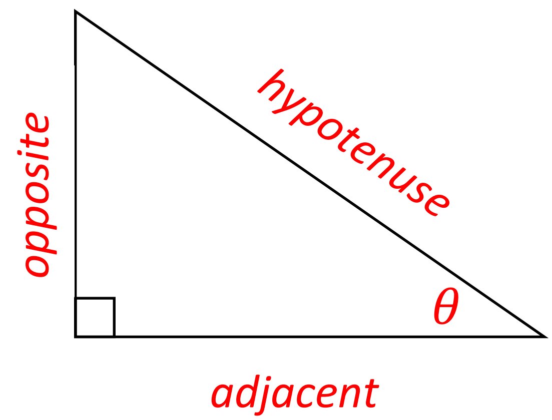
Part 1: Trig Review

Your main takeaway from grade 10 trigonometry should have been:

If we know a right triangle has an angle of θ, all other right triangles with an angle of θ are **SIMILAR** and therefore have **EQUIVALENT** ratios of corresponding sides.

There are three primary trigonometric ratios for right angled triangles. **Sine**, **Cosine**, and **Tangent**.



$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

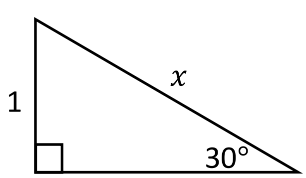
$$\tan\theta = \frac{opposite}{adjacent}$$

Acronym: SOHCAHTOA

$$S\,\frac{O}{H}\quad C\,\frac{A}{H}\quad T\,\frac{O}{A}$$

Example 1: Find the indicated missing side or angle of each triangle

a)

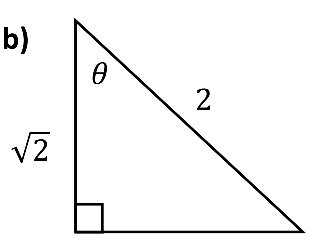


$$\sin 30 = \frac{1}{x}$$

$$x = \frac{1}{\sin 30}$$

$$\boxed{x = 2}$$

b)



$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

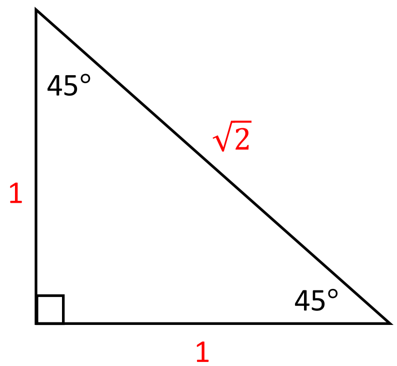
$$\boxed{\theta = 45^\circ}$$

Part 2: Special Angles

There are 2 special triangles:

i) isosceles: 45° - 45° - 90°
ii) half equilateral: 30° - 60° - 90°

i)

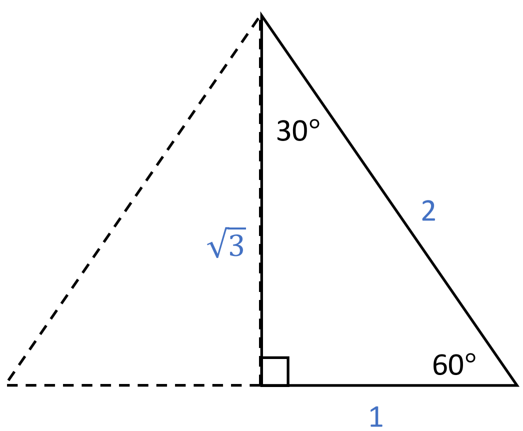


$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\tan 45° = \frac{1}{1} = 1$$

ii)



$$\sin 30° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\sin 60° = \frac{\sqrt{3}}{2}$$

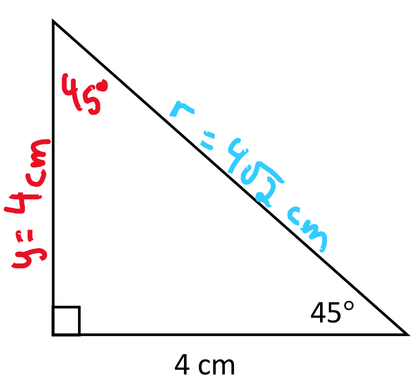
$$\cos 60° = \frac{1}{2}$$

$$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$$

All sized right triangles with these angles are SIMILAR and therefore will have the same ratios of corresponding sides. Therefore, we can use these 2 special triangles to get **EXACT** values for trig ratios involving a 30°, 45°, or 60° reference angle AND we don't need a calculator!

Example 2: Use special triangles to find the EXACT values of all sides and angles

a)



$\tan 45 = \frac{y}{4}$

$1 = \frac{y}{4}$

$4 = y$

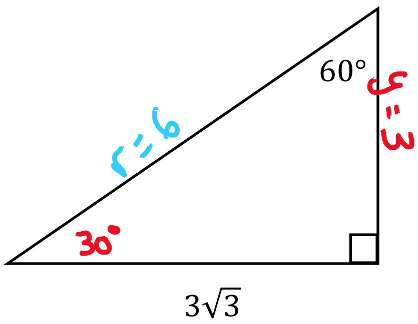
$\cos 45 = \frac{4}{r}$

$\frac{1}{\sqrt{2}} = \frac{4}{r}$

$r = 4\sqrt{2}$

6

b)



$\tan 60 = \frac{3\sqrt{3}}{y}$

$\sqrt{3} = \frac{3\sqrt{3}}{y}$

$y = \frac{3\sqrt{3}}{\sqrt{3}}$

$y = 3$

$\sin 60 = \frac{3\sqrt{3}}{r}$

$\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{r}$

$r\sqrt{3} = 6\sqrt{3}$

$r = \frac{6\sqrt{3}}{\sqrt{3}}$

$r = 6$

Example 3: Determine the exact value of…

a) $(\sin 45°)(\cos 45°) + (\sin 30°)(\sin 60°)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2} + \frac{\sqrt{3}}{4}$

$= \frac{2}{4} + \frac{\sqrt{3}}{4}$

$= \frac{2+\sqrt{3}}{4}$

b) $\frac{\sin^2 30°}{1-\cos 30°}$

$= \frac{(\sin 30)(\sin 30)}{1 - \cos 30}$

$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{1 - \frac{\sqrt{3}}{2}}$

$= \frac{\left(\frac{1}{4}\right)}{\left(\frac{2-\sqrt{3}}{2}\right)}$

$= \frac{1}{4} \times \frac{2}{2-\sqrt{3}}$

$= \frac{1}{2(2-\sqrt{3})}$

$= \frac{1}{4-2\sqrt{3}}$

Part 3: Rationalizing the Denominator

Fractions should be simplified so that the denominator contains only rational numbers.

Example 4: Rationalize the denominator for each of the following expressions

a) $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{\sqrt{4}}$

$= \frac{\sqrt{2}}{2}$

b) $\frac{3}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$

Multiplying by the 'conjugate'

$= \frac{3(1-\sqrt{5})}{(1)^2-(\sqrt{5})^2}$

$= \frac{3-3\sqrt{5}}{1-5}$

$= \frac{3-3\sqrt{5}}{-4}$

$= \frac{-3+3\sqrt{5}}{4}$