L3 – Solving Trigonometric Equations
MCR3U
Jensen

In this section, you will learn how to identify different angles that have the same trigonometric ratio, as well as learn how they are related.

To do this you will have to visualize the terminal arm rotating around a circle centred at the origin of a grid with a radius of r. This is done so that we can extend our understanding of trig functions for a broader class of angles and see how different angles are related.

http://www.mathsisfun.com/geometry/unit-circle.html

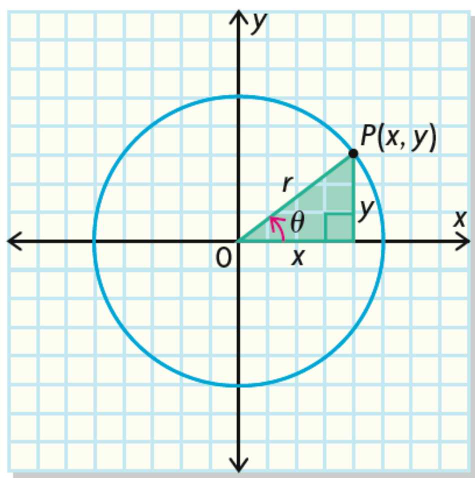
Some helpful reminders:

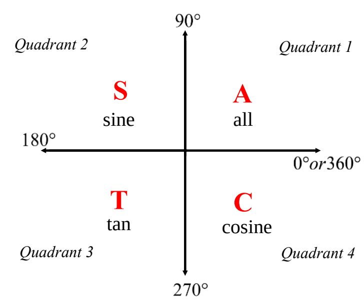
For any point $P(x, y)$ in the Cartesian plane, the trigonometric ratios for angles in standard position can be expressed in terms of x, y, and r.

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



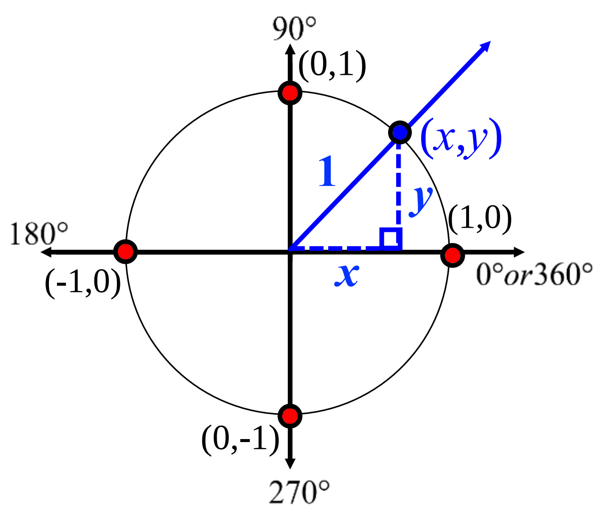


For any point $P(x, y)$ in the Cartesian plane that intersects the ***unit circle***, the trigonometric ratios for angles can be expressed in terms of x, y, and r.

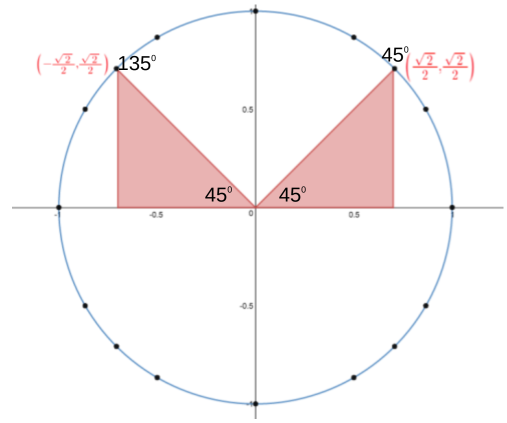
$$\sin\theta = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{1} = x$$

$$\tan\theta = \frac{y}{x}$$



We know the x-coordinate of where the terminal arm Intersects the unit circle is equivalent to the cosine ratio and the y-coordinate is equivalent to the sine ratio.



Notice that both $45°$ and $135°$ have the same reference angle. Since the angles fall in quadrants 1 and 2 respectively, they will have the exact same y-coordinate but the x-coordinates will have the same absolute value but will be opposite signs.

$$\sin 135° = \sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

The important takeaway from this is that there are 2 angles between $0°$ and $360°$ that have the exact same ratio. Using reference angles and the CAST rule, we can make sure to always find both possible angles between $0°$ and $360°$ that have the same trigonometric ratio.

