L5 – Problems in 2 and 3 Dimensions

MCR3U

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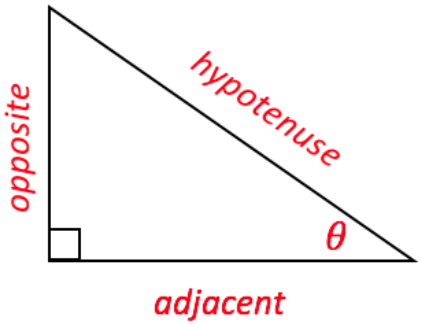
Primary trig ratios are used to solve triangles that contain a right angle.

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

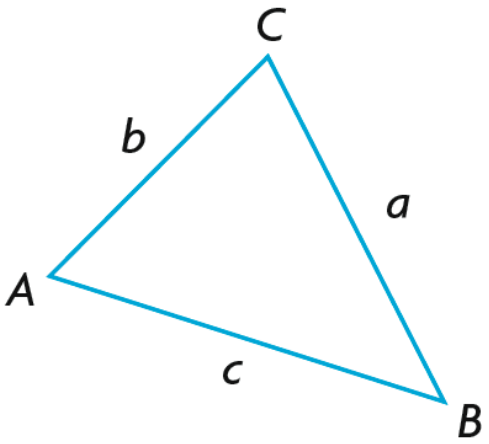


The **Sine Law** and **Cosine Law** are used to solve oblique triangles. An oblique triangle is any triangle that is NOT a right triangle.

Sine Law can be used if you know:

i) 2 sides and one angle opposite a given side
ii) 2 angles and any side

$$\frac{a}{\sin A} = \frac{b}{\sin A} = \frac{c}{\sin C}$$



The Cosine Law can be used if you know:

i) 2 sides and the angle contained by those 2 sides
ii) All 3 sides

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$

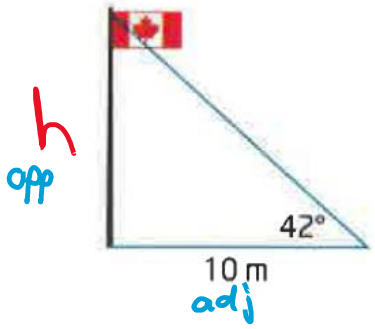
Part 1: Problems in 2 Dimensions

Example 1: Jonathan needs a new rope for his flagpole but is unsure of the length required. He measures a distance of 10m away from the base of the pole. From this point, the angle of elevation to the top of the pole is 42°. What is the height of the pole, to the nearest tenth of a meter?

$\tan(42°) = \frac{h}{10}$

$h = 10\tan(42°)$

$h \simeq 9.0\ m$



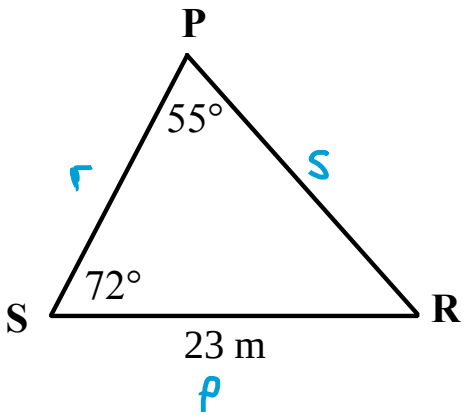
Example 2: Pam, Steven and Rachel are standing on a soccer field. Steven and Rachel are 23m apart. From Steven's point of view, the other two are separated by 72°. From Pam's point of view, the others are separated by an angle of 55°. Determine the distance from Pam to Rachel.

