L2 – Trig Ratios for Angles Greater than 90°

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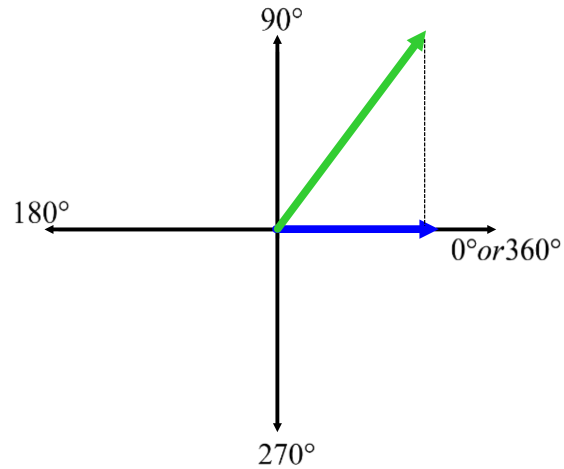
Part 1: Reference Angles

Initial Arm: Always lies on the positive x-axis at 0°. Meets the terminal arm at the origin.

Terminal Arm: The arm that rotates around the origin counter clockwise to form a positive angle or clockwise for a negative angle.

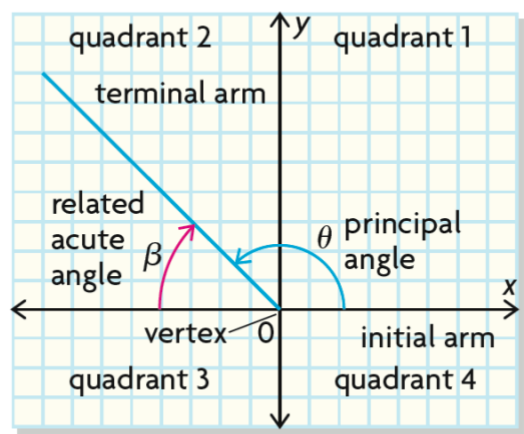
Angle θ is measured from the initial arm to the terminal arm.



Principal Angle: The counter clockwise angle between the initial arm and the terminal arm of an angle in standard position. It's value is between 0° and 360°.

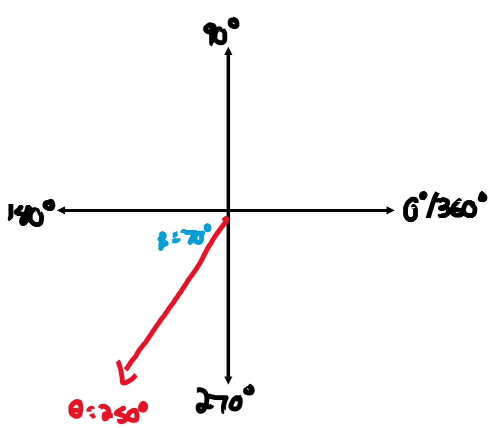
Related Acute Angle (reference angle): The acute angle between the terminal arm of an angle in standard position and the closest x-axis when the terminal arm lies in quadrant 2, 3, or 4.

The reference angle helps us determine the exact trig ratios when we are given obtuse angles.



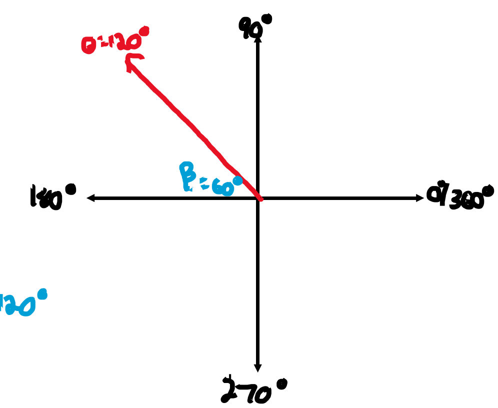
Example 1: Find the reference angle for each of the following principal angles

