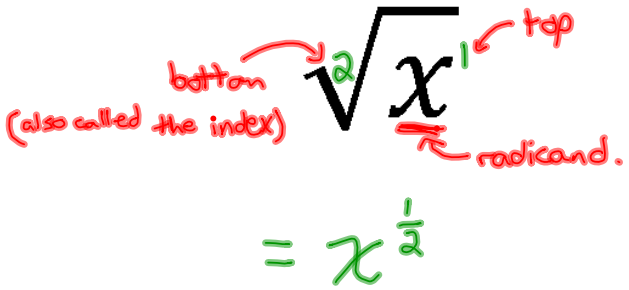
Intro to Rational exponents (Fractions):



$$= x^{\frac{1}{2}}$$

Powers with a rational exponent of the form $\frac{1}{n}$

A power involving a rational exponent with numerator 1 and denominator *n* can be interpreted as the *n*th root of the base:

$$b^{\frac{1}{n}} = \sqrt[n]{b}$$

Powers with a rational exponent of the form $\frac{1}{n}$

Example 1: Evaluate each of the following

a) $8^{\frac{1}{3}}$

$= 2$

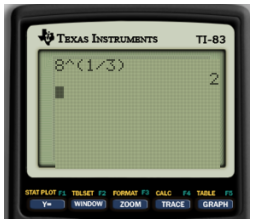
b) $\sqrt[5]{-32}$

$= (-32)^{\frac{1}{5}}$

$= -2$

c) $-16^{\frac{1}{4}}$

$= -2$



d) $\sqrt[4]{\frac{16}{81}}$

$= \left(\frac{16}{81}\right)^{\frac{1}{4}}$

$= \frac{(16)^{\frac{1}{4}}}{(81)^{\frac{1}{4}}}$

$= \frac{2}{3}$

e) $(-27)^{-\frac{1}{3}}$

$= \frac{1}{(-27)^{\frac{1}{3}}}$

$= \frac{1}{-3}$

$= -\frac{1}{3}$

Powers with a rational exponent of the form $\frac{m}{n}$

You can evaluate a power involving a rational exponent with numerator *m* and denominator *n* by taking the *n*th root of the base raised to the exponent *m*:

$$b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

Powers with a rational exponent of the form $\frac{m}{n}$

Example 2: Simplify each of the following powers

a) $\sqrt[5]{y^2}$

$= y^{\frac{2}{5}}$

b) $\sqrt[3]{x}$

$= x^{\frac{1}{3}}$

c) $\sqrt{a^{-3}b^{\frac{4}{3}}}$

$= (a^{-3}b^{4/3})^{1/2}$

$= a^{-3/2}b^{4/6}$

$= \frac{b^{2/3}}{a^{3/2}}$

d) $\sqrt[4]{x^3y^2}$

$= (x^3y^2)^{1/4}$

$= x^{3/4}y^{2/4}$

$= x^{3/4}y^{1/2}$

e) $\frac{\sqrt[3]{x^2y}y^2}{x^3}$

$= \frac{(x^2y)^{1/3}y^2}{x^3}$

$= \frac{x^{2/3}y^{1/3}y^{6/3}}{x^{9/3}}$

$= x^{-7/3}y^{7/3}$

$= \frac{y^{7/3}}{x^{7/3}}$

Example 3: Evaluate each of the following

a) $8^{\frac{2}{3}}$

$= (8^{1/3})^2$

$= (2)^2$

$= 4$

b) $81^{\frac{5}{4}}$

$= (81^{1/4})^5$

$= (3)^5$

$= 243$

c) $\left(\frac{49}{81}\right)^{-\frac{3}{2}}$

$= \left(\frac{81}{49}\right)^{3/2}$

$= \frac{(81)^{3/2}}{(49)^{3/2}}$

$= \frac{729}{343}$

If you have a power with a negative exponent and a rational base, just flip the base and make the exponent positive.

Apply Exponent Rules

Example 4: Simplify and express answer using only positive exponents

a)

$$\frac{\left(x^{\frac{2}{3}}\right)\left(x^{\frac{2}{3}}\right)}{\left(x^{\frac{1}{3}}\right)} = \frac{x^{4/3}}{x^{1/3}}$$

$$= x^{3/3}$$

$$= x$$

b)

$$\left(y^{\frac{1}{4}}\right)^2 \times \left(y^{-\frac{1}{3}}\right)^2$$

$$= y^{1/2} \times y^{-2/3}$$

$$= y^{3/6} \times y^{-4/6}$$

$$= y^{-1/6}$$

$$= \frac{1}{y^{1/6}}$$

c)

$$\left(5x^{\frac{1}{2}}\right)^2 \times 4x^{-\frac{1}{2}}$$

$= 25x \cdot 4x^{-1/2}$

$= 25x^{2/2} \cdot 4x^{-1/2}$

$= 100x^{1/2}$

d)

$$\frac{(m^{-2})^3\sqrt{m^4}}{m\sqrt{pq^{-3}}}$$

$= \dfrac{m^{-6} \cdot m^{4/2}}{m p^{1/2} q^{-3/2}}$

$= \dfrac{m^{-6} \cdot m^{2}}{m p^{1/2} q^{-3/2}}$

$= \dfrac{m^{-4}}{m p^{1/2} q^{-3/2}}$

$= \dfrac{m^{-5}}{p^{1/2} q^{-3/2}}$

$= \dfrac{q^{3/2}}{m^5 p^{1/2}}$

e) $\dfrac{(x^2)^{-4}\sqrt[5]{y^3}}{y\sqrt{x^{-2}y}}$

$= \dfrac{x^{-8}y^{3/5}}{yx^{-1}y^{1/2}}$

$= \dfrac{x^{-7}y^{3/5}}{y^{3/2}}$

$= \dfrac{x^{-7}y^{6/10}}{y^{15/10}}$

$= x^{-7}y^{-9/10}$

$= \dfrac{1}{x^7y^{9/10}}$

Complete Worksheet