1.5 Solving Quadratic Equations

Part 1: Solve by Factoring

DO IT NOW!

1. Simplify. $(\sqrt{2}+3\sqrt{3})(5\sqrt{3}-10)$

$= \sqrt{2}(5\sqrt{3}) + \sqrt{2}(-10) + 3\sqrt{3}(5\sqrt{3}) + 3\sqrt{3}(-10)$

$= 5\sqrt{6} - 10\sqrt{2} + 15\sqrt{9} - 30\sqrt{3}$

$= 5\sqrt{6} - 10\sqrt{2} - 30\sqrt{3} + 45$

2. Simplify $\dfrac{2-\sqrt{80}}{4}$

$$= \frac{2-\sqrt{16}(\sqrt{5})}{4}$$

$$= \frac{2-4\sqrt{5}}{4}$$

$$= \frac{2}{4} - \frac{\cancel{4}\sqrt{5}}{\cancel{4}}$$

$$= \frac{1}{2} - \sqrt{5}$$

3. Expand and simplify:

$$4\sqrt{10}\left(3+2\sqrt{2}\right)$$

$$= 4\sqrt{10}(3) + 4\sqrt{10}(2\sqrt{2})$$

$$= 12\sqrt{10} + 8\sqrt{20}$$

$$= 12\sqrt{10} + 8\sqrt{4}(\sqrt{5})$$

$$= 12\sqrt{10} + 16\sqrt{5}$$

Lesson Outline

Section 1: Solve a quadratic with an '*a*' value of 1 or that can be factored out

Section 2: Solve a quadratic with an '*a*' value of not 1 that can't be factored out.

*In all cases we will start with an equation in Standard Form and we will set it equal to 0:

$$ax^2+bx+c = 0$$

NOTE: If it's not in standard form, you must rearrange before factoring.

HOW TO SOLVE QUADRATICS

Solving a quadratic means to find the x-intercepts or roots.

To solve a quadratic equation:

1) It must be set to equal 0. Before factoring, it must be in the form $ax^2+bx+c = 0$
2) Factor the left side of the equation
3) Set each factor to equal zero and solve for '*x*'.

zero product rule: if two factors have a product of zero; one or both of the factors must equal zero.

Example 1: Solve the following quadratics by factoring

a) $y = x^2 - 15x + 56$

P: 56
S: -15
-8 and -7

$y = (x-8)(x-7)$

$0 = (x-8)(x-7)$

$x-8=0$ or $x-7=0$

$x=8$ $x=7$

> When factoring $ax^2+bx+c=0$ when 'a' is 1 or can be factored out
>
> **Steps to follow:**
>
> 1) Check if there is a common factor that can be divided out
> 2) Look at the 'c' value and the 'b' value
> 3) Determine what factors multiply to give 'c' and add to give 'b'
> 4) put those factors into (x+r)(x+s) for 'r' and 's'.
> 5) make sure nothing else can be factored

b) $y = x^2 - 5x + 6$

P: 6
S: -5
-2 and -3

$0 = (x-2)(x-3)$

$0 = (x-2)(x-3)$

$x-2=0$ or $x-3=0$

$x=2$ $x=3$

c) $y = 2x^2 - 8x - 42$

P: -21
S: -4
-7 and 3

$0 = 2(x^2-4x-21)$

$0 = 2(x-7)(x+3)$

$0 = (x-7)(x+3)$

$x-7=0$ or $x+3=0$

$x=7$ $x=-3$

Steps to factoring ax^2+bx+c when 'a' cannot be factored out and is not 1.

1) Look to see if there is a common factor that can be divided out
2) Take the 'a' value and multiply it to the 'c' value
3) Determine what factors of THIS number add together to get the 'b' value
4) Break the 'b' value up into THOSE factors!
5) Put parenthesis around the first two variables and the last two
6) Factor by grouping

Example 2: Solve by factoring

a) $8x^2 + 2x - 15 = 0$

P: -120
S: 2

12 and -10

$8x^2 + 12x - 10x - 15 = 0$

(factor by grouping)

$(8x^2 + 12x) + (-10x - 15) = 0$

(common factor each group)

$4x(2x+3) - 5(2x+3) = 0$

(binomial common factor)

$(2x+3)(4x-5) = 0$

(zero product rule)

$2x + 3 = 0$ or $4x - 5 = 0$

$x = -\frac{3}{2}$ $\quad$ $x = \frac{5}{4}$

b) $2x^2 - 11x = -15$

P: 30 -6 and -5
S: -11

$2x^2 - 11x + 15 = 0$

$2x^2 - 6x - 5x + 15 = 0$

$(2x^2 - 6x) + (-5x + 15) = 0$ (Factor by grouping)

(common Factor each group)

$2x(x-3) - 5(x-3) = 0$ (binomial common Factor)

$(x-3)(2x-5) = 0$ (zero product rule)

$x - 3 = 0$ or $2x - 5 = 0$

$x = 3$ $\quad$ $x = \frac{5}{2}$

Example 3: For the quadratic $y = 2x^2 - 4x - 16$

a) Find the roots of the quadratic by factoring

$0 = 2(x^2 - 2x - 8)$ P: -8 -4 and 2
S: -2

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4^r$ $\quad$ $x = -2^s$

b) Find the axis of symmetry (average of x-intercepts)

a.o.s: $x = \frac{r+s}{2} = \frac{4+(-2)}{2} = \frac{2}{2} = 1$

c) Find the coordinates of the vertex and state if it is a max or min value

$x_{vertex} = 1$

$$
\begin{aligned}
y_{vertex} &= 2x^2 - 4x - 16 \\
&= 2(1)^2 - 4(1) - 16 \\
&= 2 - 4 - 16 \\
&= -18
\end{aligned}
$$

$\therefore$ vertex is $(1, -18)$

This is a minimum value because the parabola opens up $(a > 0)$.