a) 250°



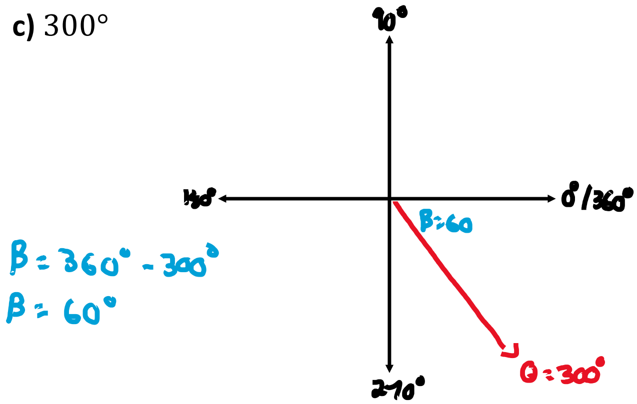
$\beta = 250° - 180°$
$\beta = 70°$

b) 120°

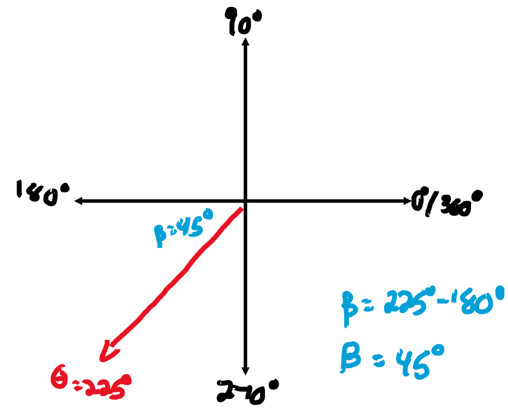


$\beta = 180° - 120°$
$\beta = 60°$

c) 300°



d) 225°



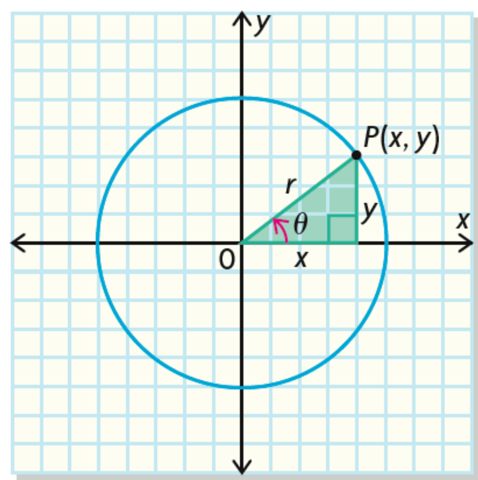
Part 2: Evaluating Trig Ratios for Any Angle

For any point $P(x, y)$ in the Cartesian plane, the trigonometric ratios for angles in standard position can be expressed in terms of $x, y,$ and r.

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



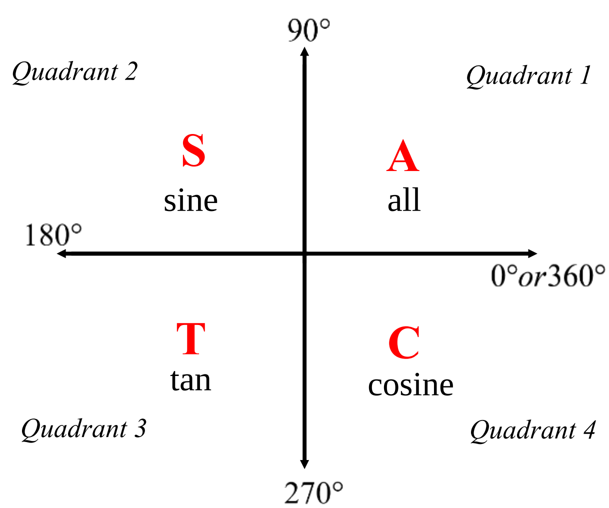
The CAST rule is an easy way to remember which primary trig ratios are positive in which quadrant. Since r is always positive, the sign of each primary ratio depends on the signs of the coordinates of the point (x, y).

In Q1, **All** ratios are positive because both x and y are positive.