$\frac{p}{\sin P} = \frac{s}{\sin S}$

$\frac{23}{\sin(55)} = \frac{s}{\sin(72)}$

$s = \frac{23\sin(72)}{\sin(55)}$

$s \simeq 26.7m$

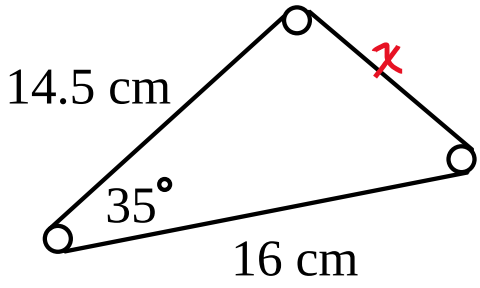


Example 3: A drive belt wraps around three pulleys as shown. Find the perimeter of the drive belt to the nearest tenth of a cm.

$x^2 = 14.5^2 + 16^2 - 2(14.5)(16)\cos(35)$

$x^2 = 86.16345145$

$x \simeq 9.3cm$



Perimeter $= 9.3 + 14.5 + 16$

Perimeter $= 39.8cm$

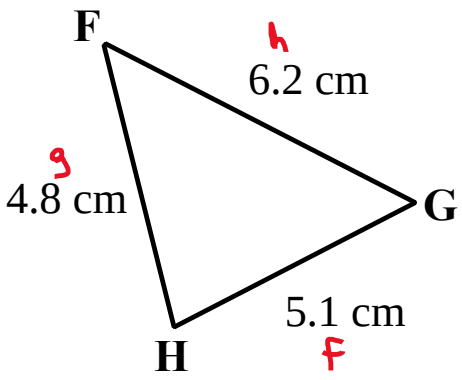
Example 4: Find the measure of angle G

$$\cos G = \frac{4.8^2 - 5.1^2 - 6.2^2}{-2(5.1)(6.2)}$$

$$\cos G = \frac{-41.41}{-63.24}$$

$$\angle G = \cos^{-1}\left(\frac{41.41}{63.24}\right)$$

$$\angle G \simeq 49.1^\circ$$



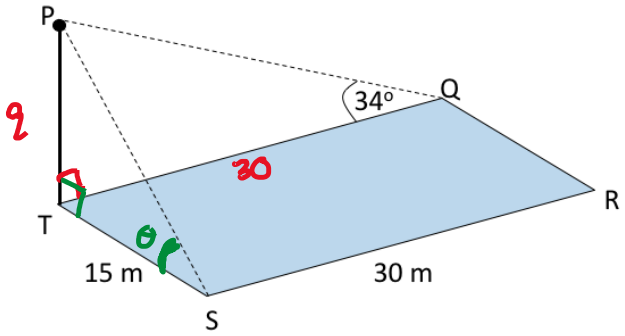
Part 2: Problems in 3 Dimensions

Example 4: A vertical flag pole TP stands in the corner of a rectangular field QRST. Using the information given in the diagram, calculate (a) The height of the flag pole and (b) The angle of elevation of P from S. Round answers to nearest tenth.

a) $\tan(34^\circ) = \frac{q}{30}$

$q = 30\tan(34^\circ)$

$q \simeq 20.2\text{ m}$



b) $\tan\theta = \frac{20.2}{15}$

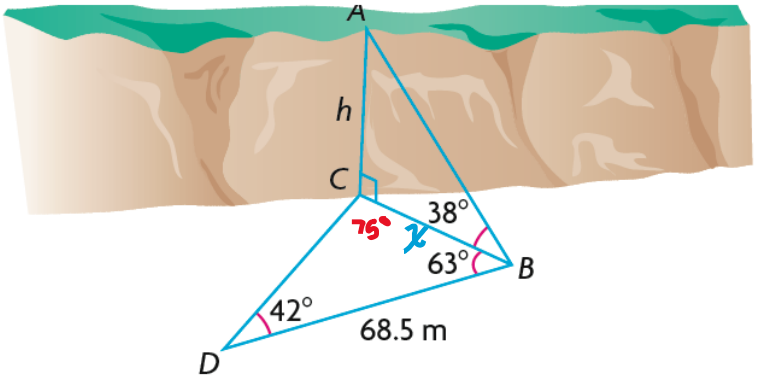
$\theta = \tan^{-1}\left(\frac{20.2}{15}\right)$

$\theta \simeq 53.4^\circ$

Example 5: From point B, Manny estimates the angle of elevation to the top of a cliff as $38°$. From point D, 68.5 meters away from Manny, Joe estimates the angle between the base of the cliff, himself, and Manny to be $42°$, while Manny estimates the angle between the base of the cliff, himself, and his friend Joe to be $63°$. What is the height of the cliff to the nearest tenth of a meter?

