L4 – Reciprocal Trig Ratios

MCR3U

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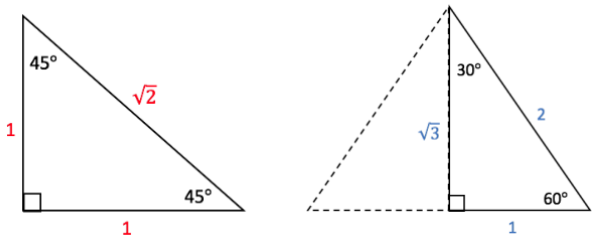
The reciprocal trigonometric ratios are reciprocals of the primary trigonometric ratios, and are defined as 1 divided by each of the primary trigonometric ratios:

Primary Trig Ratios	Reciprocal Trig Ratios
$\sin\theta = \frac{opposite}{hypotenuse}$	$cosecant = \frac{1}{\sin\theta} = \frac{hypotenuse}{opposite}$
$\cos\theta = \frac{adjacent}{hypotenuse}$	$secant = \frac{1}{\cos\theta} = \frac{hypotenuse}{adjacent}$
$\tan\theta = \frac{opposite}{adjacent}$	$cotangent = \frac{1}{\tan} = \frac{adjacent}{opposite}$

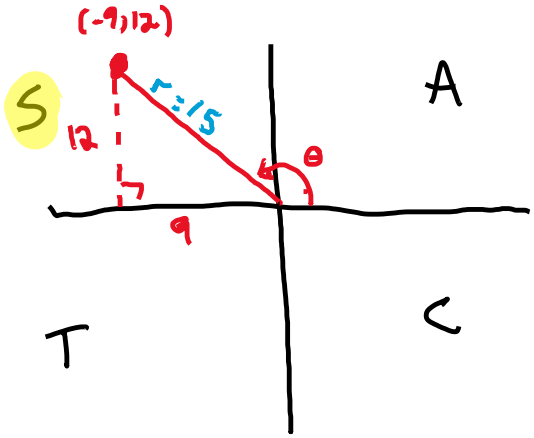
Don't forget your special triangles:



Example 1: Complete the following chart. Give exact values for each ratio.

	$\sin\theta$	$\csc\theta$	$\cos\theta$	$\sec\theta$	$\tan\theta$	$\cot\theta$
0°	0	undefined	1	1	0	undefined
30°	$\frac{1}{2}$	2	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$	1	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$	$\frac{1}{2}$	2	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
90°	1	1	0	undefined	undefined	0

Example 2: The point (-9, 12) lies on the terminal arm of an angle in standard position. Determine exact expressions for the six trigonometric ratios for the angle.



$r^2 = 12^2 + 9^2$

$r^2 = 225$

$r = 15$

$\sin\theta = \frac{12}{15}$ $\quad \csc\theta = \frac{15}{12}$

$\cos\theta = -\frac{9}{15}$ $\quad \sec\theta = -\frac{15}{9}$

$\tan\theta = -\frac{12}{9}$ $\quad \cot\theta = -\frac{9}{12}$

Example 3: Solve the following equations for $0° \leq \theta \leq 90°$

a) $\csc\theta = 8$

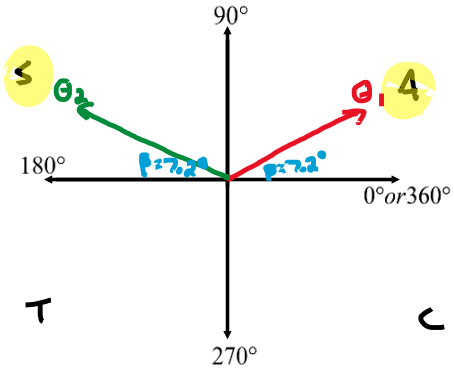
$\sin\theta = \frac{1}{8}$

$\theta_1 = \sin^{-1}\left(\frac{1}{8}\right)$

$\theta_1 \approx 7.2°$

$\theta_2 = 180° - 7.2°$

$\theta_2 \approx 172.8°$



b) $\sec\theta = \frac{5}{2}$

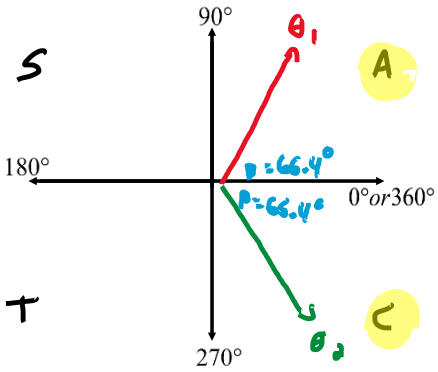
$\cos\theta = \frac{2}{5}$

$\theta_1 = \cos^{-1}\left(\frac{2}{5}\right)$

$\theta_1 \approx 66.4°$

$\theta_2 = 360 - 66.4$

$\theta_2 = 293.6°$



Example 4: Solve the following equation for $0° \leq \theta \leq 360°$.

$\csc\theta + 2 = 0$

$\csc\theta = -2$

$\sin\theta = -\frac{1}{2}$

$\theta_1 = \sin^{-1}\left(-\frac{1}{2}\right)$

$\theta_1 = -30 + 360$

$\theta_1 = 330°$

$\theta_2 = 180° + 30°$

$\theta_2 = 210°$