L2 - 1.2 Functions and Function Notation
MCR3U
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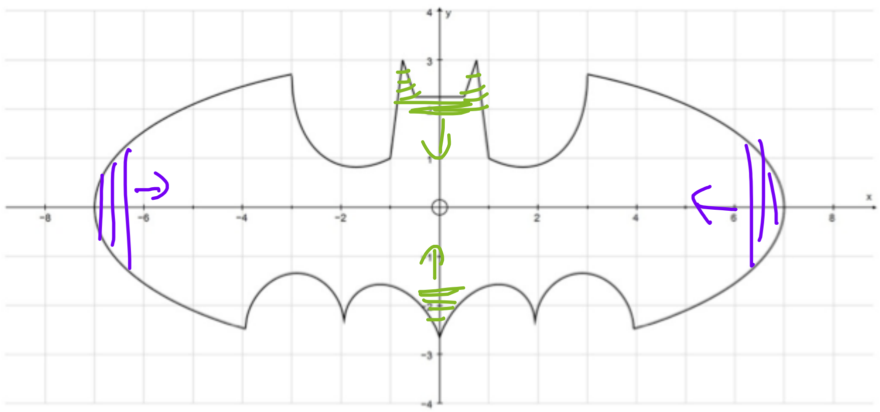
Part 1: Domain & Range Review

a) State the domain and range of the relation shown in the following graph:



D: $\{X \in \mathbb{R} \mid -7 \le x \le 7\}$

R: $\{Y \in \mathbb{R} \mid -2.5 \le y \le 3\}$

b) Is this a function?

No, it does NOT pass the vertical line test.

c) What determines if a relation is a function or not?

For each value of *x*, there can only be one corresponding value of *y*.

d) How does the vertical line test help us determine if a relation is a function?

If any vertical line touches the graph of the relation in more than one spot, it is NOT a function.

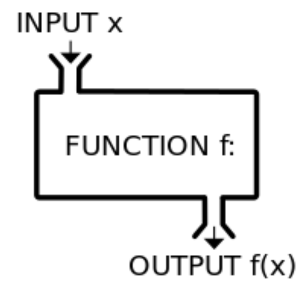
e) What is domain?

The values *x* may take.

f) What is range?

The value *y* may take.

Part 2: Find Values Using Function Notation



What does a function do?

Takes an input (x), performs operations on it and then gives an output (y).

What does function notation look like?

read as 'f of x' or 'f at x'

replaces 'y'

$f(x) =$ some operations applied to x

Example 1: For each of the following functions, determine $f(2)$, $f(-5)$, and $f(1/2)$

a) $f(x) = 2x - 4$

$f(2) = 2(2) - 4$
$f(2) = 0$
$(2, 0)$

$f(-5) = 2(-5) - 4$
$f(-5) = -14$
$(-5, -14)$

$f(\frac{1}{2}) = 2(\frac{1}{2}) - 4$
$f(\frac{1}{2}) = -3$
$(\frac{1}{2}, -3)$

b) $f(x) = 3x^2 - x + 7$

$f(2) = 3(2)^2 - 2 + 7$
$f(2) = 17$
$(2, 17)$

$f(-5) = 3(-5)^2 - (-5) + 7$
$f(-5) = 75 + 5 + 7$
$f(-5) = 87$
$(-5, 87)$

$f(\frac{1}{2}) = 3(\frac{1}{2})^2 - \frac{1}{2} + 7$
$f(\frac{1}{2}) = \frac{3}{4} - \frac{2}{4} + \frac{28}{4}$
$f(\frac{1}{2}) = \frac{29}{4}$
$(\frac{1}{2}, \frac{29}{4})$

c) $f(x) = 87$

$f(2) = 87$
$(2, 87)$

$f(-5) = 87$
$(-5, 87)$

$f(\frac{1}{2}) = 87$
$(\frac{1}{2}, 87)$

d) $f(x) = \frac{2x}{x^2-3}$

$f(2) = \frac{2(2)}{(2)^2-3}$

$= 4$

$(2, 4)$

$f(-5) = \frac{2(-5)}{(-5)^2-3}$

$= \frac{-10}{22}$

$= -\frac{5}{11}$

$\left(-5, -\frac{5}{11}\right)$

$f\left(\frac{1}{2}\right) = \frac{2\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2-3}$

$= \frac{1}{\frac{1}{4}-\frac{12}{4}}$

$= \frac{1}{\left(-\frac{11}{4}\right)}$

$= -\frac{4}{11}$

$\left(\frac{1}{2}, -\frac{4}{11}\right)$

<u>Part 3: Applications of Function Notation</u>

Example 3: For the function $h(t) = \overset{a}{-3}(t + \overset{h}{1})^2 + \overset{k}{5}$

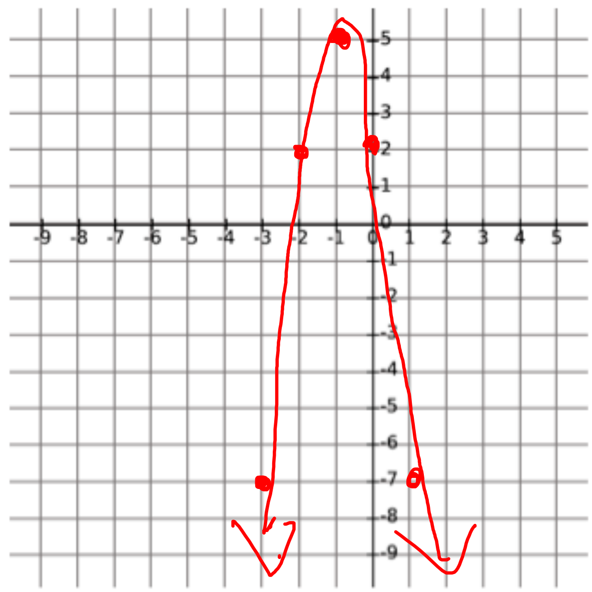
$\boxed{\text{vertex form: } f(x) = a(x-h)^2+k}$

i) Graph it and find the domain and range

opens down $(a<0)$

vertex at $(\overset{h}{-1}, \overset{k}{5})$

t	$h(t)$
-3	-7
-2	2
-1	5
0	2
1	-7



ii) Find $h(-7)$

$h(-7) = -3[(-7)+1]^2 + 5$

$= -3(-6)^2 + 5$

$= -3(36) + 5$

$= -103$

Example 4: The temperature of the water at the surface of a lake is 22 degrees Celsius. As Geno scuba dives to the depths of the lake, he finds that the temperature decreases by 1.5 degrees for every 8 meters he descends.

a) Model the water temperature at any depth using function notation.

Notice it is a constant rate of change making it a linear function of the form y = mx + b

$m = \frac{\Delta T}{\Delta d} = \frac{-1.5}{8} = \frac{-3}{16}$

$T(d) = -\frac{3}{16} d + 22$

$b = 22$

b) What is the water temperature at a depth of 40 meters?

$T(40) = \frac{-3}{16}(40) + 22$

$= 14.5^\circ C$

c) At the bottom of the lake the temperature is 5.5 degrees Celsius. How deep is the lake?

$5.5 = \frac{-3}{16} d + 22$

$(16)\ -16.5 = \frac{-3}{16} d\ (16)$

$-264 = -3d$

$d = 88$ meters deep