Example 1: Solve the following equations for $0° \leq \theta \leq 360°$. Round answers to the nearest tenth of a degree.

a) $\sin\theta = -0.7$

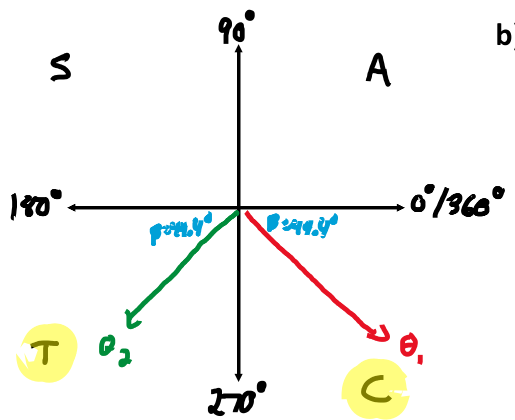
$\theta_1 = \sin^{-1}(-0.7)$

$\theta_1 = -44.4° + 360°$

$\boxed{\theta_1 = 315.6°}$

$\theta_2 = 180° + 44.4°$

$\boxed{\theta_2 \approx 224.4°}$



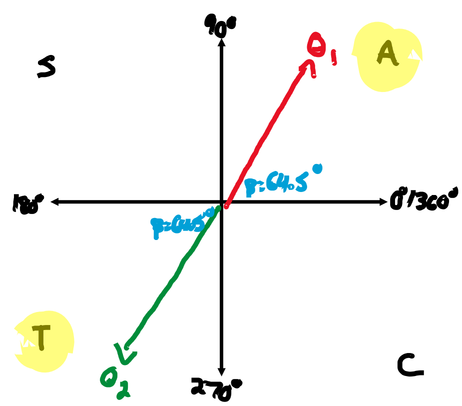
b) $\tan\theta = 2.1$

$\theta_1 = \tan^{-1}(2.1)$

$\boxed{\theta_1 \approx 64.5°}$

$\theta_2 = 180° + 64.5°$

$\boxed{\theta_2 \approx 244.5°}$



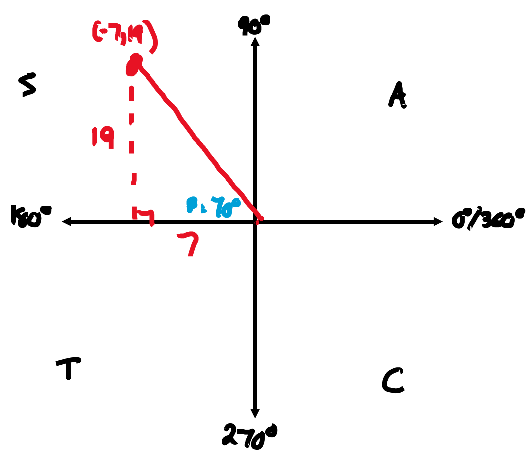
Example 2: The point $(-7, 19)$ lies on the terminal arm. Find the angle to the terminal arm (principal angle, θ) and find the related acute angle (reference angle, β).

$\beta = \tan^{-1}\left(\frac{19}{7}\right)$

$\beta \approx 70°$

$\theta = 180° - 70°$

$\theta \approx 110°$



Example 3: The point $P(5, 11)$ lies on the terminal arm of angle θ in standard position. Draw a sketch of angle θ, determine the exact value of r, determine the primary trig ratios for angle θ, then calculate θ to the nearest tenth of a degree.

$5^2 + 11^2 = r^2$

$r^2 = 146$

$r = \sqrt{146}$

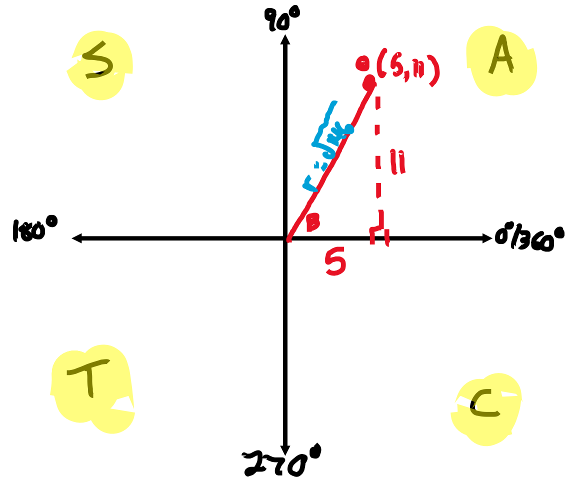
$\sin\theta = \frac{11}{\sqrt{146}}$

$\cos\theta = \frac{5}{\sqrt{146}}$

$\tan\theta = \frac{11}{5}$

$\theta = \tan^{-1}\left(\frac{11}{5}\right)$

$\theta \simeq 65.6°$



Example 4: Solve the following equation for $0° \leq \theta \leq 360°$.

$3\cos\theta + 1 = 0$

$\cos\theta = \frac{-1}{3}$

$\theta_1 = \cos^{-1}\left(\frac{-1}{3}\right)$

$\boxed{\theta_1 \simeq 109.5°}$

$\theta_2 = 180° + 70.5°$

$\boxed{\theta_2 \simeq 250.5°}$