In Q2, only **Sine** is positive, since x is negative and y is positive.

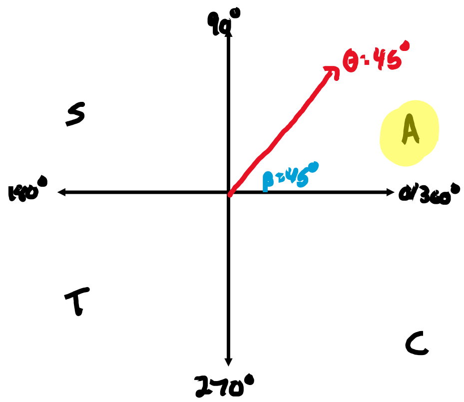
In Q3, only **Tangent** is positive, since both x and y are negative.

In Q4, only **Cosine** is positive, since x is positive but y is negative.



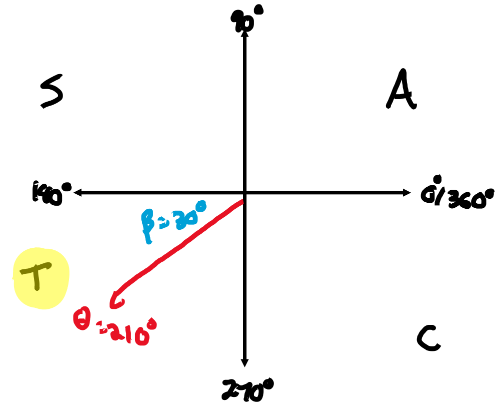
Example 2: Find the EXACT value of each of the following

a) $\sin 45°$



$\sin 45 = \frac{1}{\sqrt{2}}$

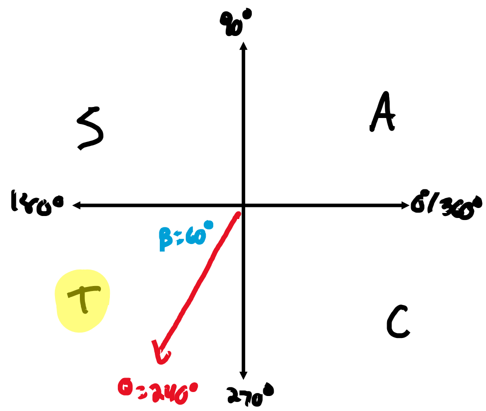
b) $\sin 210°$



$\sin 210 = -\sin 30$

$= -\frac{1}{2}$

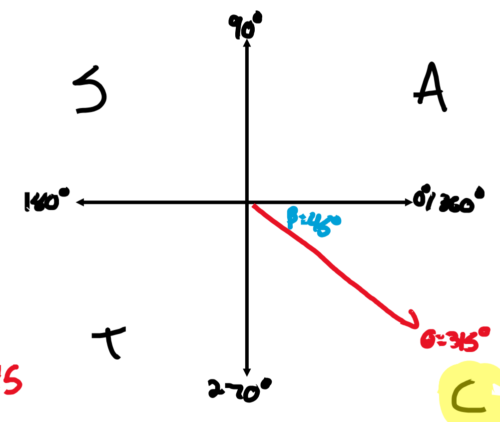
c) $\cos 240°$



$\cos 240 = -\cos 60$

$= -\frac{1}{2}$

d) $\tan 315°$

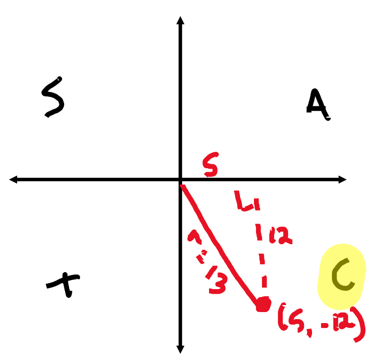


$\tan 315 = -\tan 45$

$= -1$

Example 3: Each point lies on the terminal arm of angle θ in standard position. Determine each of the primary trig ratios for angle θ.

a) (5, -12)



