## L2 - Trig Ratios for Angles Greater than $90^{\circ}$

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## Part 1: Reference Angles




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| between the initial arm and the terminal arm of an <br> angle in standard position. It's value is between $0^{\circ}$ <br> and $360^{\circ}$. <br> : The acute <br> angle between the terminal arm of an angle in <br> standard position and the closest $x$-axis when the <br> terminal arm lies in quadrant 2, 3, or 4. <br> The reference angle helps us determine the exact <br> trig ratios when we are given obtuse angles. |



Example 1: Find the reference angle for each of the following principal angles
a) $250^{\circ}$
b) $120^{\circ}$
c) $300^{\circ}$


## Part 2: Evaluating Trig Ratios for Any Angle

For any point $P(x, y)$ in the Cartesian plane, the trigonometric ratios for angles in standard position can be expressed in terms of $x, y$, and $r$.

$$
\begin{aligned}
& \sin \theta= \\
& \cos \theta= \\
& \tan \theta=
\end{aligned}
$$

The CAST rule is an easy way to remember which primary trig ratios are positive in which quadrant. Since $r$ is always positive, the sign of each primary ratio depends on the signs of the coordinates of the point $(x, y)$.

In Q1, $\qquad$ ratios are positive because both $x$ and $y$ are positive.

In Q2, only $\qquad$ is positive, since $x$ is negative and $y$ is positive.

In Q3, only $\qquad$ is positive, since both $x$ and $y$ are negative.

In Q4, only $\qquad$ is positive, since $x$ is positive but $y$ is negative.


Example 2: Find the EXACT value of each of the following
a) $\sin 45^{\circ}$

b) $\sin 210^{\circ}$
c) $\cos 240^{\circ}$

d) $\tan 315^{\circ}$



Example 3: Each point lies on the terminal arm of angle $\theta$ in standard position. Determine each of the primary trig ratios for angle $\theta$.
a) $(5,-12)$

b) $(-8,3)$


## Part 3: Unit Circle

The unit circle, a circle with a radius of 1 unit, is very useful since the $x$ and $y$ coordinates of where the terminal intersects it tell us the Cosine and Sine ratios respectively.

http://www.mathsisfun.com/geometry/unit-circle.html

Example 4: Find the EXACT value of each of the following
a) $\sin 270^{\circ}$
b) $\cos 360^{\circ}$
b)


## Part 4: Negative and Co-terminal Angles

Co-terminal angles are angles in standard position that have the $\qquad$ .

Starting at $30^{\circ}$ and rotating $360^{\circ}$ counter clockwise will bring you back to the same terminal arm.

$$
30^{\circ}+360^{\circ}=390^{\circ}
$$

Therefore, $30^{\circ}$ and $390^{\circ}$ are co-terminal.






A negative angle is an angle measured $\qquad$ from the positive $x$-axis.

You can find an equivalent (co-terminal) positive angle by adding $360^{\circ}$ to the negative angle.
$-210^{\circ}$ and $150^{\circ}$ have the same terminal arm (coterminal) and therefore have the same trigonometric ratios.

Example 5: Find three co-terminal angles of $60^{\circ}$

Example 6: Find the EXACT value of each of the following
a) $\sin \left(-45^{\circ}\right)$

b) $\cos \left(-60^{\circ}\right)$


