1.4 Working With Radicals
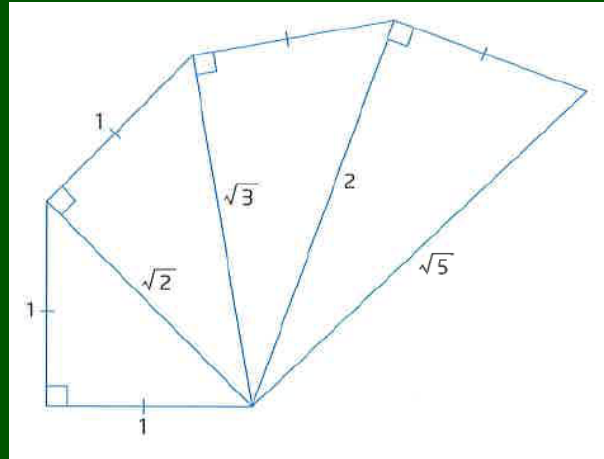

Lesson Outline

Section 1: Investigation

Section 2: Definitions

Section 3: Entire radicals to mixed radicals

Section 4: Add/Subtract radicals

Section 5: Multiply Radicals

Section 6: Application

Investigation

a) Complete the following table:

A	B
$\sqrt{4} \times \sqrt{4} =$ 4	$\sqrt{4 \times 4} =$ 4
$\sqrt{81} \times \sqrt{81} =$ 81	$\sqrt{81 \times 81} =$ 81
$\sqrt{225} \times \sqrt{225} =$ 225	$\sqrt{225 \times 225} =$ 225
$\sqrt{5} \times \sqrt{5} =$ 5	$\sqrt{5 \times 5} =$ 5
$\sqrt{31} \times \sqrt{31} =$ 31	$\sqrt{31 \times 31} =$ 31
$\sqrt{12} \times \sqrt{9} =$ 10.39	$\sqrt{12 \times 9} =$ 10.39
$\sqrt{23} \times \sqrt{121} =$ 52.75	$\sqrt{23 \times 121} =$ 52.75

b) What do you notice about the results in each row?

The results are the same in each row.

c) Make a general conclusion about an equivalent expression for $\sqrt{a} \times \sqrt{b}$

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

Definitions

Radicand:

a number or expression under a radical sign

Entire Radical:

a radical in the form $\sqrt{n}$, where $n > 0$, such as $\sqrt{45}$

Mixed Radical:

a radical in the form $a\sqrt{b}$, where $a \neq 1$ or -1 and $b > 0$, such as $3\sqrt{5}$

More About Radicals

Some numbers cannot be expressed as fractions. These are called ___irrational___ numbers. One type of irrational number is of the form $\sqrt{n}$ where n is not a perfect square. These numbers are sometimes called ___radicals___.

An approximate value can be found for these irrational numbers using a calculator but it is better to work with an exact value. Answers should be left in radical form when an EXACT answer is needed. Sometimes entire radicals can be simplified by removing perfect square factors. The resulting expression is called a ___mixed radical___.

PERFECT SQUARES

1^2	=	1	11^2 =	121
2^2	=	4	12^2 =	144
3^2	=	9	13^2 =	169
4^2	=	16	14^2 =	196
5^2	=	25	15^2 =	225
6^2	=	36	16^2 =	256
7^2	=	49	17^2 =	289
8^2	=	64	18^2 =	324
9^2	=	81	19^2 =	361
10^2	=	100	20^2 =	400

Example 1: Express each radical as a mixed radical in simplest form.

Hint: remove perfect square factors and then simplify

a) $\sqrt{50}$

$= \sqrt{25 \times 2}$

$= (\sqrt{25})(\sqrt{2})$

$= 5\sqrt{2}$

b) $\sqrt{27}$

$= \sqrt{9 \times 3}$

$= (\sqrt{9})(\sqrt{3})$

$= 3\sqrt{3}$

c) $\sqrt{180}$

$= \sqrt{36 \times 5}$

$= (\sqrt{36})(\sqrt{5})$

$= 6\sqrt{5}$

Adding and Subtracting Radicals

Adding and subtracting radicals works in the same way as adding and subtracting polynomials. You can only add ___like___ terms or, in this case, ___like___ ___radicals___.

Example:

$2\sqrt{3} + 5\sqrt{7}$ cannot be added because they do not have the same radical.

However, $3\sqrt{5} + 6\sqrt{5}$ have a common radical, so they can be added. $3\sqrt{5} + 6\sqrt{5} = 9\sqrt{5}$

Note, the radical stays the same when adding and subtracting expressions with like radicals.