$\sin\theta = -\frac{12}{13}$

$\cos\theta = \frac{5}{13}$

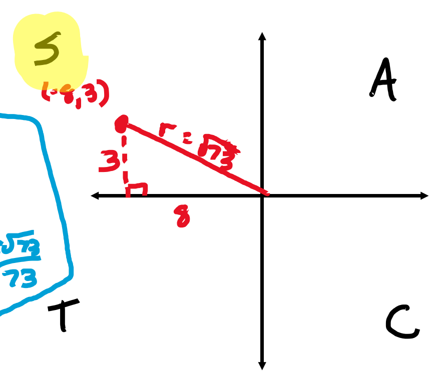
$\tan\theta = \frac{-12}{5}$

$5^2 + 12^2 = r^2$

$169 = r^2$

$r = 13$

b) (-8, 3)



$\sin\theta = \frac{3}{\sqrt{73}} = \frac{3\sqrt{73}}{73}$

$\cos\theta = -\frac{8}{\sqrt{73}} = -\frac{8\sqrt{73}}{73}$

$\tan\theta = -\frac{3}{8}$

$3^2 + 8^2 = r^2$

$73 = r^2$

$r = \sqrt{73}$

Part 3: Unit Circle

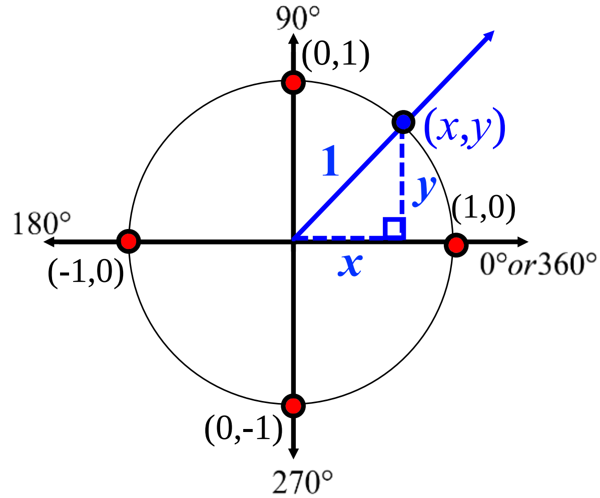
The unit circle, a circle with a radius of **1 unit**, is very useful since the x and y coordinates of where the terminal intersects it tell us the Cosine and Sine ratios respectively.

For any point $P(x, y)$ in the Cartesian plane that intersects the ***unit circle***, the trigonometric ratios for angles can be expressed in terms of x, y, and r.