$\angle BCD = 180 - 63 - 42$

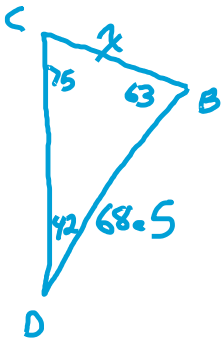
$= 75°$



$$\frac{x}{\sin(42)} = \frac{68.5}{\sin(75)}$$

$$x = \frac{68.5\sin(42)}{\sin(75)}$$

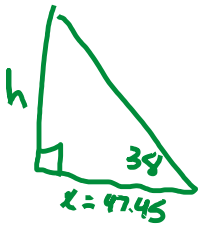
$$x \simeq 47.45$$



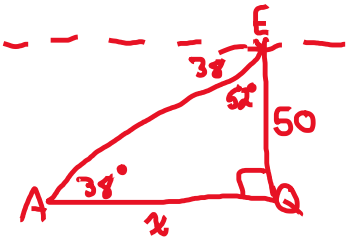
$$\tan(38) = \frac{h}{47.45}$$

$$h = 47.45\tan(38)$$

$$h \simeq 37.1\text{m}$$



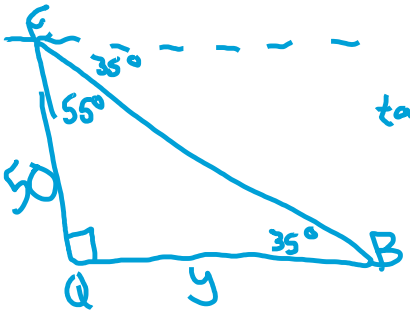
Example 6: Emma is on a 50 meter high bridge and sees two boats anchored below. From her position, boat A has a bearing of $230°$ and boat B has a bearing of $120°$. Emma estimates the angles of depression to be $38°$ for boat A and $35°$ for boat B. How far apart are the boats to the nearest meter?



$$\tan(38°) = \frac{50}{x}$$

$$x = \frac{50}{\tan(38)}$$

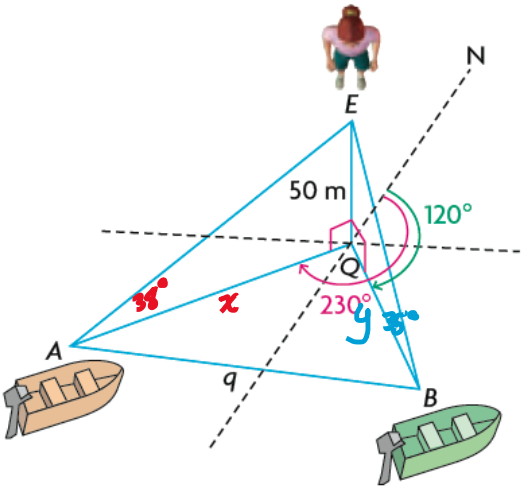
$$x \simeq 64\text{m}$$

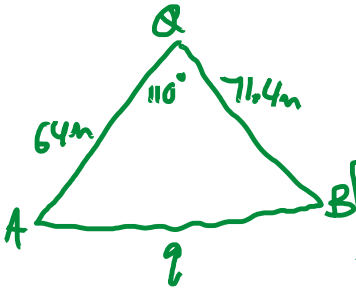


$$\tan(35°) = \frac{50}{y}$$

$$y = \frac{50}{\tan(35°)}$$

$$y \simeq 71.4\text{m}$$





$$q^2 = 64^2 + 71.4^2 - 2(64)(71.4)\cos(110°)$$

$$q^2 = 12319.75049$$

$$q \simeq 111\text{ m}$$