Example 2: Simplify the following

a) $9\sqrt{7} - 3\sqrt{7}$

$$= 6\sqrt{7}$$

b) $4\sqrt{3} - 2\sqrt{27}$

$$= 4\sqrt{3} - 2(\sqrt{9})(\sqrt{3})$$
$$= 4\sqrt{3} - 2(3)(\sqrt{3})$$
$$= 4\sqrt{3} - 6\sqrt{3}$$
$$= -2\sqrt{3}$$

c) $5\sqrt{8} + 3\sqrt{18}$

$= 5(\sqrt{4})(\sqrt{2}) + 3(\sqrt{9})(\sqrt{2})$

$= 5(2)(\sqrt{2}) + 3(3)(\sqrt{2})$

$= 10\sqrt{2} + 9\sqrt{2}$

$= 19\sqrt{2}$

d) $\frac{1}{4}\sqrt{28} - \frac{3}{4}\sqrt{63} + \frac{2}{3}\sqrt{50}$

$= \frac{1}{4}(\sqrt{4})(\sqrt{7}) - \frac{3}{4}(\sqrt{9})(\sqrt{7}) + \frac{2}{3}(\sqrt{25})(\sqrt{2})$

$= \frac{2}{4}\sqrt{7} - \frac{9}{4}\sqrt{7} + \frac{10}{3}\sqrt{2}$

$= \frac{-7}{4}\sqrt{7} + \frac{10}{3}\sqrt{2}$

Multiplying Radicals

Example 3: Simplify fully

> Multiply the coefficients together and then multiply the radicands together. Then simplify!

a) $(2\sqrt{3})(3\sqrt{6})$

$= (2)(3)(\sqrt{3})(\sqrt{6})$

$= 6\sqrt{18}$

$= 6(\sqrt{9})(\sqrt{2})$

$= 6(3)(\sqrt{2})$

$= 18\sqrt{2}$

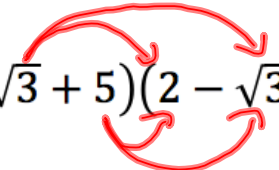**b)** $2\sqrt{3}(4 + 5\sqrt{3})$

$$= 2\sqrt{3}(4) + 2\sqrt{3}(5\sqrt{3})$$

$$= 8\sqrt{3} + 10\sqrt{9}$$

$$= 8\sqrt{3} + 10(3)$$

$$= 8\sqrt{3} + 30$$

Don't forget the distributive property:

$a(x+y) = ax + ay$

c) $-7\sqrt{2}(6\sqrt{8} - 11)$

$$= -7\sqrt{2}(6\sqrt{8}) - 7\sqrt{2}(-11)$$

$$= -42\sqrt{16} + 77\sqrt{2}$$

$$= -168 + 77\sqrt{2}$$

d) $(\sqrt{3} + 5)(2 - \sqrt{3})$

> *Don't forget FOIL. Each term in the first binomial must be multiplied by each term in the second binomial.*

$= \sqrt{3}(2) + \sqrt{3}(-\sqrt{3}) + 5(2) + 5(-\sqrt{3})$

$= 2\sqrt{3} - \sqrt{9} + 10 - 5\sqrt{3}$

$= 2\sqrt{3} - 5\sqrt{3} - 3 + 10$

$= -3\sqrt{3} + 7$

$(a + b) \quad (a - b)$

e) $(2\sqrt{2} + 3\sqrt{3})(2\sqrt{2} - 3\sqrt{3})$

$\quad a \qquad\quad b$

> *There is a shortcut! This is a difference of squares.*
>
> $(a + b)(a - b) = a^2 - b^2$

$= (2\sqrt{2})^2 - (3\sqrt{3})^2$

$= 4(2) - 9(3)$

$= 8 - 27$

$= -19$

Key Concepts

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ for $a \geq 0$ and $b \geq 0$.

- An entire radical can be simplified to a mixed radical in simplest form by removing the largest perfect square from under the radical to form a mixed radical.

  For example, $\sqrt{50} = \sqrt{25 \times 2}$
  $$= 5\sqrt{2}$$

- Like radicals can be combined through addition and subtraction. For example,
  $3\sqrt{7} + 2\sqrt{7} = 5\sqrt{7}$.

- Radicals can be multiplied using the distributive property.

  For example, $4\sqrt{2}(5\sqrt{3} - 3) = 20\sqrt{6} - 12\sqrt{2}$ and
  $(\sqrt{2} - 3)(\sqrt{2} + 1) = \sqrt{4} + \sqrt{2} - 3\sqrt{2} - 3$
  $$= 2 - 2\sqrt{2} - 3$$
  $$= -2\sqrt{2} - 1$$