$$\sin\theta = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{1} = x$$

$$\tan\theta = \frac{y}{x}$$

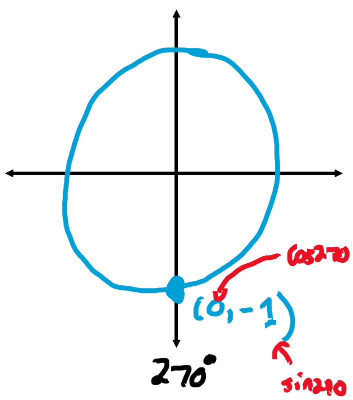


http://www.mathsisfun.com/geometry/unit-circle.html

Example 4: Find the EXACT value of each of the following

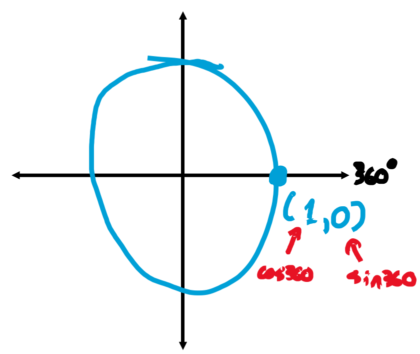
a) $\sin 270°$

sin270 = -1



b) $\cos 360°$

cos360 = 1



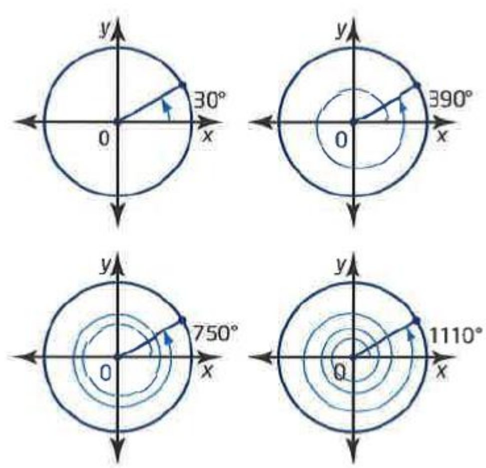
Part 4: Negative and Co-terminal Angles

Co-terminal angles are angles in standard position that have the **same terminal arm**.

Starting at 30° and rotating 360° counter clockwise will bring you back to the same terminal arm.

$$30° + 360° = 390°$$

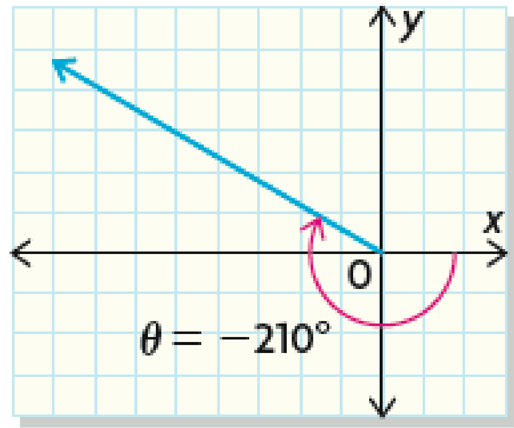
Therefore, 30° and 390° are co-terminal.



A negative angle is an angle measured **clockwise** from the positive x-axis.

You can find an equivalent (co-terminal) positive angle by adding $360°$ to the negative angle.

$-210°$ and $150°$ have the same terminal arm (co-terminal) and therefore have the same trigonometric ratios.



Example 5: Find three co-terminal angles of $60°$

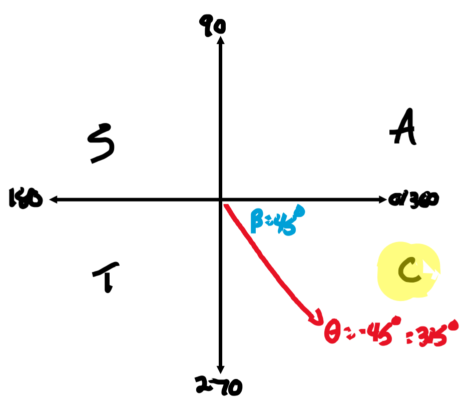
$\theta_1 = 60° + 360°$
$\theta_1 = 420°$

$\theta_2 = 60° + 2(360°)$
$\theta_2 = 780$

$\theta_3 = 60° + 3(360°)$
$\theta_3 = 1140°$

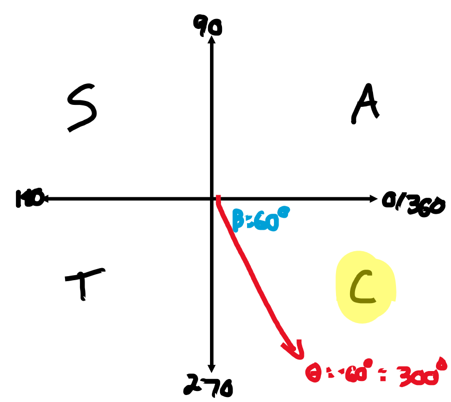
Example 6: Find the EXACT value of each of the following

a) $\sin(-45°)$



$\sin(-45°) = \sin(-45° + 360°)$
$= \sin(315°)$
$= -\sin(45°)$
$= -\frac{1}{\sqrt{2}}$
$= -\frac{\sqrt{2}}{2}$

b) $\cos(-60°)$



$\cos(-60°) = \cos(-60° + 360°)$
$= \cos(300°)$
$= \cos(60)$
$= \frac{1}{2}$